NDA 2 2024

LIVE

MATHS

INDEFINITE & DEFINITE INTEGRATION

CLASS 2

NAVJYOTI SIR

Crack
EXAMS

28 June 2024 Live Classes Schedule

8:00AM	28 JUNE 2024 DAILY CURRENT AFFAIRS	RUBY MA'AM
9:00AM	28 JUNE 2024 DAILY DEFENCE UPDATES	DIVYANSHU SIR

AFCAT 2 2024 LIVE CLASSES

2:30PM	STATIC GK - COUNTRY CAPITAL CURRENCY - CLASS 1	DIVYANSHU SIR

NDA 2 2024 LIVE CLASSES

11:30AM	GK - MODERN HISTORY - CLASS 2	RUBY MA'AM
2:30PM	GS - CHEMISTRY MCQS - CLASS 5	SHIVANGI MA'AM
5:30PM	ENGLISH - ORDERING OF WORDS - CLASS 3	ANURADHA MA'AM
6:30PM	MATHS - INDEFINITE & DEFINITE INTEGRATION - CLASS 2	NAVJYOTI SIR

CDS 2 2024 LIVE CLASSES

11:30AM	GK - MODERN HISTORY - CLASS 2	RUBY MA'AM
2:30PM	GS - CHEMISTRY MCQS - CLASS 5	SHIVANGI MA'AM
5:30PM	ENGLISH - ORDERING OF WORDS - CLASS 3	ANURADHA MA'AM

PARTIAL FRACTIONS

$$\frac{2x+3}{x^2+5x+6} \rightarrow \frac{A}{x+a} + \frac{B}{x+b}$$

$$\frac{px+q}{(x+a)(x+b)} = \left\{ \frac{A}{x+a} + \frac{B}{x+b} \right\} \rightarrow \int \frac{A}{x+a}\,dx + \int \frac{B}{x+b}\,dx$$

$$A \log|x+a| + B \log|x+b| + C$$

find values of A & B.

$$\text{———} \; x \; \text{———}$$

NDA 1 2024 LIVE CLASS - MATHS - PART 2

PARTIAL FRACTIONS

Form of the proper rational function	Form of the partial fraction
$\dfrac{px \pm q}{(x \pm a)(x \pm b)}$, $a \neq b$	$\dfrac{A}{x \pm a} + \dfrac{B}{x \pm b}$
$\dfrac{px \pm q}{(x \pm a)^2}$	$\dfrac{A}{x \pm a} + \dfrac{B}{(x \pm a)^2}$
$\dfrac{px^2 \pm qx \pm r}{(x \pm a)(x \pm b)(x \pm c)}$	$\dfrac{A}{x \pm a} + \dfrac{B}{x \pm b} + \dfrac{C}{x \pm c}$
$\dfrac{px^2 \pm qx \pm r}{(x \pm a)^2(x \pm b)}$	$\dfrac{A}{x \pm a} + \dfrac{B}{(x \pm a)^2} + \dfrac{C}{x \pm b}$
$\dfrac{px^2 \pm qx \pm r}{(x \pm a)(x^2 \pm bx \pm c)}$	$\dfrac{A}{x \pm a} + \dfrac{Bx + C}{x^2 \pm bx \pm c}$
$\dfrac{Px^2 \pm qx \pm r}{(x \pm a)^3}$	$\dfrac{A}{(x \pm a)} + \dfrac{B}{(x \pm a)^2} + \dfrac{C}{(x \pm a)^3}$
Where $x^2 + bx + c$ can not be factorised further	

SSBCrack EXAMS

STANDARD SUBSTITUTION

S.No.	Functions	Substitution
(i)	$(a^2 + x^2), \sqrt{x^2 + a^2}, \frac{1}{\sqrt{x^2 + a^2}}$	$x = a \tan\theta$ or $a \cot\theta$ or $a \sinh\theta$
(ii)	$(a^2 - x^2), \sqrt{a^2 - x^2}, \frac{1}{\sqrt{a^2 - x^2}}$	$x = a \sin\theta$ or $a \cos\theta$
(iii)	$(x \pm \sqrt{x^2 \pm a^2})^n = t^n$	expression inside the bracket $= t$
(iv)	$\frac{2x}{a^2 - x^2}, \frac{2x}{a^2 + x^2}, \frac{a^2 - x^2}{a^2 + x^2}$	$x = a \tan\theta$

($\sin 2\theta$, $\cos 2\theta$, $\tan 2\theta$

SSBCrack EXAMS

STANDARD SUBSTITUTION

(v)	$\dfrac{1}{(x+a)^{1-\frac{1}{n}}(x+b)^{1+\frac{1}{n}}}$ $(n \in N, n > 1)$	$\dfrac{x+a}{x+b} = t$
(vi)	$(x^2 - a^2), \sqrt{x^2 - a^2}, \dfrac{1}{\sqrt{x^2 - a^2}}$	$x = a\sec\theta$ or $a\,\mathrm{cosec}\,\theta$ or $a\cosh\theta$
(vii)	$\sqrt{\dfrac{a-x}{a+x}}$ or $\sqrt{\dfrac{a+x}{a-x}}$	$x = a\cos 2\theta$

SSBCrack EXAMS

STANDARD SUBSTITUTION

(viii)	$\sqrt{\dfrac{x-\alpha}{\beta-x}}$ or $\sqrt{(x-\alpha)(\beta-x)}$	$x = \alpha\cos^2\theta + \beta\sin^2\theta$
(ix)	$\sqrt{2ax - x^2}$	$x = a(1-\cos\theta)$
(x)	$\sqrt{\dfrac{x}{a+x}}, \sqrt{\dfrac{a+x}{x}}, \sqrt{x(a+x)},$	$x = a\tan^2\theta$ or $a\cot^2\theta$
(xi)	$\sqrt{\dfrac{x}{a-x}}; \sqrt{\dfrac{a-x}{x}}, \sqrt{x(a-x)}, \dfrac{1}{\sqrt{x(a-x)}}$	$x = a\sin^2\theta$ or $a\cos^2\theta$
(xii)	$\sqrt{\dfrac{x}{x-a}}; \sqrt{\dfrac{x-a}{x}}, \sqrt{x(x-a)}, \dfrac{1}{\sqrt{x(x-a)}}$	$x = a\sec^2\theta$ or $a\operatorname{cosec}^2\theta$

SSBCrack EXAMS

Q) What is $\int \frac{dx}{\sec^2(\tan^{-1} x)}$ equal to?

(a) $\sin^{-1} x + c$
~~(b)~~ $\tan^{-1} x + c$
(c) $\sec^{-1} x + c$
(d) $\cos^{-1} x + c$

$\tan^2 x = (\tan x)^2$

$$\int \frac{dx}{1+\tan^2(\tan^{-1}x)} = \int \frac{dx}{1+[\tan(\tan^{-1}x)]^2} = \int \frac{dx}{1+x^2}$$

$$= \tan^{-1}x + c$$

Q) What is $\int \frac{dx}{\sec^2(\tan^{-1} x)}$ equal to?

(a) $\sin^{-1} x + c$

(b) $\tan^{-1} x + c$

(c) $\sec^{-1} x + c$

(d) $\cos^{-1} x + c$

Ans: (b)

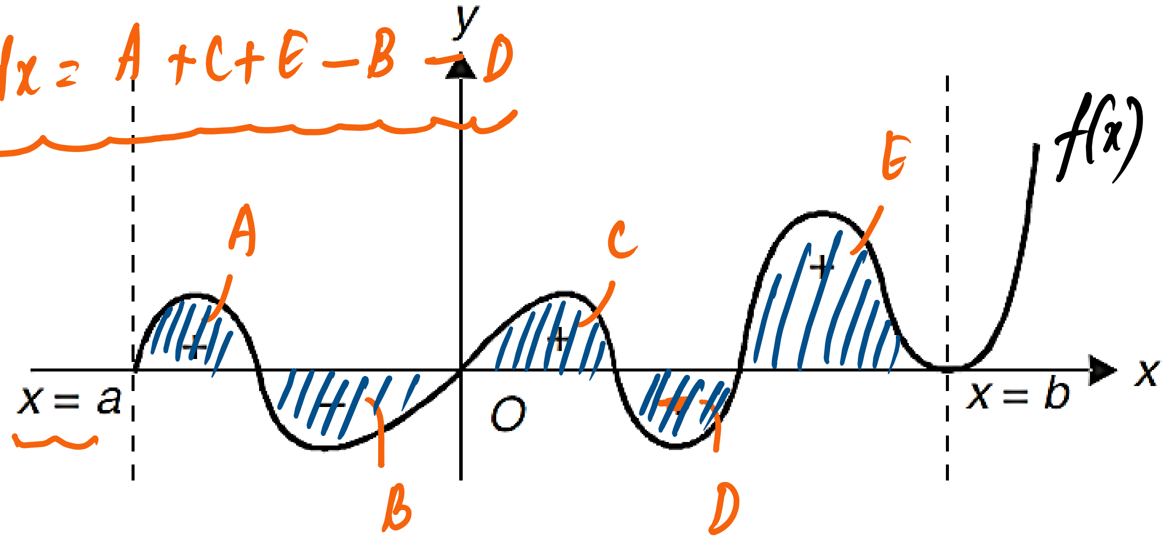
DEFINITE INTEGRATION

Let $f(x)$ be a continuous function defined on a closed interval $[a, b]$ and $\int f(x)\,dx = F(x) + C$, then

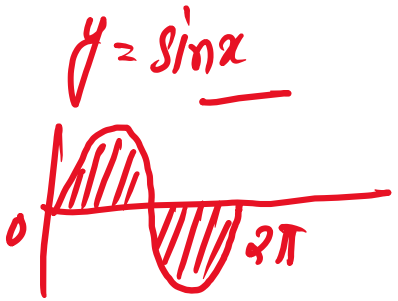
$$\int_a^b f(x)\,dx = [F(x)]_a^b \quad \text{or} \quad \int_a^b f(x)\,dx = F(b) - F(a)$$

Geometrically it represents an algebraic sum of the areas of regions bounded by graph of the function $y = f(x)$, the x-axis and the straight lines $x = a$ and $x = b$.

$$\int_a^b f(x)\,dx = A + C + E - B - D$$



$$\int_0^{2\pi} (\sin x)\,dx$$



$$= [-\cos x]_0^{2\pi}$$

$$= \{-\cos(2\pi)\} - \{(-\cos(0))\}$$

$$= \{-1\} - \{-1\}$$

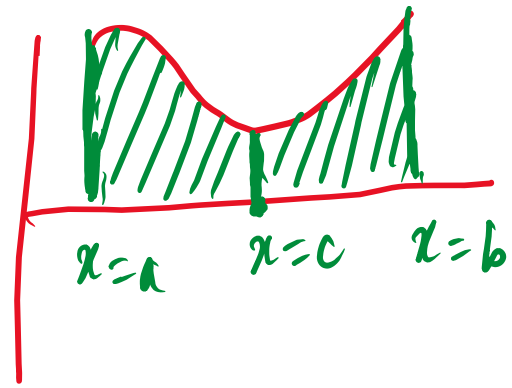
$$= -1 + 1 = 0$$

IMPORTANT PROPERTIES

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

$$\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$$

$a < b < c$



$$\int_0^a f(x)dx = \int_0^a f(a-x)dx$$

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

EXAMPLE

The value of $\int_{-1}^{1} |x|\,dx$ is

(a) 1 (b) -1

(c) 0 (d) None of these

$|x| = \begin{cases} +x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

$I = \int_{-1}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx$

$= \int_{-1}^{0} (-x)\,dx + \int_{0}^{1} (x)\,dx = \left[-\frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_{0}^{1}$

$\left[\frac{x^2}{2}\right]_{0}^{-1} + \left[\frac{x^2}{2}\right]_{0}^{1}$

$= \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right) = 1$

EXAMPLE

The value of $\int_{-1}^{1} |x| dx$ is

(a) 1
(b) –1
(c) 0
(d) None of these

Ans: (a)

EXAMPLE

The value of $\int_0^{\pi/2} \dfrac{\sin x - \cos x}{1+\sin x \cos x} dx$ is — 1

$I =$

(a) 0 (b) 1

(c) –1 (d) –2

$$I = \int_0^{\pi/2} \frac{\sin(\pi/2 - x) - \cos\left(\frac{\pi}{2} - x\right)}{1 + \sin\left(\frac{\pi}{2} - x\right)\cos\left(\frac{\pi}{2} - x\right)}$$

$$I = \int_0^{\pi/2} \frac{\cos x - \sin x}{1 + \cos x \sin x} \quad \text{—— } 2$$

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

1 + 2

$$2I = \int_0^{\pi/2} \frac{0}{1+\cos x \sin x} dx$$

$$\boxed{I = 0}$$

$$\int_0^a f(x)\,dx + \int_0^a g(x)\,dx = \int_0^a [f(x) + g(x)]\,dx$$

EXAMPLE

The value of $\int_0^{\pi/2} \dfrac{\sin x - \cos x}{1 + \sin x \cos x}\, dx$ is

(a) 0 (b) 1

(c) -1 (d) -2

Ans: (a)

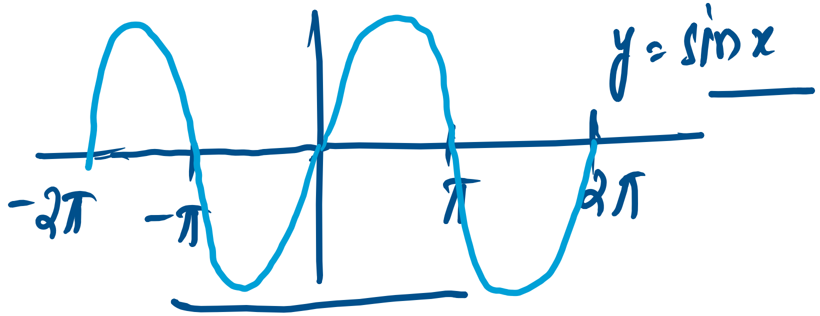
IMPORTANT PROPERTIES

$$\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_{0}^{a} f(x)\,dx, & \text{if } f(-x) = f(x) \\ 0, & \text{if } f(-x) = -f(x) \end{cases}$$

even function, | cos x, $x^{\text{even no.}}$ $(x^2, x^4 \ldots)$

odd function, $(\sin x, \tan x, x^{\text{odd no.}})$

$$\int_{0}^{2a} f(x)dx = \begin{cases} 2\int_{0}^{a} f(x), & if\ f(2a-x) = f(x) \\ 0\ , & if\ f(2a-x) = -f(x) \end{cases}$$



IMPORTANT PROPERTIES

$$\int_0^{\pi/2} \log(\sin x)\, dx = \int_0^{\pi/2} \log(\cos x) dx = -\frac{\pi}{2}\log 2$$

$$\int_0^{\pi/2} \log\ (\tan x)\, dx = \int_0^{\pi/2} \log\ (\cot x)\, dx = 0$$

$$\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\, dx = \frac{\pi}{4}$$

EXAMPLE

The value of $\int_0^{2\pi} \cos^5 x \, dx$ is

(a) 1

(b) 0

(c) -1

(d) 2

$\int_0^{2a} f(x)\, dx$ — $f(2a-x)$ — $-f(x)$ — 0

$+f(x)$ — $2\int_0^{a} f(x)\, dx$

$f(2\pi - x)$

$= \cos^5(2\pi - x)$

$= \underline{\cos^5 x}$

$2\int_0^{\pi} \cos^5 x \, dx$

$= 2\int_0^{\pi} \cos^3 x\,(1-\sin^2 x)\, dx$

$$I = 2\int_0^{\pi} \cos^5 x \, dx$$

$$= 2\int_0^{\pi} \cos^3 x \left(1 - \sin^2 x\right) dx$$

$$= 2\int_0^{\pi} \cos^3 x \, dx - 2\int_0^{\pi} \cos^3 x \sin^2 x \, dx$$

$$= \frac{2}{4}\int_0^{\pi} \left(\cos 3x + 3\cos x\right) dx - 2$$

$$\cos 3x = 4\cos^3 x - 3\cos x$$

$$\cos^3 x = \frac{\cos 3x + 3\cos x}{4}$$

$$t = \cos^3 x$$

$$dt = 3\cos^2 x \sin x$$

$$\int_0^{\pi} \cos^4 x \cdot \cos x \, dx$$

$$= \int_0^{\pi} (\cos^2 x)^2 \cdot \cos x \, dx$$

$$= \int_0^{\pi} (1 - \sin^2 x)^2 \cdot \cos x \, dx$$

$t = \sin x$

$dt = \cos x \, dx$

$x = 0 \Rightarrow t = 0$

$x = \pi \Rightarrow t =$

$$= \int (1 - t^2)^2 \cdot dt$$

EXAMPLE

The value of $\int_0^{2\pi} \cos^5 x \, dx$ is

(a) 1

(b) 0

(c) -1

(d) 2

Ans: (b)

EXAMPLE

Find the value of $\int_{-\pi/4}^{\pi/4} x^3 \sin^4 x \, dx$.

(a) -1 (b) 0

(c) 2 (d) 3

$\int_{-a}^{a} f(x)\,dx \Rightarrow$ check if $f(-x) = -f(x) \Rightarrow \int_{-a}^{a} f(x)\,dx = 0$

(or)

$f(-x) = +f(x) \Rightarrow 2\int_{0}^{a} f(x)\,dx.$

$f(x) = x^3 \sin^4 x$

$f(-x) = (-x)^3 \sin^4(-x) = (-x^3)(\sin(-x))^4 = (-x^3)(-\sin x)^4 = (-x^3)(\sin^4 x) = -f(x)$

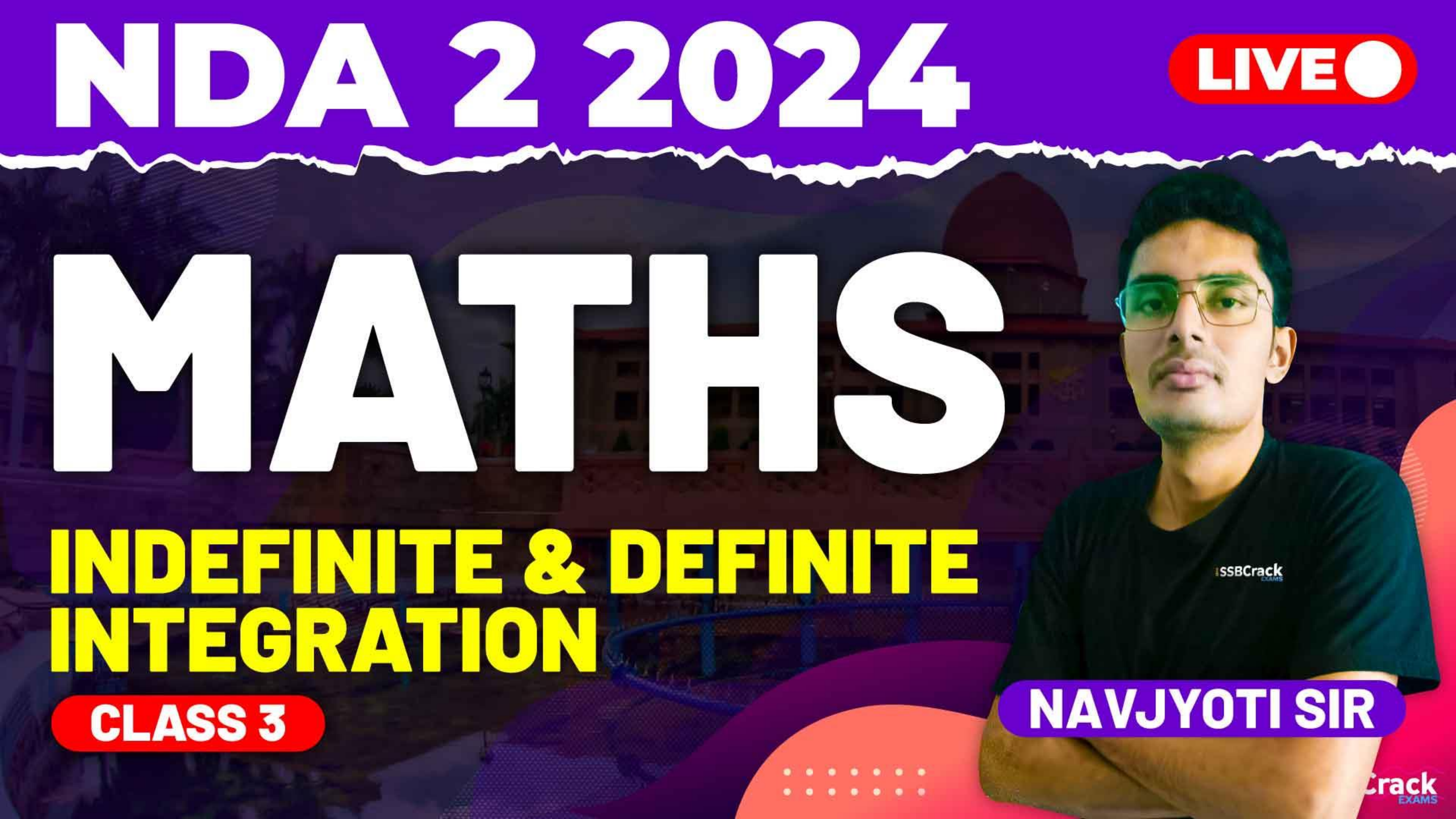
EXAMPLE

Find the value of $\int_{-\pi/4}^{\pi/4} x^3 \sin^4 x \, dx$.

(a) -1 (b) 0

(c) 2 (d) 3

Ans: (b)


NDA 2 2024
LIVE
MATHS
INDEFINITE & DEFINITE INTEGRATION
CLASS 3
SSBCrack EXAMS
NAVJYOTI SIR
Crack EXAMS