

1. Let  $A$  and  $B$  be matrices of order  $3 \times 3$ .

If  $|A| = \frac{1}{2\sqrt{2}}$  and  $|B| = \frac{1}{729}$ , then what

is the value of  $|2B(\text{adj}(3A))|$ ?

(a) 27

(b)  $\frac{27}{2\sqrt{2}}$

(c)  $\frac{27}{2}$

(d) 1

2. If  $z$  is any complex number and

$iz^3 + z^2 - z + i = 0$ , where  $i = \sqrt{-1}$ , then

what is the value of  $(|z|+1)^2$ ?

(a) 1

(b) 4

(c) 81

(d) 121

3. What is the sum of all four digit numbers formed by using all digits 0, 1, 4, 5 without repetition of digits?

(a) 44440

(b) 46460

(c) 46440

(d) 64440

4. If  $x, y$  and  $z$  are the cube roots of unity, then what is the value of  $xy + yz + zx$ ?

(a) 0

(b) 1

(c) 2

(d) 3

5. A man has 7 relatives (4 women and 3 men). His wife also has 7 relatives (3 women and 4 men). In how many ways can they invite 3 women and 3 men so that 3 of them are man's relatives and 3 of them are his wife's relatives?

(a) 340

(b) 484

(c) 485

(d) 469

6. A triangle  $PQR$  is such that 3 points lie on the side  $PQ$ , 4 points on  $QR$  and 5 points on  $RP$  respectively. Triangles are constructed using these points as vertices. What is the number of triangles so formed?

(a) 205

(b) 206

(c) 215

(d) 220

7. If  $\log_b a = p$ ,  $\log_d c = 2p$  and  $\log_f e = 3p$ ,

then what is  $(ace)^{\frac{1}{p}}$  equal to ?

- (a)  $bd^2f^3$
- (b)  $ddf$
- (c)  $b^3d^2f$
- (d)  $b^2d^2f^2$

8. If  $-\sqrt{2}$  and  $\sqrt{3}$  are roots of the equation  $a_0 + a_1x + a_2x^2 + a_3x^3 + x^4 = 0$  where  $a_0, a_1, a_2, a_3$  are integers, then which one of the following is correct ?

- (a)  $a_2 = a_3 = 0$
- (b)  $a_2 = 0$  and  $a_3 = -5$
- (c)  $a_0 = 6, a_3 = 0$
- (d)  $a_1 = 0$  and  $a_2 = 5$

9. Let  $z_1$  and  $z_2$  be two complex numbers

such that  $\left| \frac{z_1 + z_2}{z_1 - z_2} \right| = 1$ , then what is

$\operatorname{Re}\left(\frac{z_1}{z_2}\right) + 1$  equal to ?

- (a)  $-1$
- (b)  $0$
- (c)  $1$
- (d)  $5$

10. If  $26! = n8^k$ , where  $k$  and  $n$  are positive integers, then what is the maximum value of  $k$  ?

- (a) 6
- (b) 7
- (c) 8
- (d) 9

Handwritten notes for Q10:  $26 \times 25 = \dots$ ,  $26 = 2 \times 13$ ,  $26 = 2 \times 13$ ,  $k = 2$

11. Consider the following statements in respect of two non-singular matrices  $A$  and  $B$  of the same order  $n$  :



1.  $\operatorname{adj}(AB) = (\operatorname{adj}A)(\operatorname{adj}B)$
2.  $\operatorname{adj}(AB) = \operatorname{adj}(BA)$
3.  $(AB)\operatorname{adj}(AB) - |AB|I_n$  is a null matrix of order  $n$

How many of the above statements are correct ?

- (a) None
- (b) Only one statement
- (c) Only two statements
- (d) All three statements

Handwritten notes for Q11:  $27$ ,  $60$ ,  $20$ ,  $30$

12. Consider the following statements in respect of a non-singular matrix  $A$  of order  $n$  :

1.  $A(\text{adj}A^T) = A(\text{adj}A)^T$
2. If  $A^2 = A$ , then  $A$  is identity matrix of order  $n$
3. If  $A^3 = A$ , then  $A$  is identity matrix of order  $n$

Which of the statements given above are correct ?

- (a) 1 and 2 only
- (b) 2 and 3 only
- (c) 1 and 3 only
- (d) 1, 2 and 3

13. How many four-digit natural numbers are there such that all of the digits are even ?

- (a) 625
- (b) 500
- (c) 400
- (d) 256

2160

14. If  $\omega \neq 1$  is a cube root of unity, then what are the solutions of  $(z - 100)^3 + 1000 = 0$  ?

- (a)  $10(1 - \omega), 10(10 - \omega^2), 100$
- (b)  $10(10 - \omega), 10(10 - \omega^2), 90$

$2^3 -$

(c)  $10(1 - \omega), 10(10 - \omega^2), 1000$

(d)  $(1 + \omega), (10 + \omega^2), -1$

15. What is  $(1 + i)^4 + (1 - i)^4$  equal to, where  $i = \sqrt{-1}$  ?

- (a) 4
- (b) 0
- (c) -4
- (d) -8

$(1+i)^2 + (1-i)^2$   
 $(2i + 2i)^2$   
 $(4i)^2$   
 $16i^2$   
 $-16$   
 $(1-i)^2 + (1+i)^2$   
 $(-2i + 2i)^2$   
 $(-4i)^2$   
 $16i^2$   
 $-16$   
 $-16 + -16 = -32$   
 $-32 / 2 = -16$   
 $-16 + 16 = 0$

16. Consider the following statements in respect of a skew-symmetric matrix  $A$  of order 3 :

1. All diagonal elements are zero.
2. The sum of all the diagonal elements of the matrix is zero.
3.  $A$  is orthogonal matrix.

Which of the statements given above are correct ?

- (a) 1 and 2 only
- (b) 2 and 3 only
- (c) 1 and 3 only
- (d) 1, 2 and 3

$0^2 + 2^2 + 5^2$   
 $0 + 4 + 25 = 29$



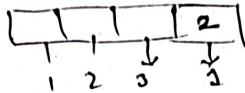
17. Four digit numbers are formed by using the digits 1, 2, 3, 5 without repetition of digits. How many of them are divisible by 4?  $1, 2, 3, 5$  (w.p)

(a) 120

(b) 24

(c) 12

(d) 6



20. If  $ABC$  is a triangle, then what is the value of the determinant

$$\begin{vmatrix} \cos C & \sin B & 0 \\ \tan A & 0 & \sin B \\ 0 & \tan(B+C) & \cos C \end{vmatrix} ?$$

(a) -1

(b) 0

(c) 1

(d) 3

18. What is the remainder when  $2^{120}$  is divided by 7?

(a) 1

(b) 3

(c) 5

(d) 6

$$\frac{2^{100} \cdot 2^{20}}{7}$$

21. What is the number of different matrices, each having 4 entries that can be formed using 1, 2, 3, 4 (repetition is allowed)?

(a) 72

(b) 216

(c) 254

(d) 768

19. For what value of  $n$  is the determinant

$$\begin{vmatrix} C(9,4) & C(9,3) & C(10,n-2) \\ C(11,6) & C(11,5) & C(12,n) \\ C(m,7) & C(m,6) & C(m+1,n+1) \end{vmatrix} = 0$$

for every  $m > n$ ?

(a) 4

(b) 5

(c) 6

(d) 7

22. Let  $A = \{x \in \mathbb{R} : -1 < x < 1\}$ . Which of the following is/are bijective functions from  $A$  to itself?

1.  $f(x) = x|x|$

2.  $g(x) = \cos(\pi x)$

Select the correct answer using the code given below:

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

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$n=6$



$$|x+4| < 2$$

$$x+4 > 0$$

$$x+4 > 2$$

23. Let  $R$  be a relation on the open interval  $(-1, 1)$  and is given by

$R = \{(x, y) : |x+y| < 2\}$ . Then which one of the following is correct ?

- (a)  $R$  is reflexive but neither symmetric nor transitive
- (b)  $R$  is reflexive and symmetric but not transitive
- (c)  $R$  is reflexive and transitive but not symmetric

(d)  $R$  is an equivalence relation.

24. For any three non-empty sets  $A, B, C$ , what is

$(A \cup B) - \{(A - B) \cup (B - A) \cup (A \cap B)\}$  equal to ?

(a) Null set

(b)  $A$

(c)  $B$

(d)  $(A \cup B) - (A \cap B)$



25. If  $a, b, c$  are the sides of triangle  $ABC$ , then what is

$\begin{vmatrix} a^2 & b \sin A & c \sin A \\ b \sin A & 1 & \cos A \\ c \sin A & \cos A & 1 \end{vmatrix}$  equal to ?

(a) Zero

(b) Area of triangle

(c) Perimeter of triangle

(d)  $a^2 + b^2 + c^2$



26. If  $a, b, c$  are in AP;  $b, c, d$  are in GP;  $c, d, e$  are in HP, then which of the following is/are correct ?

1.  $a, c$  and  $e$  are in GP

2.  $\frac{1}{a}, \frac{1}{c}, \frac{1}{e}$  are in GP

Select the correct answer using the code given below :

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

$2a \neq \frac{b}{c}$   
 $c = \frac{a+b}{2} = bc$

$b-a, c-b$  AP  
 $\frac{b}{c} = \frac{c}{d} = \frac{AP}{AP}$

27. What is the number of solutions of  $\log_4(x-1) = \log_2(x-3)$  ?

(a) Zero

(b) One

(c) Two

(d) Three

$\log_2(x-1) = \log_2(x-3)$   
 $x-1 = x-3$

28. For  $x \geq y > 1$ ,

let  $\log_x\left(\frac{x}{y}\right) + \log_y\left(\frac{y}{x}\right) = k$ ,

then the value of  $k$  can never be equal to

(a) -1

(b)  $-\frac{1}{2}$

(c) 0

(d) 1

$\log_x(\log_x x - \log_x y)$   
 $\log_y(\log_y y - \log_y x)$   
 $\log_x x = \log_y y = k$

29. If  $A = \begin{bmatrix} \sin 2\theta & 2\sin^2 \theta - 1 & 0 \\ \cos 2\theta & 2\sin \theta \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ , then

which of the following statements is/are correct?

1.  $A^{-1} = \text{adj}A$
2.  $A$  is skew-symmetric matrix
3.  $A^{-1} = A^T$

Select the correct answer using the code given below :

- (a) 1 only
- (b) 1 and 2
- (c) 1 and 3
- (d) 2 and 3

30. What is the coefficient of  $x^{10}$  in the expansion of  $(1-x^2)^{20} \left(2-x^2-\frac{1}{x^2}\right)^{-5}$ ?

- (a) -1
- (b) 1
- (c) 10
- (d) Coefficient of  $x^{10}$  does not exist

31. If the 4th term in the expansion of  $\left(mx + \frac{1}{x}\right)^n$  is  $\frac{5}{2}$ , then what is the value of  $mn$ ?

- (a) -3
- (b) 3
- (c) 6
- (d) 12

32. If  $a, b$  and  $c$  ( $a > 0, c > 0$ ) are in GP, then consider the following in respect of the equation  $ax^2 + bx + c = 0$ :

1. The equation has imaginary roots.
2. The ratio of the roots of the equation is  $1 : \omega$  where  $\omega$  is a cube root of unity.
3. The product of roots of the equation is  $\left(\frac{b^2}{a^2}\right)$ .

Which of the statements given above are correct?

- (a) 1 and 2 only
- (b) 2 and 3 only
- (c) 1 and 3 only
- (d) 1, 2 and 3

33. If  $x^2 + mx + n$  is an integer for all integral values of  $x$ , then which of the following is/are correct?

1.  $m$  must be an integer
2.  $n$  must be an integer

Select the correct answer using the code given below:

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

34. In a binomial expansion of  $(x+y)^{2n+1}(x-y)^{2n+1}$ , the sum of middle terms is zero. What

is the value of  $\left(\frac{x^2}{y^2}\right)$ ?

- (a) 1
- (b) 2
- (c) 4
- (d) 8

35. Let  $A = \{1, 2, 3, 4, 5\}$  and  $B = \{6, 7\}$ . What is the number of onto functions from  $A$  to  $B$ ?

- (a) 10
- (b) 20

(c) 30

(d) 32

(Ans original)

36. What is  $\frac{\sqrt{3} \cos 10^\circ - \sin 10^\circ}{\sin 25^\circ \cos 25^\circ}$  equal to?

- (a) 1
- (b)  $\sqrt{3}$
- (c) 2
- (d) 4

$\sqrt{3} \cos 10 - \sin 10$   
 $\frac{\sqrt{3} \cos 10 - \sin 10}{\sin 25 \cos 25} = 1$

37. What is  $(\sin 9^\circ - \cos 9^\circ)$  equal to?

- (a)  $-\frac{\sqrt{5}-\sqrt{5}}{2}$
- (b)  $-\frac{\sqrt{5}-\sqrt{3}}{2}$
- (c)  $\frac{\sqrt{5}-\sqrt{5}}{2}$
- (d)  $\frac{\sqrt{5}-\sqrt{5}}{4}$

0.15  
 15  
 9  
 6  
 15  
 9  
 6  
 15  
 9  
 6  
 15  
 9  
 6

38. If in a triangle  $ABC$ ,  $\sin^3 A + \sin^3 B + \sin^3 C = 3 \sin A \sin B \sin C$ , then what is

the value of the determinant  $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ ;

where  $a, b, c$  are sides of the triangle?

- (a)  $a + b + c$
- (b)  $ab + bc + ca$
- (c)  $(a+b)(b+c)(c+a)$
- (d) 0



39. If  $\cos^{-1}x = \sin^{-1}x$ , then which one of the following is correct?

- (a)  $x = 1$
- (b)  $x = \frac{1}{2}$
- (c)  $x = \frac{1}{\sqrt{2}}$
- (d)  $x = \frac{1}{\sqrt{3}}$

$\sin \theta + \cos \theta = 2 \sin \theta \cos \theta$   
 $\sin \theta + \cos \theta = 2 \sin \theta \cos \theta$   
 $\sin \theta + \cos \theta = 2 \sin \theta \cos \theta$   
 $\sin \theta + \cos \theta = 2 \sin \theta \cos \theta$

42. What is  $\sqrt{15 + \cot^2\left(\frac{\pi}{4} - 2 \cot^{-1} 3\right)}$  equal to?

- (a) 1
- (b) 7
- (c) 8
- (d) 16

$625 + 1 - 2$   
 $\sin 60^\circ$

43. What is the value of  $\sin 10^\circ \cdot \sin 50^\circ + \sin 50^\circ \cdot \sin 250^\circ + \sin 250^\circ \cdot \sin 10^\circ$  equal to?

- (a)  $-\frac{1}{4}$
- (b)  $-\frac{3}{4}$
- (c)  $\frac{3 \sin 10^\circ}{4}$
- (d)  $-\frac{3 \cos 10^\circ}{4}$

$\frac{180}{60}$   
 $\frac{180}{60}$   
 $\frac{180}{60}$

40. What is the number of solutions of  $(\sin \theta - \cos \theta)^2 = 2$  where  $-\pi < \theta < \pi$ ?

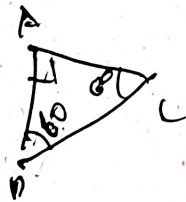
- (a) Only one
- (b) Only two
- (c) Four
- (d) No solution

$1 - \sin 2\theta = 2$   
 $1 - \sin 2\theta = 2$   
 $1 - \sin 2\theta = 2$

41.  $ABC$  is a triangle such that angle  $C = 60^\circ$ , then what is  $\frac{\cos A + \cos B}{\cos\left(\frac{A-B}{2}\right)}$  equal to?

equal to?

- (a) 2
- (b)  $\sqrt{2}$
- (c) 1
- (d)  $\frac{1}{\sqrt{2}}$



44. What is  $\tan^{-1}\left(\frac{a}{b}\right) - \tan^{-1}\left(\frac{a-b}{a+b}\right)$  equal to?

- (a)  $-\frac{\pi}{4}$
- (b)  $\frac{\pi}{4}$
- (c)  $\tan^{-1}\left(\frac{a^2 - b^2}{a^2 + b^2}\right)$
- (d)  $\tan^{-1}\left(\frac{2ab}{a^2 + b^2}\right)$

$\frac{\cos 60 + \cos 30}{\cos 30}$   
 $\frac{\cos 60 + \cos 30}{\cos 30}$   
 $\frac{\cos 60 + \cos 30}{\cos 30}$   
 $\frac{1 + \frac{1}{2}}{\frac{\sqrt{3}}{2}}$

45. Under which one of the following conditions does the equation  $(\cos\beta - 1)x^2 + (\cos\beta)x + \sin\beta = 0$  in  $x$  have a real root for  $\beta \in [0, \pi]$ ?

- (a)  $1 - \cos\beta < 0$
- (b)  $1 - \cos\beta \leq 0$
- (c)  $1 - \cos\beta > 0$
- (d)  $1 - \cos\beta \geq 0$

46. In a triangle  $ABC$ ,  $AB = 16$  cm,  $BC = 63$  cm and  $AC = 65$  cm. What is the value of  $\cos 2A + \cos 2B + \cos 2C$ ?

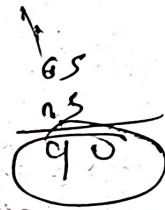
- (a)  $-1$
- (b)  $0$
- (c)  $1$
- (d)  $\frac{76}{65}$

47. If  $f(\theta) = \frac{1}{1 + \tan\theta}$  and  $\alpha + \beta = \frac{5\pi}{4}$ , then what is the value of  $f(\alpha)f(\beta)$ ?

- (a)  $-\frac{1}{2}$
- (b)  $\frac{1}{2}$
- (c)  $1$
- (d)  $2$

48. If  $\tan\alpha$  and  $\tan\beta$  are the roots of the equation  $x^2 - 6x + 8 = 0$ , then what is the value of  $\cos(2\alpha + 2\beta)$ ?

- (a)  $\frac{13}{75}$
- (b)  $\frac{13}{85}$
- (c)  $\frac{17}{85}$
- (d)  $\frac{19}{85}$



49. What is the value of  $\tan 65^\circ + 2\tan 45^\circ - 2\tan 40^\circ - \tan 25^\circ$ ?

- (a)  $0$
- (b)  $1$
- (c)  $2$
- (d)  $4$

50. Consider the following statements:

1. In a triangle  $ABC$ , if  $\cot A \cdot \cot B \cdot \cot C > 0$ , then the triangle is an acute angled triangle.
2. In a triangle  $ABC$ , if  $\tan A \cdot \tan B \cdot \tan C > 0$ , then the triangle is an obtuse angled triangle.

Which of the statements given above is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

Handwritten notes and calculations for question 47:

$$f(\theta) = \frac{1}{1 + \tan\theta}$$

$$f(\alpha)f(\beta) = \frac{1}{(1 + \tan\alpha)(1 + \tan\beta)}$$

$$= \frac{1}{1 + \tan(\alpha + \beta)}$$

$$= \frac{1}{1 + \tan(\frac{5\pi}{4})}$$

$$= \frac{1}{1 + 1} = \frac{1}{2}$$

Handwritten notes for question 49:

$$\cos = \frac{adj}{hyp}$$

$$\sin = \frac{opp}{hyp}$$

Handwritten notes at the bottom right:

$$\frac{1}{1+1} \left( \frac{1}{2} \right) +$$

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51. If  $(a, b)$  is the centre and  $c$  is the radius of the circle  $x^2 + y^2 + 2x + 6y + 1 = 0$ , then what is the value of  $a^2 + b^2 + c^2$ ?

- (a) 19
- (b) 18
- (c) 17
- (d) 11

52. If  $(1, -1, 2)$  and  $(2, 1, -1)$  are the end points of a diameter of a sphere  $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz - 1 = 0$ , then what is  $u + v + w$  equal to?

- (a) -2
- (b) -1
- (c) 1
- (d) 2

53. The number of points represented by the equation  $x = 5$  on the  $xy$ -plane is

- (a) Zero
- (b) One
- (c) Two
- (d) Infinitely many

54. If  $\langle l, m, n \rangle$  are the direction cosines of a normal to the plane  $2x - 3y + 6z + 4 = 0$ , then what is the value of  $49(7l^2 + m^2 - n^2)$ ?

- (a) 0
- (b) 1
- (c) 3
- (d) 71

55. A line through  $(1, -1, 2)$  with direction ratios  $\langle 3, 2, 2 \rangle$  meets the plane  $x + 2y + 3z = 18$ . What is the point of intersection of line and plane?

- (a)  $(4, 4, 1)$
- (b)  $(2, 4, 1)$
- (c)  $(4, 1, 4)$
- (d)  $(3, 4, 7)$

56. If  $p$  is the perpendicular distance from origin to the plane passing through  $(1, 0, 0)$ ,  $(0, 1, 0)$  and  $(0, 0, 1)$ , then what is  $3p^2$  equal to?

- (a) 4
- (b) 3
- (c) 2
- (d) 1



57. If the direction cosines  $\langle l, m, n \rangle$  of a line are connected by relation  $l + 2m + n = 0$ ,  $2l - 2m + 3n = 0$ , then what is the value of  $l^2 + m^2 - n^2$ ?

(a)  $\frac{1}{101}$

(b)  $\frac{29}{101}$

(c)  $\frac{41}{101}$

(d)  $\frac{92}{101}$

58. If a variable line passes through the point of intersection of the lines  $x + 2y - 1 = 0$  and  $2x - y - 1 = 0$  and meets the coordinate axes in  $A$  and  $B$ , then what is the locus of the mid-point of  $AB$ ?

(a)  $3x + y = 10xy$

(b)  $x + 3y = 10xy$

(c)  $3x + y = 10$

(d)  $x + 3y = 10$

59. What is the equation to the straight line passing through the point  $(-\sin\theta, \cos\theta)$  and perpendicular to the line  $x\cos\theta + y\sin\theta = 9$ ?

(a)  $x\sin\theta - y\cos\theta - 1 = 0$

(b)  $x\sin\theta - y\cos\theta + 1 = 0$

(c)  $x\sin\theta - y\cos\theta = 0$

(d)  $x\cos\theta - y\sin\theta + 1 = 0$

60. Two points  $P$  and  $Q$  lie on line  $y = 2x + 3$ . These two points  $P$  and  $Q$  are at a distance 2 units from another point  $R(1, 5)$ . What are the coordinates of the points  $P$  and  $Q$ ?

(a)  $\left(1 + \frac{2}{\sqrt{5}}, 5 + \frac{4}{\sqrt{5}}\right), \left(1 - \frac{2}{\sqrt{5}}, 5 - \frac{4}{\sqrt{5}}\right)$

(b)  $\left(3 + \frac{2}{\sqrt{5}}, 5 + \frac{4}{\sqrt{5}}\right), \left(-1 - \frac{2}{\sqrt{5}}, 5 - \frac{4}{\sqrt{5}}\right)$

(c)  $\left(1 - \frac{2}{\sqrt{5}}, 5 + \frac{4}{\sqrt{5}}\right), \left(1 + \frac{2}{\sqrt{5}}, 5 - \frac{4}{\sqrt{5}}\right)$

(d)  $\left(3 - \frac{2}{\sqrt{5}}, 5 + \frac{4}{\sqrt{5}}\right), \left(-1 + \frac{2}{\sqrt{5}}, 5 - \frac{4}{\sqrt{5}}\right)$

61. If two sides of a square lie on the lines  $2x + y - 3 = 0$  and  $4x + 2y + 5 = 0$ , then what is the area of the square in square units?

(a) 6.05

(b) 6.15

(c) 6.25

(d) 6.35

62.  $ABC$  is a triangle with  $A(3, 5)$ . The mid-points of sides  $AB, AC$  are at  $(-1, 2), (6, 4)$  respectively. What are the coordinates of centroid of the triangle  $ABC$ ?

(a)  $\left(\frac{8}{3}, \frac{11}{3}\right)$

(b)  $\left(\frac{7}{3}, \frac{7}{3}\right)$

(c)  $\left(2, \frac{8}{3}\right)$

(d)  $\left(\frac{8}{3}, 2\right)$

63.  $ABC$  is an acute angled isosceles triangle. Two equal sides  $AB$  and  $AC$  lie on the lines  $7x - y - 3 = 0$  and  $x + y - 5 = 0$ . If  $\theta$  is one of the equal angles, then what is  $\cot\theta$  equal to?

(a)  $\frac{1}{3}$

(b)  $\frac{1}{2}$

(c)  $\frac{2}{3}$

(d) 2

64. In the parabola  $y^2 = 8x$ , the focal distance of a point  $P$  lying on it is 8 units. Which of the following statements is/are correct?

1. The coordinates of  $P$  can be  $(6, 4\sqrt{3})$ .

2. The perpendicular distance of  $P$  from the directrix of parabola is 8 units.

Select the correct answer using the code given below:

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

65. What is the eccentricity of the ellipse if the angle between the straight lines joining the foci to an extremity of the minor axis is  $90^\circ$ ?

(a)  $\frac{1}{3}$

(b)  $\frac{1}{2}$

(c)  $\frac{1}{\sqrt{3}}$

(d)  $\frac{1}{\sqrt{2}}$

$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{pmatrix}$

66. Let  $\vec{a} = \hat{i} - \hat{j} + \hat{k}$  and  $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ . If  $\vec{a} \times (\vec{b} \times \vec{a}) = \alpha\hat{i} - \beta\hat{j} + \gamma\hat{k}$ , then what is the value of  $\alpha + \beta + \gamma$ ?

(a) 8

(b) 7

(c) 6

(d) 1

$\vec{a} \times (\vec{b} \times \vec{a}) = (\vec{a} \cdot \vec{a})\vec{b} - (\vec{a} \cdot \vec{b})\vec{a}$

$\vec{a} \cdot \vec{a} = 1 + 1 + 1 = 3$

$\vec{a} \cdot \vec{b} = 1 + (-2) + (-1) = -2$

$\vec{a} \times (\vec{b} \times \vec{a}) = 3(\hat{i} + 2\hat{j} - \hat{k}) - (-2)(\hat{i} - \hat{j} + \hat{k})$

$= 3\hat{i} + 6\hat{j} - 3\hat{k} + 2\hat{i} - 2\hat{j} + 2\hat{k}$

$= 5\hat{i} + 4\hat{j} - \hat{k}$

$\alpha = 5, \beta = 4, \gamma = -1$

$\alpha + \beta + \gamma = 5 + 4 - 1 = 8$

$\alpha(-1-2) = \beta(0+2)$

$3\alpha = 2\beta$

$\alpha = \frac{2\beta}{3}$

$3\left(\frac{2\beta}{3}\right) + 6\beta - 3\gamma = -2\left(\frac{2\beta}{3}\right) + 2\beta - 2\gamma$

$2\beta + 6\beta - 3\gamma = -\frac{4\beta}{3} + 2\beta - 2\gamma$

$8\beta - 3\gamma = -\frac{4\beta}{3} + 2\beta - 2\gamma$

$8\beta - 3\gamma + \frac{4\beta}{3} - 2\beta + 2\gamma = 0$

$\frac{10\beta}{3} - \gamma = 0$

$\gamma = \frac{10\beta}{3}$

$\alpha + \beta + \gamma = \frac{2\beta}{3} + \beta + \frac{10\beta}{3} = 8\beta$

$\alpha + \beta + \gamma = 8$

67. If a vector of magnitude 2 units makes an angle  $\frac{\pi}{3}$  with  $2\hat{i}$ ,  $\frac{\pi}{4}$  with  $3\hat{j}$  and an acute angle  $\theta$  with  $4\hat{k}$ , then what are the components of the vector?

- (a)  $(1, \sqrt{2}, 1)$
- (b)  $(1, -\sqrt{2}, 1)$
- (c)  $(1, -\sqrt{2}, -1)$
- (d)  $(1, \sqrt{2}, -1)$

68. Consider the following in respect of moment of a force :

1. The moment of force about a point is independent of point of application of force.
2. The moment of a force about a line is a vector quantity.

Which of the statements given above is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

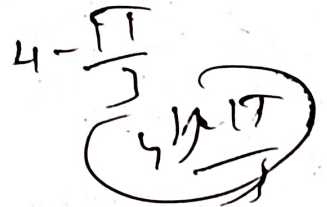
69. For any vector  $\vec{r}$ , what is

\*  $(\vec{r} \cdot \hat{i})(\vec{r} \times \hat{i}) + (\vec{r} \cdot \hat{j})(\vec{r} \times \hat{j}) + (\vec{r} \cdot \hat{k})(\vec{r} \times \hat{k})$  equal to?

- ~~(a) 0~~
- (b)  $\vec{r}$
- (c)  $2\vec{r}$
- (d)  $3\vec{r}$

70. Let  $\vec{a}$  and  $\vec{b}$  are two vectors of magnitude 4 inclined at an angle  $\frac{\pi}{3}$ , then what is the angle between  $\vec{a}$  and  $\vec{a} - \vec{b}$ ?

- (a)  $\frac{\pi}{2}$
- (b)  $\frac{\pi}{3}$
- (c)  $\frac{\pi}{4}$
- (d)  $\frac{\pi}{6}$



71. Let  $y_1(x)$  and  $y_2(x)$  be two solutions of the differential equation  $\frac{dy}{dx} = x$ . If  $y_1(0) = 0$  and  $y_2(0) = 4$ , then what is the number of points of intersection of the curves  $y_1(x)$  and  $y_2(x)$ ?

- (a) No point
- (b) One point
- (c) Two points
- (d) More than two points

72. The differential equation, representing the curve  $y = e^x(a \cos x + b \sin x)$  where  $a$  and  $b$  are arbitrary constants, is

- (a)  $\frac{d^2y}{dx^2} + 2y = 0$
- (b)  $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$
- (c)  $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$
- (d)  $\frac{d^2y}{dx^2} + y = 0$



$$f \circ g(x) = f(g(x)) = a(cx+d) - b = c(ax-b) + d$$

$$acx + ad - b = cax - cb + d$$

73. If  $f(x) = ax - b$  and  $g(x) = cx + d$  are such that  $f(g(x)) = g(f(x))$ , then which one of the following holds?

- (a)  $f(d) = g(b)$   
 (b)  $f(b) + g(d) = 0$   
 (c)  $f(a) + g(c) = 2a$   
 (d)  $f(d) + g(b) = 2d$

$f(x) = ax - b$   
 $g(x) = cx + d$

76. If  $\frac{dy}{dx} = 2e^x y^3$ ,  $y(0) = \frac{1}{2}$  then what is  $4y^2(2 - e^x)$  equal to?

- (a) 1  
 (b) 2  
 (c) 3  
 (d) 4

$$= a \cdot d - b + c \cdot d + d$$

74. What is  $\int_{-1}^1 (3\sin x - \sin 3x) \cos^2 x dx$  equal to?

- (a)  $-\frac{1}{4}$   
 (b) 0  
 (c)  $\frac{1}{2}$   
 (d)  $\frac{1}{4}$

77. Let  $p = \int_a^b f(x) dx$  and  $q = \int_a^b |f(x)| dx$ .

If  $f(x) = e^{-x}$ , then which one of the following is correct?

- (a)  $p = 2q$   
 (b)  $p = -q$   
 (c)  $4p = q$   
 (d)  $p = q$



78. What is  $\int_0^{\frac{\pi}{2}} \frac{a + \sin x}{2a + \sin x + \cos x} dx$  equal to?

- (a)  $\frac{\pi}{4}$   
 (b)  $\frac{\pi}{2}$   
 (c) 1  
 (d) 0

$$\frac{16x^3}{3} - 4bx^2 + x$$

75. What are the order and degree respectively of the differential equation

$$\left\{ 2 - \left( \frac{dy}{dx} \right)^2 \right\}^{0.6} = \frac{d^2 y}{dx^2} ?$$

- (a) 2, 2  
 (b) 2, 3  
 (c) 5, 2  
 (d) 2, 5

79. The non-negative values of  $b$  for which the function  $\frac{16x^3}{3} - 4bx^2 + x$  has neither maximum nor minimum in the range  $x > 0$  is

- (a)  $0 < b < 1$   
 (b)  $1 < b < 2$   
 (c)  $b > 2$   
 (d)  $0 \leq b < 1$

80. Which one of the following is correct

in respect of  $f(x) = \frac{1}{\sqrt{|x|-x}}$  and

$g(x) = \frac{1}{\sqrt{x-|x|}}$  ?

- (a)  $f(x)$  has some domain and  $g(x)$  has no domain
- (b)  $f(x)$  has no domain and  $g(x)$  has some domain
- (c)  $f(x)$  and  $g(x)$  have the same domain
- (d)  $f(x)$  and  $g(x)$  do not have any domain

*Handwritten notes:*  
 $x > 0$   
 $x > 0$   
 $x - |x| > 0$   
 $x > |x|$   
 $(x) - x$   
 $x + |x|$

Consider the following for the next two (02) items that follow :

Given that  $\int \frac{3\cos x + 4\sin x}{2\cos x + 5\sin x} dx =$

$\frac{\alpha x}{29} + \frac{\beta}{29} \ln|2\cos x + 5\sin x| + c$

81. What is the value of  $\alpha$  ?

- (a) 7
- (b) 13
- (c) 17
- (d) 26

82. What is the value of  $\beta$  ?

- (a) 7
- (b) 13
- (c) 17
- (d) 26

Consider the following for the next two (02) items that follow :

Let  $f(x) = \frac{x}{\ln x}$ ; ( $x > 1$ )

83. Consider the following statements :

- 1.  $f(x)$  is increasing in the interval  $(e, \infty)$
- 2.  $f(x)$  is decreasing in the interval  $(1, e)$
- 3.  $9\ln 7 > 7\ln 9$  *not understood*

Which of the statements given above are correct ?

- (a) 1 and 2 only
- (b) 2 and 3 only
- (c) 1 and 3 only
- (d) 1, 2 and 3

*Handwritten note:*  $f'' = e$

84. Consider the following statements :

- 1.  $f''(e) = \frac{1}{e}$
- 2.  $f(x)$  attains local minimum value at  $x = e$
- 3. A local minimum value of  $f(x)$  is  $e$

Which of the statements given above are correct ?

- (a) 1 and 2 only
- (b) 2 and 3 only
- (c) 1 and 3 only
- (d) 1, 2 and 3

Consider the following for the next two (02) items that follow :

Let  $f(x)$  and  $g(x)$  be two functions such that

$$g(x) = x - \frac{1}{x} \text{ and } f \circ g(x) = x^3 - \frac{1}{x^3}.$$

85. What is  $g[f(x) - 3x]$  equal to ?

(a)  $x^3 - \frac{1}{x^3}$

(b)  $x^3 + \frac{1}{x^3}$

(c)  $x^2 - \frac{1}{x^2}$

(d)  $x^2 + \frac{1}{x^2}$

Handwritten work for Q85:  $(x - \frac{1}{x})^3 - \frac{1}{(x - \frac{1}{x})^3}$   
 $= \frac{(x^2 - 1)^3}{x^3} - \frac{1}{(\frac{x^2 - 1}{x})^3}$

86. What is  $f''(x)$  equal to ?

(a)  $\frac{2}{x^3}$

(b)  $2x + \frac{2}{x^3}$

(c)  $6x + 3$

(d)  $6x$

Handwritten work for Q86:  $\frac{x^2 - 1}{x^3}$   
 $\frac{2x}{x^3} - \frac{3}{x^4} = \frac{2x^2 - 3}{x^4}$   
 $\frac{2x^2 - 3}{x^4} = \frac{2x^2}{x^4} - \frac{3}{x^4} = \frac{2}{x^2} - \frac{3}{x^4}$   
 $\frac{d}{dx} (\frac{2}{x^2} - \frac{3}{x^4}) = -\frac{4}{x^3} + \frac{12}{x^5} = \frac{-4x^2 + 12}{x^5}$

Consider the following for the next two (02) items that follow :

Let  $f(x) = |x| + 1$  and  $g(x) = [x] - 1$ , where  $[.]$  is the greatest integer function.

Let  $h(x) = \frac{f(x)}{g(x)}$

87. Consider the following statements :

1.  $f(x)$  is differentiable for all  $x < 0$
2.  $g(x)$  is continuous at  $x = 0.0001$
3. The derivative of  $g(x)$  at  $x = 2.5$  is 1

Which of the statements given above are correct ?

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

88. What is  $\lim_{x \rightarrow 0^-} h(x) + \lim_{x \rightarrow 0^+} h(x)$  equal to ?

(a)  $-\frac{3}{2}$

(b)  $-\frac{1}{2}$

(c)  $\frac{1}{2}$

(d)  $\frac{3}{2}$

Handwritten work for Q88:  $h(x) = \frac{|x| + 1}{[x] - 1}$   
 $\lim_{x \rightarrow 0^-} h(x) = \frac{0 + 1}{-1} = -1$   
 $\lim_{x \rightarrow 0^+} h(x) = \frac{0 + 1}{0 - 1} = -1$   
 $-1 + (-1) = -2$

Consider the following for the next two (02) items that follow :

Let  $\varphi(a) = \int_a^{a+100\pi} |\sin x| dx$

89. What is  $\varphi(a)$  equal to ?

(a) 0

(b)  $a$

(c)  $100a$

(d) 200

90. What is  $\varphi'(a)$  equal to ?

(a) 0

(b)  $\pi$

(c) 100

(d) 200



Consider the following for the next two (02) items that follow :

A differentiable function  $f(x)$  has a local maximum at  $x = 0$ . Let  $y = 2f(x) + ax - b$ .

91. Which of the following is/are correct ?

1.  $f'(0) = 0$
2.  $f''(0) < 0$

Select the correct answer using the code given below :

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

92. The function  $y$  has a relative maxima at  $x = 0$  for

- (a)  $a > 0, b = 0$
- (b) for all  $b$  and  $a = 0$
- (c) for all  $b > 0$  only
- (d) for all  $a$  and  $b = 0$

Consider the following for the next two (02) items that follow :

Let  $f(x) = |x - 1|$ ,  $g(x) = [x]$  and  $h(x) = f(x)g(x)$  where  $[.]$  is greatest integer function.

93. What is  $\int_{-1}^0 h(x)dx$  equal to ?

- (a)  $-\frac{3}{2}$
- (b)  $-1$

(c) 0

(d)  $\frac{1}{2}$

94. What is  $\int_0^2 h(x)dx$  equal to ?

(a)  $-\frac{3}{2}$

(b)  $-1$

(c) 0

(d)  $\frac{1}{2}$

Consider the following for the next two (02) items that follow :

Let  $\int \frac{dx}{\sqrt{x+1} - \sqrt{x-1}} = \alpha(x+1)^{\frac{3}{2}} + \beta(x-1)^{\frac{3}{2}} + c$

95. What is the value of  $\alpha$  ?

(a)  $\frac{1}{3}$

(b)  $\frac{2}{3}$

(c) 1

(d)  $\frac{4}{3}$

96. What is the value of  $\beta$  ?

(a)  $-\frac{2}{3}$

(b)  $-\frac{1}{3}$

(c)  $\frac{1}{3}$

(d)  $\frac{2}{3}$

Consider the following for the next two (02) items that follow :

The circle  $x^2 + y^2 - 2x = 0$  is partitioned by line  $y = x$  in two segments. Let  $A_1, A_2$  be the areas of major and minor segments respectively.

(c)  $\frac{3}{8x} + \frac{x}{8} + \frac{1}{4}$

(d)  $\frac{3}{8x} - \frac{x}{8} + \frac{1}{4}$

97. What is the value of  $A_1$  ?

(a)  $\frac{\pi - 2}{4}$

(b)  $\frac{\pi + 2}{4}$

(c)  $\frac{3\pi - 2}{4}$

(d)  $\frac{3\pi + 2}{4}$

100. What is  $8 \int_1^2 f(x) dx$  equal to ?

(a)  $\ln(8\sqrt{e})$

(b)  $\ln(4\sqrt{e})$

(c)  $\ln 2$

(d)  $\ln 2 - 1$

98. What is the value of  $\frac{2(A_1 + A_2)}{A_1 - 3A_2}$  ?

(a)  $\pi$

(b) 1

(c) -1

(d)  $-\pi$

101. A bag contains 5 black and 4 white balls. A man selects two balls at random. What is the probability that both of these are of the same colour ?

(a)  $\frac{1}{6}$

(b)  $\frac{5}{108}$

(c)  $\frac{4}{9}$

(d)  $\frac{5}{18}$

Consider the following for the next two (02) items that follow :

Let  $3f(x) + f\left(\frac{1}{x}\right) = \frac{1}{x} + 1$

99. What is  $f(x)$  equal to ?

(a)  $\frac{1}{8x} - \frac{x}{8} + \frac{1}{4}$

(b)  $\frac{3}{8x} - \frac{x}{8} + \frac{3}{4}$

102. If a random variable (x) follows binomial distribution with mean 5 and variance 4, and  $5^{23}P(X=3) = \lambda 4^2$ , then what is the value of  $\lambda$  ?

(a) 3

(b) 5

(c) 23

(d) 25

103. From data  $(-4, 1)$ ,  $(-1, 2)$ ,  $(2, 7)$  and  $(3, 1)$ , the regression line of  $y$  on  $x$  is obtained as  $y = a + bx$ , then what is the value of  $2a + 15b$ ?

- (a) 6
- (b) 11
- (c) 17
- (d) 21

104. Let  $x + 2y + 1 = 0$  and  $2x + 3y + 4 = 0$  are two lines of regression computed from some bivariate data. If  $\theta$  is the acute angle between them, then what is the value of  $488 \tan 3\theta$ ?

- (a) 191
- (b) 161
- (c) 131
- (d) 121

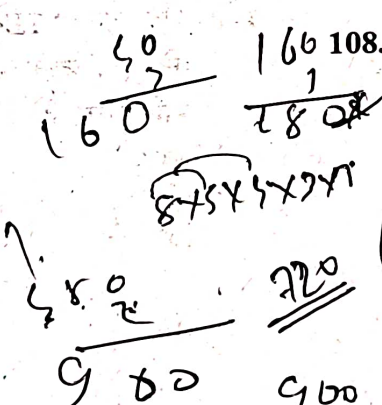


105. If two random variables  $X$  and  $Y$  are connected by relation

$$\frac{2X - 3Y}{5X + 4Y} = 4 \text{ and } X \text{ follows Binomial distribution with parameters } n = 10 \text{ and}$$

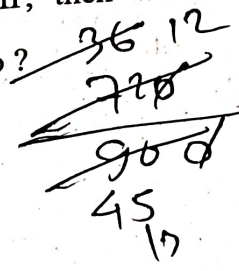
$p = \frac{1}{2}$ , then what is the variance of  $Y$ ?

- (a)  $\frac{810}{361}$
- (b)  $\frac{9}{19}$
- (c)  $\frac{21}{361}$
- (d)  $\frac{121}{361}$



106. If  $a, b, c$  are in HP, then what is

$$\frac{1}{b-a} + \frac{1}{b-c} \text{ equal to?}$$



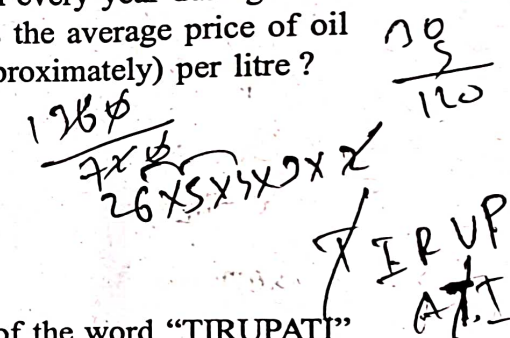
- 1.  $\frac{2}{b}$
- 2.  $\frac{1}{a} + \frac{1}{c}$
- 3.  $\frac{1}{2} \left( \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right)$

Select the correct answer using the code given below :

- (a) 1 only
- (b) 2 only
- (c) 3 only
- (d) 1, 2 and 3

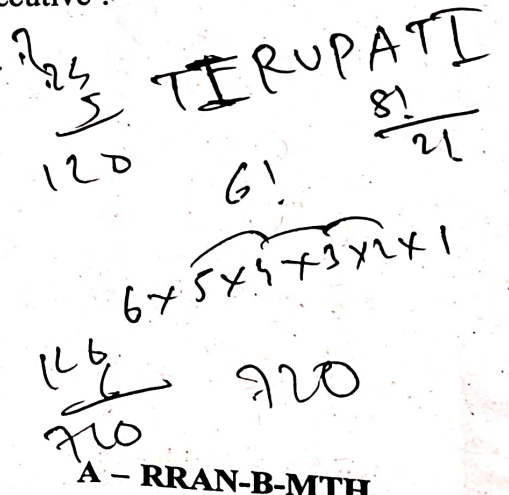
107. An edible oil is sold at the rates 150, 200, 250, 300 rupees per litre in four consecutive years. Assuming that an equal amount of money is spent on oil by a family in every year during these years, what is the average price of oil in rupees (approximately) per litre?

- (a) 210
- (b) 220
- (c) 230
- (d) 240



108. If the letters of the word "TIRUPATI" are written down at random, then what is the probability that both Ts are always consecutive?

- (a)  $\frac{1}{2}$
- (b)  $\frac{1}{4}$
- (c)  $\frac{1}{7}$
- (d)  $\frac{1}{14}$





109. Let  $m = 77^n$ . The index  $n$  is given a positive integral value at random. What is the probability that the value of  $m$  will have 1 in the units place?

(a)  $\frac{1}{2}$

(b)  $\frac{1}{3}$

(c)  $\frac{1}{4}$

(d)  $\frac{1}{n}$

110. Three different numbers are selected at random from the first 15 natural numbers. What is the probability that the product of two of the numbers is equal to third number?

(a)  $\frac{1}{91}$

(b)  $\frac{2}{455}$

(c)  $\frac{1}{65}$

(d)  $\frac{6}{455}$

Consider the following for the next two (02) items that follow :

Let  $A$  and  $B$  be two events such that  $P(A \cup B) \geq 0.75$  and  $0.125 \leq P(A \cap B) \leq 0.375$ .

111. What is the minimum value of  $P(A) + P(B)$ ?

(a) 0.625

(b) 0.750

(c) 0.825

(d) 0.875

112. What is the maximum value of  $P(A) + P(B)$ ?

(a) 0.75

(b) 1.125

(c) 1.375

(d) 1.625

Consider the following for the next two (02) items that follow :

$A, B$  and  $C$  are three events such that  $P(A) = 0.6, P(B) = 0.4, P(C) = 0.5, P(A \cup B) = 0.8, P(A \cap C) = 0.3$  and  $P(A \cap B \cap C) = 0.2$  and  $P(A \cup B \cup C) \geq 0.85$ .

113. What is the minimum value of  $P(B \cap C)$ ?

(a) 0.1

(b) 0.2

(c) 0.35

(d) 0.45

Handwritten calculation for Q113:  

$$P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C) \geq 0.85$$

$$0.6 + 0.4 + 0.5 - 0.2 - P(B \cap C) + 0.2 \geq 0.85$$

$$1.5 - P(B \cap C) \geq 0.85$$

$$P(B \cap C) \leq 1.5 - 0.85 = 0.65$$
 (Note: The handwritten calculation in the image shows a different path, possibly using a different inequality or a typo, resulting in 1.5 - 0.2 = 1.3, then 1.3 - 0.2 = 1.1, then 1.1 - 0.2 = 0.9, then 0.9 - 0.2 = 0.7, then 0.7 - 0.2 = 0.5, then 0.5 - 0.2 = 0.3, then 0.3 - 0.2 = 0.1. This seems to be a miscalculation. The correct calculation is as shown above.)

114. What is the maximum value of  $P(B \cap C)$ ?

(a) 0.1

(b) 0.2

(c) 0.35

(d) 0.45

Consider the following for the next two (02) items that follow :

An unbiased coin is tossed  $n$  times. The probability of getting at least one tail is  $p$  and the probability of at least two tails is  $q$  and  $p - q = \frac{5}{32}$ .

115. What is the value of  $n$  ?

- (a) 4  
(b) 5  
(c) 6  
(d) 7

116. What is the value of  $p + q$  ?

- (a)  $\frac{57}{32}$   
(b)  $\frac{53}{32}$   
(c)  $\frac{51}{32}$   
(d) 1

Handwritten calculations:  
 $\frac{50}{41}$   
 $\frac{41}{35}$   
 $\frac{32}{42}$   
 41

Consider the following for the next two (02) items that follow :

|       |   |          |          |     |              |
|-------|---|----------|----------|-----|--------------|
| $x_i$ | 1 | 2        | 3        | ... | $n$          |
| $f_i$ | 1 | $2^{-1}$ | $2^{-2}$ | ... | $2^{-(n-1)}$ |

117. What is  $\sum_i^n x_i f_i$  equal to ?

- (a)  $\frac{2^{n+1} - n + 2}{2^{n-1}}$   
(b)  $\frac{2^{n+1} - n - 2}{2^{n-1}}$

Handwritten calculations:  
 $\frac{2^2 - 1 + 2}{2^0}$   
 $\frac{2 - 1 + 2}{2^0}$   
 $\frac{3 - 1 + 2}{2^0}$   
 $\frac{4}{2^0}$   
 $\frac{4}{1} = 4$

(c)  $\frac{2^{n+1} + n + 2}{2^{n-1}}$

(d)  $\frac{2^{n+1} - n - 2}{2^n}$

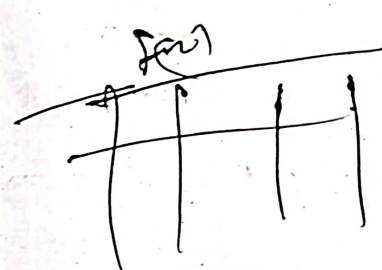
118. What is the mean of the distribution ?

(a)  $\frac{2^{n+1} - n + 2}{2^n - 1}$

(b)  $\frac{2^{n+1} - n - 2}{2^{n-1}}$

(c)  $\frac{2^{n+1} - n - 2}{2^n - 1}$

(d)  $\frac{2^{n+1} - n + 2}{2^n}$



Consider the following for the next two (02) items that follow :

The marks obtained by 10 students in a Statistics test are 24, 47, 18, 32, 19, 15, 21, 35, 50 and 41.

119. What is the mean deviation of the largest five observations ?

- (a) 4.8  
(b) 5.5  
(c) 6  
(d) 7.5

120. What is the variance of the largest five observations ?

- (a) 14.6  
(b) 21.8  
(c) 25.2  
(d) 46.8