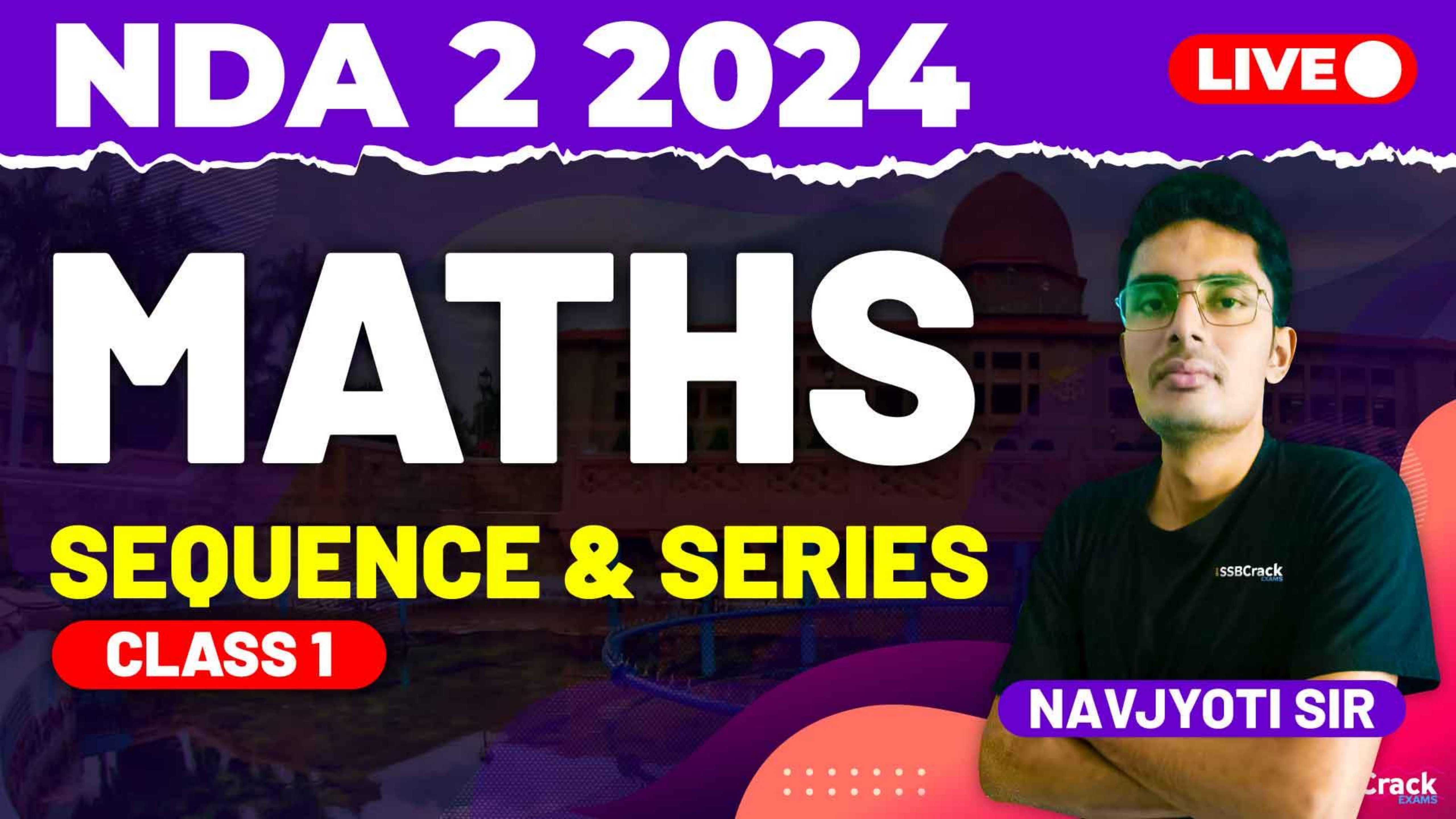
NDA 2 2024
LIVE
MATHS
SEQUENCE & SERIES
CLASS 1
NAVJYOTI SIR
SSBCrack EXAMS
Crack EXAMS

12 June 2024 Live Classes Schedule

8:00AM	12 JUNE 2024 DAILY CURRENT AFFAIRS	RUBY MA'AM
9:00AM	12 JUNE 2024 DAILY DEFENCE UPDATES	DIVYANSHU SIR

SSB INTERVIEW LIVE CLASSES

9:00AM	OVERVIEW OF PIQ & PERSONAL INTERVIEW	ANURADHA MA'AM

AFCAT 2 2024 LIVE CLASSES

2:30PM	STATIC GK - HIGHEST SMALLEST IN INDIA & WORLD	DIVYANSHU SIR
4:00PM	MATHS - TRIGONOMETRY - CLASS 2	NAVJYOTI SIR
5:30PM	ENGLISH - CLOZE TEST - CLASS 1	ANURADHA MA'AM

NDA 2 2024 LIVE CLASSES

11:30AM	GK - INDIAN GEOGRAPHY - CLASS 1	RUBY MA'AM
2:30PM	GS - CHEMISTRY - CLASS 3	SHIVANGI MA'AM
5:30PM	ENGLISH - CLOZE TEST - CLASS 1	ANURADHA MA'AM
6:30PM	MATHS - SEQUENCE & SERIES - CLASS 1	NAVJYOTI SIR

CDS 2 2024 LIVE CLASSES

11:30AM	GK - INDIAN GEOGRAPHY - CLASS 1	RUBY MA'AM
2:30PM	GS - CHEMISTRY - CLASS 3	SHIVANGI MA'AM
4:00PM	MATHS - TRIGONOMETRY - CLASS 2	NAVJYOTI SIR
5:30PM	ENGLISH - CLOZE TEST - CLASS 1	ANURADHA MA'AM

WHAT WILL WE STUDY ?

- Sequence
- Series and Progression
- Arithmetic Progression (AP)
- Arithmetic Mean (AM)
- Geometric Progression (GP)
- Geometric Mean (GM)
- Special Series

SEQUENCE

$$t_n = 3 + 2n$$

$n = 0 \longrightarrow 3$

$n = 1 \longrightarrow 5$

$n = 2 \longrightarrow 7$

$n = 3 \longrightarrow 9$

$3, 5, 7, 9 \ldots$

$0, 1, 1, 2, 3, 5, 8 \ldots$

SERIES & PROGRESSION

sum of numbers in sequence $\longrightarrow$ series,

$\underline{3} + \underline{5} + \underline{7} + \underline{9} = \_\_\_\_$

numbers in sequence are increasing or decreasing
(sequence is called a progression)

AP —
GP —
HP —

ARITHMETIC PROGRESSION

+3 — common difference (d)

2 5 8 11 14 17

+3 +3 +3 +3 +3

2 | 2+3 | 2+3+3 | 2+3+3+3 | 2+3+3+3+3 | 2+3+3+3+3+3

2 | 2+3×1 | 2+3×2 | 2+3×3 | 2+3×4 | 2+3×5

2 | 2+1×3 | 2+2×3 | 2+3×3 | 2+4×3 | 2+5×3

d = 3

1st | 2nd | 3rd | 4th | 5th

a | a+1d | a+2d | a+3d | a+4d | a+5d

a

nth term, $a_n = a + (n-1)d$

SUM OF TERMS IN AP

$a_1 \quad a_2 \quad a_3 \quad a_4 \quad \dots \quad a_n$

$a \quad a+d \quad a+2d \quad a+3d \quad \dots \quad a+(n-1)d$

a – first term

d – common difference

n – no. of terms

$$\boxed{S_n = \frac{n}{2}\left[2a + (n-1)d\right]}$$

(OR)

$$S_n = \frac{n}{2}\left[a + (a + (n-1)d)\right] = \frac{n}{2}\left[a + a_n\right] = \boxed{\frac{n}{2}\left[\text{first term} + \text{last term}\right]}$$

SUPPOSITION OF TERMS IN A.P.

When number of terms be odd then we take,

3 terms as : a – d, a, a + d

5 terms as : a – 2d, a – d, a, a + d, a + 2d

middle term = a

common difference = d

When number of terms be even then we take

4 terms as : a – 3d, a – d, a + d, a + 3d

6 terms as : a – 5d, a – 3d, a – d, a + d, a + 3d ,a + 5d

middle term = a-d, a+d

common difference = 2d.

IMPORTANT RESULTS

If a constant is added to or subtracted from each term of an AP, then the resulting sequence is also an AP with the same common difference.

3 +3 +3

2 5 8 11 14

+2

4 7 10 13 16

+3 +3 +3 +3

$d = 7 - 4 = 3 \; (a_2 - a_1)$

$a_3 - a_2 = 10 - 7 = 3$

2 5 8 11 14

-2

0 3 6 9 12

$d = 3$

IMPORTANT RESULTS

If each term of a given AP is multiplied or divided by a non-zero constant k, then the resulting sequence is also an AP with common difference kd or d/k, where d is the common difference of the given AP.

2 5 8 11 14 } $d = 3$ ✓

×2

4 10 16 22 28 } $d' = \underbrace{3 \times 2}_{} = \underbrace{6}_{}$

$\underbrace{4 \quad 10}_{6} \quad \underbrace{10 \quad 16}_{6} \quad \underbrace{16 \quad 22}_{6}$

÷2

1 2.5 4 5.5 7

$\underbrace{1 \quad 2.5}_{1.5} \quad \underbrace{2.5 \quad 4}_{1.5} \quad \underbrace{4 \quad 5.5}_{1.5} \quad \underbrace{5.5 \quad 7}_{1.5}$

$d' = \frac{3}{2} = 1.5$

IMPORTANT RESULT

If $a_1, a_2, a_3, \ldots$ and $b_1, b_2, b_3, \ldots$ are two A.P., then

(i) $a_1 \pm b_1, a_2 \pm b_2, a_3 \pm b_3 \ldots$ are also in A.P.

(ii) $a_1b_1, a_2b_2, a_3b_3 \ldots$ and $\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}, \ldots$ are not in A.P.

2 5 8 11 14 $d_1 = 3$

11 9 7 5 3 $d_2 = -2$

Adding - 13 14 15 16 17 (+1 +1 +1 +1) $\} \; d = d_1 + d_2 = 3 - 2 = 1$

subtracting - −9 −4 1 6 11 (+5 5 +5 +5) $\} \; d = d_1 - d_2 = 3 - (-2) = 5$

Multiply : 22 45 56 (23, 11) X

divide : $\frac{2}{11}$ $\frac{5}{9}$ $\frac{8}{7}$ X

(0.19..) (0.35..) (1.14..)

~0.35 ~0.65

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IMPORTANT RESULT

If $a_1, a_2, a_3, \ldots\ldots a_n$ are in A.Ps., then

(i) $a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2} = \ldots\ldots$

19

2 5 8 11 14 17

19

7 19

(ii) $a_r = \dfrac{a_{r-k} + a_{r+k}}{2} \ \forall\, k, 0 \le k \le n - r$

2 5 8 11 14 17

$2 \times 5 = 8 + 2$

a, b, c in AP,

$b - a = c - b \Rightarrow \boxed{2b = a + c}$

IMPORTANT RESULT

If n^{th} term of any sequence is linear expression in n, then the sequence is an A. P.

$a_n = 4 + 3n \;|\; 5 + 6n$ (power of $n = 1$)

If sum of n terms of any sequence is a quadratic expression in n, then sequence is an A. P.

$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

(n^2)

$S_n = 4n^2 + 3n + 2$ (power of $n = 2$)

$$\underbrace{\underbrace{a_1, a_2, a_3 \ldots\ldots a_{n-1}}_{S_{n-1}}, a_n}$$

$$S_n = a_1 + a_2 + \ldots a_n$$

$$S_n - S_{n-1} = a_n$$

ARITHMETIC MEAN (AM)

a & b be 2 numbers $\Rightarrow$ $AM = \frac{a+b}{2}$

'n' numbers $(a_1, a_2 \cdots a_n)$ $\Rightarrow$ $\boxed{AM = \frac{a_1 + a_2 \cdots a_n}{n}}$

Let $A_1, A_2 \cdots A_n$ be n Arithmetic Means between a & b,

$a, A_1, A_2, A_3 \cdots A_n, b$ will form an AP.

a (1), $(a+d)$ (2), $(a+2d)$ (3), $a+3d$ (4), $a+nd$ $(n+1)$, $a+(n+1)d$ $(n+2)$

$b = a + (n+1)d$

$\Rightarrow \boxed{d = \frac{b-a}{n+1}}$

GEOMETRIC PROGRESSION (GP)

$2, \quad 6, \quad 18, \quad 36, \quad 72$

$2 \qquad 2\times3 \qquad 2\times3\times3 \qquad 2\times3\times3\times3 \qquad 2\times3\times3\times3\times3$

$2 \qquad 2\times3^1 \qquad 2\times3^2 \qquad 2\times3^3 \qquad 2\times3^4$

(1st) 2nd (3rd) (4th) (5th)

$a \qquad ar^1 \qquad ar^2 \qquad ar^3 \qquad ar^4$

nth term, $\boxed{a_n = ar^{n-1}}$

$\frac{6}{2} = 3 \qquad \frac{36}{18}$

$\frac{18}{6} = 3$ common ratio (r)

SUM OF TERMS IN GP

$r > 1 \longrightarrow S_n = \dfrac{a(r^n - 1)}{r - 1}$

$r < 1 \longrightarrow S_n = \dfrac{a(1 - r^n)}{1 - r}$

→ (special case) : sum to infinite terms, (only for $r<1$)

$n \to \infty$, $r^n \to 0$

$S_\infty = \dfrac{a(1-0)}{1-r} = \dfrac{a}{1-r}$

$S_\infty = \dfrac{a}{1-r}$

$|r| < 1$

SUPPOSITION OF TERMS IN GP

(1) odd no. of terms,

middle term = a ; common ratio = r

3 terms – $\frac{a}{r}, a, ar$

5 terms : $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$

(2) even no. of terms,

middle term : $\frac{a}{r}, ar$

common ratio : r^2

4 terms : $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$

6 terms : $\frac{a}{r^5}, \frac{a}{r^3}, \frac{a}{r}, ar, ar^3, ar^5$

IMPORTANT RESULT

If all the terms of a GP be multiplied or divided by the same non-zero constant, then it remains a GP with the same common ratio.

$r=2$

3 6 12 24 48

$\div 3$

$r=2$ ✓

1 2 4 8 16

$r=2$

$\times 2$

2 2 2 2

6 12 24 48 96

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IMPORTANT RESULT

The reciprocals of the terms of a given GP form a GP. (new comm ratio $= \frac{1}{r}$)

$2, \; 4, \; 8, \; 16 \; \ldots$ (ratio 2)

$\frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{8} \quad \frac{1}{16}$ (ratio $\frac{1}{2}$)

If each terms of a GP be raised to the same power, the resulting sequence also forms a GP.

$3 \quad 6 \quad 12 \qquad r = 2$

$3^2 \quad 6^2 \quad 12^2 \Rightarrow 9 \xrightarrow{\times 4} 36 \xrightarrow{\times 4} 144 \qquad r' = r^2$

IMPORTANT RESULT

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Three non-zero numbers, a, b and c are in GP, if $b^2 = ac$.

$a, \quad b, \quad c$

$$\frac{b}{a} = \frac{c}{b} \Rightarrow \boxed{b^2 = ac}$$

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IMPORTANT RESULT

If $a_1, a_2, a_3, \ldots, a_n, \ldots$ is a GP of non-zero, non-negative terms, then $\log a_1, \log a_2, \ldots, \log a_n, \ldots$ is an AP and *vice-versa*.

$a_1 \quad a_2 \quad a_3$ — GP

$$\frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = \ldots$$

$$\log\left(\frac{a_2}{a_1}\right) = \log\left(\frac{a_3}{a_2}\right) =$$

$$\underbrace{\log a_2 - \log a_1}_{} = \underbrace{\log a_3 - \log a_2}_{}$$

(common difference is same)

$\log a_1, \log a_2, \log a_3$ — AP

GEOMETRIC MEAN (G)

For a & b, $G = \sqrt[2]{ab}$

$a_1, a_2, a_3 \,---\, a_n \Rightarrow$ $\boxed{\text{Geometric mean} = \sqrt[n]{a_1 a_2 a_3 \,---\, a_n}}$

$a, \underbrace{A_1, A_2 \,---\, A_n}_{n\text{-geometric means}}, b \Rightarrow$ GP

$a \qquad ar \qquad ar^2 \qquad ar^n \qquad ar^{n+1}$

$b = ar^{n+1} \Rightarrow \boxed{r = \sqrt[n+1]{\frac{b}{a}}}$

$$\begin{cases} A_1 = ar = a\sqrt[n+1]{b/a} = a\left(\frac{b}{a}\right)^{\frac{1}{n+1}} \\ A_2 = ar^2 = a\left(\frac{b}{a}\right)^{\frac{2}{n+1}} \\ \vdots \\ A_n = ar^n = \underbrace{a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}} \end{cases}$$

SSBCrack EXAMS

SUM TO n TERMS OF SPECIAL SERIES

Sum of first n natural numbers

$$= 1 + 2 + 3 + 4 + \ldots + n = \Sigma n = \frac{n(n+1)}{2}$$

Sum of squares of n natural numbers

$$= 1^2 + 2^2 + 3^2 + 4^2 + \ldots + n^2 = \Sigma n^2 = \frac{n(n+1)(2n+1)}{6}$$

Sum of cubes of n natural numbers

$$= 1^3 + 2^3 + 3^3 + 4^3 + \ldots + n^3$$

$$= \Sigma n^3 = \frac{n^2(n+1)^2}{4} = \left[\frac{n(n+1)}{2}\right]^2 = [\Sigma n]^2$$

$$a_n = A\underline{\underline{n}}^3 + Bn^2 + Cn + \underline{\underline{D}}$$

$$S_n = \sum a_n$$

$$= A\sum \underline{n^3} + B\sum \underline{\underline{n^2}} + C\sum n + Dn$$

OTHER RESULTS

$1 + 3 + 5 + \ldots$ to n terms $= n^2$.

Sum of n odd terms $= n^2$

$2 + 4 + 6 + \ldots.$ to $\cancel{n}^{2n}$ terms $= n(n + 1)$

$2(1+2+3+ \ldots n)$

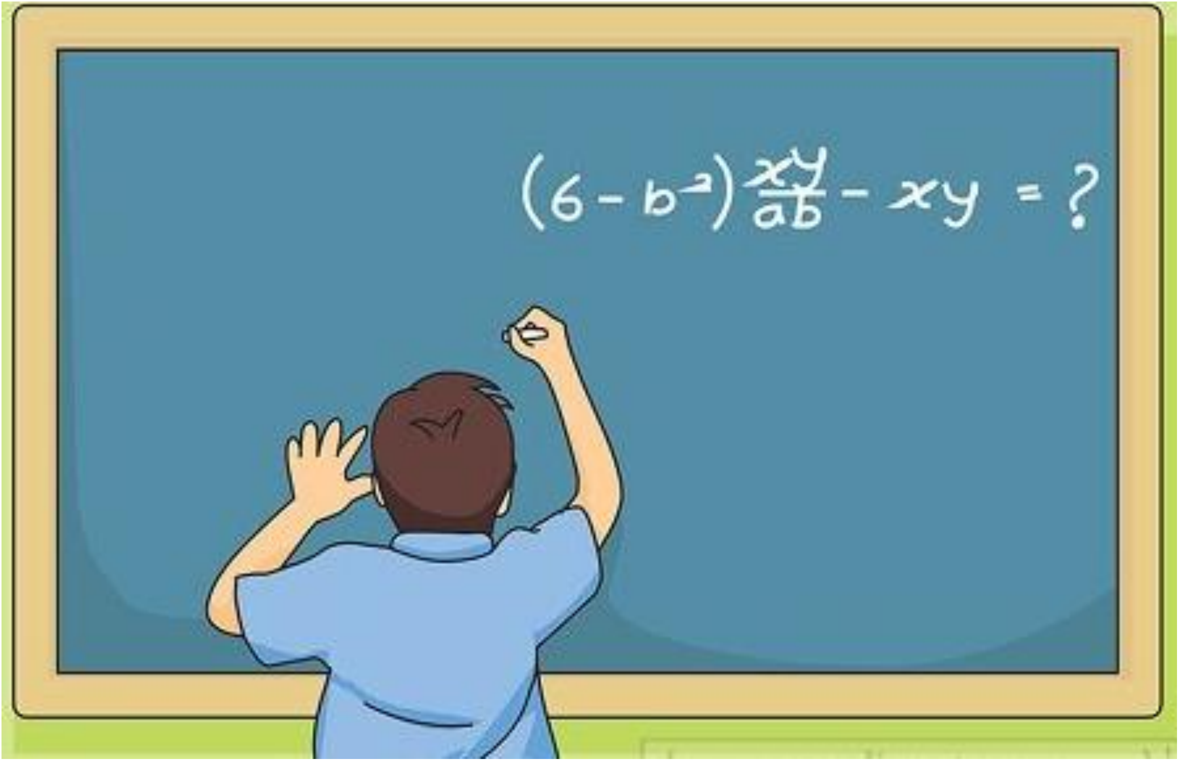
$2 \dfrac{n(n+1)}{2} = n(n+1)$

SSBCrack EXAMS

OTHER RESULTS

If a, b and c are in AP as well as in GP, then $a = b = c$.

PRACTISE TIME !



[2024 (I)]

If a, b and c ($a > 0$, $c > 0$) are in GP, then consider the following in respect of the equation $ax^2 + bx + c = 0$:

1. The equation has imaginary roots.
2. The ratio of the roots of the equation is $1 : \omega$ where ω is a cube root of unity.
3. The product of roots of the equation is $\left(\frac{b^2}{a^2}\right)$.

Which of the statements given above are correct ?

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

[2024 (I)]

If a, b and c $(a > 0, c > 0)$ are in GP, then consider the following in respect of the equation $ax^2 + bx + c = 0$:

1. The equation has imaginary roots.
2. The ratio of the roots of the equation is $1 : \omega$ where ω is a cube root of unity.
3. The product of roots of the equation is $\left(\frac{b^2}{a^2}\right)$.

Which of the statements given above are correct ?

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

Ans: (d)

If p^2, q^2 and r^2 (where $p, q, r > 0$) are in GP, then which of the following is/are correct? **[NDA/NA 2020-I]**

1. p, q and r are in GP.
2. $\ln p$, $\ln q$ and $\ln r$ are in AP.

Select the correct answer using the code given below :

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

If p^2, q^2 and r^2 (where $p, q, r > 0$) are in GP, then which of the following is/are correct? **[NDA/NA 2020-I]**

1. p, q and r are in GP.
2. $\ln p$, $\ln q$ and $\ln r$ are in AP.

Select the correct answer using the code given below :

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

Ans: (c)

Let a, b, c be in AP and $k \neq 0$ be a real number. Which of the following are correct? **[NDA/NA 2019-II]**

1. ka, kb, kc are in AP
2. $k-a, k-b, k-c$ are in AP
3. $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}$ are in AP

Select the correct answer using the code given below :

(a) 1 and 2 only
(b) 2 and 3 only
(c) 1 and 3 only
(d) 1, 2 and 3

Let a, b, c be in AP and $k \neq 0$ be a real number. Which of the following are correct? **[NDA/NA 2019-II]**

1. ka, kb, kc are in AP
2. $k-a, k-b, k-c$ are in AP
3. $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}$ are in AP

Select the correct answer using the code given below :

(a) 1 and 2 only (b) 2 and 3 only

(c) 1 and 3 only (d) 1, 2 and 3

Ans: (d)

Q) What is the product of first $2n + 1$ terms of a geometric progression ?

(a) The $(n + 1)$th power of the nth term of the GP

(b) The $(2n + 1)$th power of the nth term of the GP

(c) The $(2n + 1)$th power of the $(n + 1)$th term of the GP

(d) The nth power of the $(n + 1)$th terms of the GP

Q) What is the product of first $2n + 1$ terms of a geometric progression ?

(a) The $(n + 1)$th power of the nth term of the GP

(b) The $(2n + 1)$th power of the nth term of the GP

(c) The $(2n + 1)$th power of the $(n + 1)$th term of the GP

(d) The nth power of the $(n + 1)$th terms of the GP

Ans: (c)

SSBCrack EXAMS

Q) The equation $(a^2 + b^2) x^2 - 2b (a + c) x + (b^2 + c^2) = 0$ has equal roots. Which one of the following is correct about a, b, and c ?

(a) They are in AP

(b) They are in GP

(c) They are in HP

(d) They are neither in AP, nor in GP, nor in HP

Q) The equation $(a^2 + b^2)x^2 - 2b(a + c)x + (b^2 + c^2) = 0$ has equal roots. Which one of the following is correct about a, b, and c ?

(a) They are in AP

(b) They are in GP

(c) They are in HP

(d) They are neither in AP, nor in GP, nor in HP

Ans: (b)

SSBCrack EXAMS

Q) If a, b, c are in geometric progression and a, 2b, 3c are in arithmetic progression, then what is the common ratio r such that $0 < r < 1$?

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) $\frac{1}{8}$

Q) If a, b, c are in geometric progression and a, 2b, 3c are in arithmetic progression, then what is the common ratio r such that $0 < r < 1$?

(a) $\frac{1}{3}$

(b) $\frac{1}{2}$

(c) $\frac{1}{4}$

(d) $\frac{1}{8}$

Ans: (a)

Q) If a, b and c are three positive numbers in an arithmetic progression, then:

(a) $ac > b^2$

(b) $b^2 > a + c$

(c) $ab + bc \leq 2ac$

(d) $ab + bc \geq 2ac$

Q) If a, b and c are three positive numbers in an arithmetic progression, then:

(a) $ac > b^2$

(b) $b^2 > a + c$

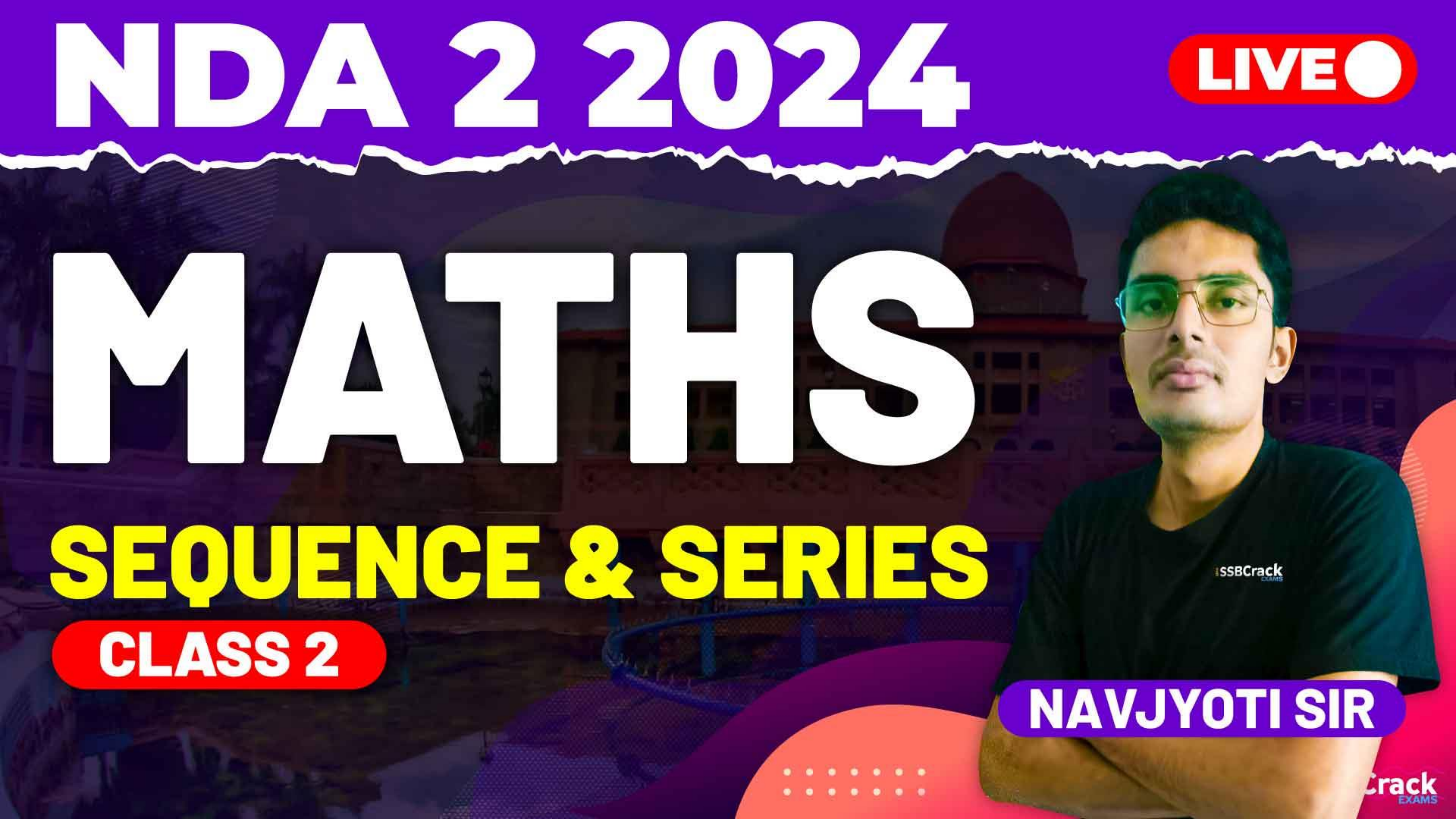
(c) $ab + bc \leq 2ac$

(d) $ab + bc \geq 2ac$

Ans: (d)

Summary

- Sequence
- Series and Progression
- Arithmetic Progression (AP)
- Arithmetic Mean (AM)
- Geometric Progression (GP)
- Geometric Mean (GM)
- Special Series
- Practise MCQs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