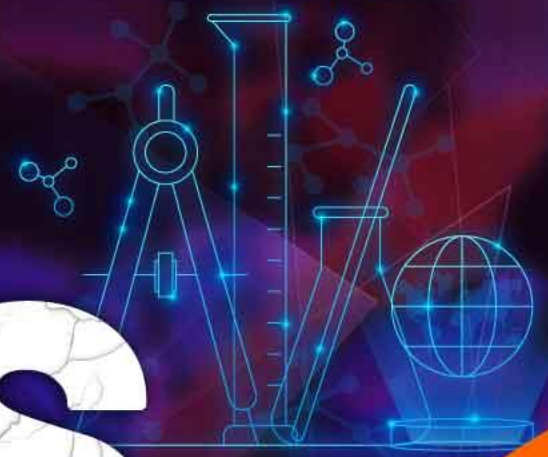
CDS-AFCAT 2 2024

SSBCrack EXAMS

LIVE

MATHS

LOGARITHMS



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LOGARITHMS

If a is a positive real number, other than 1 and $a^x = m$, then x is called the logarithm of m to the base a. It is denoted by $\log_a m$. Thus, $a^x = m \Leftrightarrow x = \log_a m$

e.g., $2^3 = 8 \Leftrightarrow \log_2 8 = 3$

If $x < 0$, then $\log_a x$ is imaginary.

If $x = 0$, then $\log_a x$ is meaningless.

$\log_a x$ exists, if and only if $x > 0$, $a > 0$ and $a \neq 1$.

$4^3 = 64 \rightarrow \log_4 64 = 3$

(base)

PROPERTIES OF LOGARITHMS

1. $a^{\log_a x} = x\,;\ a \neq 0, \neq 1, x > 0$

2. $a^{\log_b x} = x^{\log_b a}\,;\ a > 0,\ b > 0, \neq 1, x > 0$

3. $\log_a a = 1, \log_a 1 = 0\,;\ a > 0, \neq 1$

4. $\log_a x = \dfrac{1}{\log_x a}\,;\ x, a > 0, \neq 1$

5. ~~$\log_a x = \log_b x$~~ $\log_a b = \dfrac{\log_b x}{\log_b a}\,;\ a, b > 0, \neq 1, x > 0$

(1) $a^{\log_a x} = y$ → x

$\log_a(a^{\log_a x}) = \log_a y$

$= \log_a x \log_a a = \log_a y$ $\quad \log(a^m)$

$\log_a x(1) = \log_a y$ $\quad = m\log a$

$x = y$

$2^1 = 2$

$\log_2 2 = 1$

(2) $a^{\log_b x} = x^{\log_b a}$

$\log_b (a^{\log_b x}) = \log_b (x^{\log_b a})$

$= \log_b x \cdot \log_b a = \log_b a \cdot \log_b x$

(4) $\log_a b = \dfrac{1}{\log_b a}$

(5) $\log_a b = \dfrac{\log_m b}{\log_m a}$

PROPERTIES OF LOGARITHMS

6. For $m, n > 0, a > 0$ and $\neq 1$

(a) $\log_a (m \cdot n) = \log_a m + \log_a n$

(b) $\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$

(c) $\log_a (m^n) = n \log_a (m)$

7. For $x > 0, a > 0, \neq 1$

(a) $\log_{a^n} x = \frac{1}{n} \log_a x$ $\{\log_a x^m = m \log_a x\}$

(b) $\log_{a^n} x^m = \left(\frac{m}{n}\right) \log_a x$

PROPERTIES OF LOGARITHMS

8. For $x > y > 0$

   (a) $\log_a x > \log_a y$, if $a > 1$

   (b) $\log_a x < \log_a y$, if $0 < a < 1$

9. $a > 1$, then

   (a) $\log_a x > p \Rightarrow x > a^p$

   (b) $0 < \log_a x < p \Rightarrow 1 < x < a^p$

10. If $0 < a < 1$, then

   (a) $\log_a x > p \Rightarrow 0 < x < a^p$

   (b) $0 < \log_a x < p \Rightarrow a^p < x < 1$

$\log_a b = m$

$b = a^m$

$\log_3 27 = 3$

$3^3 = 27$

Q) What is the value of $\dfrac{(\log_{27} 9)(\log_{16} 64)}{\log_4 \sqrt{2}}$?

(a) 1 (b) 2 (c) 4 (d) 8

$$\frac{\left(\frac{\log_3 9}{\log_3 27}\right)\left(\frac{\log_2 64}{\log_2 16}\right)}{\frac{1}{\log_{\sqrt{2}} 4}} = \frac{\frac{2}{3} \times \frac{6}{4}}{\frac{1}{4}} = 4$$

$(\sqrt{2})^1 = \sqrt{2}$

$(\sqrt{2})^2 = 2$

$(\sqrt{2})^3 = 2\sqrt{2}$

$(\sqrt{2})^4 = 4$

Q) What is the value of $\frac{(\log_{27} 9)(\log_{16} 64)}{\log_4 \sqrt{2}}$?

(a) 1 (b) 2 (c) 4 (d) 8

Ans: (c)

Q) If $(\log_x x)(\log_3 2x)(\log_{2x} y) = \log_x x^2$, then what is the value of y?

(a) $\frac{9}{2}$ (b) 9 (c) 18 (d) 27

(1) $(\log_3(2x))(\log_{2x} y) = \log_x x^2$

$(\log_3 2x)(\log_{2x} y) = 2$

$\frac{1}{\log_{2x} 3} \times \log_{2x} y = 2$

$\frac{\log_{2x} y}{\log_{2x} 3} = 2$

$\log_{2x} y = 2\log_{2x} 3$

$\log_{2x} y = \log_{2x} 3^2$

$y = 3^2 = 9$

Q) If $(\log_x x)(\log_3 2x)(\log_{2x} y) = \log_x x^2$, then what is the value of y?

(a) $\frac{9}{2}$ (b) 9 (c) 18 (d) 27

Ans: (b)

Q) If $4^{\log_9 3} + 9^{\log_2 4} = 10^{\log_x 83}$, then x is

(a) 4
(b) 9
(c) 10
(d) None of these

$\log_b a^m = m \log_b a$

$4^{\frac{1}{\log_3 9}} + 9^2 = 10^{\log_x 83}$

$4^{\frac{1}{2}} + 81 =$

$2 + 81 = 10^{\log_x 83}$

$83 = 10^{\log_x 83}$

$\log_{10} 83 = \log_x 83 (\log_{10} 10)$

$\log_{10} 83 = \log_x 83 (1)$

$\Rightarrow x = 10$

Q) If $4^{\log_9 3} + 9^{\log_2 4} = 10^{\log_x 83}$, then x is

(a) 4
(b) 9
(c) 10
(d) None of these

Ans: (c)

Q) What is the value of $2\log_8 2 - \dfrac{\log_3 9}{3}$?

(a) 0 (b) 1 (c) 2 (d) 1/3

$$2 \times \frac{1}{\log_2 8} - \frac{2}{3}$$

$$= \frac{2}{3} - \frac{2}{3} = 0$$

Q) What is the value of $2\log_8 2 - \frac{\log_3 9}{3}$?

(a) 0 (b) 1 (c) 2 (d) 1/3

Ans: (a)

Q) If $\log_{10}(x+1) + \log_{10} 5 = 3$, then what is the value of x?

(a) 199 (b) 200

(c) 299 (d) 300

$\log_a b = m$

$\Rightarrow b = a^m$

$\log_{10}(x+1) + \log_{10} 5 = 3$

$\log_a A + \log_a B = \log_a (A \cdot B)$

$\log_{10}(5(x+1)) = 3$

$5x + 5 = 10^3$

$5x = 995$

$x = 199$

Q) If $\log_{10}(x+1) + \log_{10} 5 = 3$, then what is the value of x?

(a) 199 (b) 200

(c) 299 (d) 300

Ans: (a)

Q) If $\log_e\left(\frac{a+b}{2}\right) = \frac{1}{2}(\log_e a + \log_e b)$, then

(a) $a = b$

(b) $a = \frac{b}{2}$

(c) $2a = b$

(d) $a = \frac{b}{3}$

e — natural logarithm

$e = 2.7\ldots$

$\log_e \rightarrow \ln$

$\log_e x = \ln x$

$\log_e\left(\frac{a+b}{2}\right) = \frac{1}{2}\left(\log_e(ab)\right)$

$\log_e\left(\frac{a+b}{2}\right) = \log_e(ab)^{1/2}$

$\Rightarrow \frac{a+b}{2} = \sqrt{ab}$

$AM(a,b) = GM(a,b)$

$\Rightarrow a = b$

Q) If $\log_e\left(\frac{a+b}{2}\right) = \frac{1}{2}(\log_e a + \log_e b)$, then

(a) $a = b$

(b) $a = \frac{b}{2}$

(c) $2a = b$

(d) $a = \frac{b}{3}$

Ans: (a)

Q) The value of $e^{(\log_{10} \tan 1^\circ + \log_{10} \tan 2^\circ + ... + \log_{10} \tan 89^\circ)}$ is equal to

(a) 0 (b) 1

(c) e (d) $\frac{1}{e}$

$\log_{10} \tan 1^\circ + \log_{10} \tan 2^\circ + \dots + \log_{10} \tan 89^\circ$

$\log_{10} (\tan 1^\circ \tan 2^\circ \dots \tan 89^\circ)$

$\tan 1^\circ \tan 2^\circ \dots \tan 45^\circ \dots \cot 3^\circ \cot 2^\circ \cot 1^\circ$

$\log_{10} (1) = 0$

$e^0 = 1$

$\log_a (1) = 0 = \log_{10} (1) = \log_e (1)$

at any base $(1) = 0$

Q) The value of $e^{(\log_{10} \tan 1^\circ + \log_{10} \tan 2^\circ + ... + \log_{10} \tan 89^\circ)}$ is equal to

(a) 0

(b) 1

(c) e

(d) $\frac{1}{e}$

Ans: (b)

Q) If $x < 0, y < 0$, then $\log(xy)$ is equal to

(a) $\log x + \log y$

(b) $\log(-x) + \log(-y)$

(c) $-\log x - \log y$

(d) None of these

$\log(xy) = \log x + \log y$

log, not defined for x, y = -ve

or, $x < 0, y < 0$

Q) If $x < 0, y < 0$, then $\log(xy)$ is equal to

(a) $\log x + \log y$

(b) $\log(-x) + \log(-y)$

(c) $-\log x - \log y$

(d) None of these

Ans: (b)

Q) What is $\log\left(a+\sqrt{a^2+1}\right)+\log\left(\frac{1}{a+\sqrt{a^2+1}}\right)$ equals to?

(a) 1 (b) 0

(c) 2 (d) $\frac{1}{2}$

$\log x \times \log \frac{1}{x} = 1$

$\log x + \log \frac{1}{x} = 0$

$\log\left(a+\sqrt{a^2+1}\right) + \log\left(a+\sqrt{a^2+1}\right)^{-1}$

$\log(x)^m = m\log x$

$\log\left(a+\sqrt{a^2+1}\right) - \log\left(a+\sqrt{a^2+1}\right) = 0$

Q) What is $\log\left(a+\sqrt{a^2+1}\right)+\log\left(\frac{1}{a+\sqrt{a^2+1}}\right)$ equals to?

(a) 1
(b) 0
(c) 2
(d) $\frac{1}{2}$

Ans: (b)

Q) If $\log_k x \log_5 k = 3$, then what is the value of x?

(a) k^5 (b) $5k^3$

(c) 243 (d) 125

$\log_k x \times \frac{1}{\log_k 5} = 3$

$\log_k x = 3 \log_k 5$

$\log_k x = \log_k 5^3$ → $m \log_a b = \log_a b^m$

$\Rightarrow x = 5^3 = 125$

Q) If $\log_k x \log_5 k = 3$, then what is the value of x?

(a) k^5

(b) $5k^3$

(c) 243

(d) 125

Ans: (d)

Q) For what value(s) of x is

$$\log_{10}\left(999+\sqrt{x^2-3x+3}\right)=3?$$

(a) 0
(b) Only 1
(c) Only 2
(d) 1, 2

HW

Q) For what value(s) of x is

$$\log_{10}\left(999+\sqrt{x^2-3x+3}\right)=3?$$

(a) 0
(b) Only 1
(c) Only 2
(d) 1, 2

Ans: (d)

Q) If $\frac{\log x}{\log 5} = \frac{\log 36}{\log 6} = \frac{\log 64}{\log y}$, what are the values of x and y, respectively?

(a) 8, 25 (b) 25, 8 (c) 8, 8 (d) 25, 25

$\frac{\log x}{\log 5} = 2$

$\log x = \log 5^2$

$x = 25$

$\frac{\log 64}{\log y} = 2$

$\log 64 = \log y^2$

$y^2 = 64$

$y = 8$

(if nothing, means same base,

$\log b + \log a$

(same bases)

Q) If $\frac{\log x}{\log 5} = \frac{\log 36}{\log 6} = \frac{\log 64}{\log y}$, what are the values of x and y, respectively?

(a) 8, 25 (b) 25, 8 (c) 8, 8 (d) 25, 25

Ans: (b)

Q) How many number of digits are there in 2^{98}?
(Given that $\log_{10} 2 = 0.30103$)

(a) 98 (b) 99
(c) 30 (d) 29

$\log_{10} 2^{98} = 98 \times \log_{10} 2$

$= 98 \times 0.30103$

$= (100-2) \times 0.30103$

$= 30.103 - 0.60206$

$= 29$ (Integer part)

no. of digits $= 29 + 1$ (left side of decimal)

x	$\log_{10} x$	no. of digits
10	1	1+1 = 2
100	2	2+1 = 3
1000	3	3+1 = 4

no. of digits of x = Integral value $(\log_{10} x) + 1$

Q) How many number of digits are there in 2^{98}?
(Given that $\log_{10} 2 = 0.30103$)

(a) 98 (b) 99

(c) 30 (d) 29

Ans: (c)

Q) What is the number of digits in the numeral form of 8^{17} ?

(a) 51 (b) 16 (c) 15 (d) 14

Q) What is the number of digits in the numeral form of 8^{17} ?

(a) 51 (b) 16 (c) 15 (d) 14

Ans: (b)

Q) What is the least integral value of $2\log_{10} x - \log_x (0.01)$?

(a) 0 (b) 2 (c) 4 (d) 3

$$\log_{a^n} b = \frac{1}{n}\log_a b$$

$2\log_{10} x - \log_x\left(\frac{1}{100}\right)$

$2\left(\frac{1}{\log_x 10}\right) - [\log_x(1) - \log_x(100)]$

$2\left(\frac{1}{\log_x 10}\right) - 0 + \log_x 100$

$2\log_{10} x + \frac{1}{\log_{100} x}$

$2\log_{10} x + \frac{1}{\frac{1}{2}(\log_{10} x)}$

$2\log_{10} x + \frac{2}{\log_{10} x} = 2\left(A + \frac{1}{A}\right)$

min value of $A + \frac{1}{A} = 2$ (when $A = 1$)

$$2\log_{10} x - \log_x (0.01)$$

$$\frac{2}{\log_x 10} - \left[\log_x (1) - \log_x (100)\right]$$

$$\frac{2}{\log_x 10} + 2\log_x 10$$

$$2\left(\frac{1}{A} + A\right)$$

$$2 \times 2 = 4$$

For $A + \frac{1}{A}$ to be minimum

$A = 1$

$A + \frac{1}{A} = 2$

Q) What is the least integral value of $2\log_{10} x - \log_x (0.01)$?

(a) 0 (b) 2 (c) 4 (d) 3

Ans: (c)

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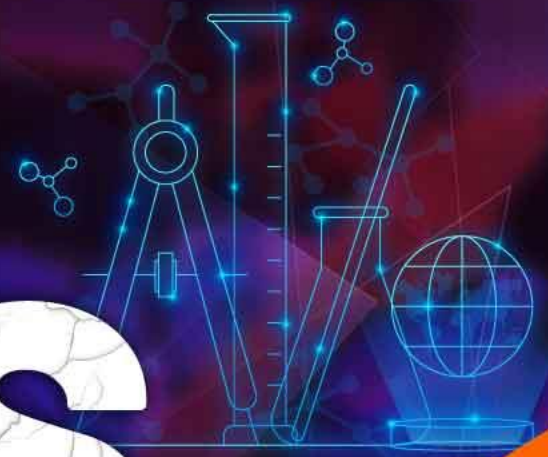

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