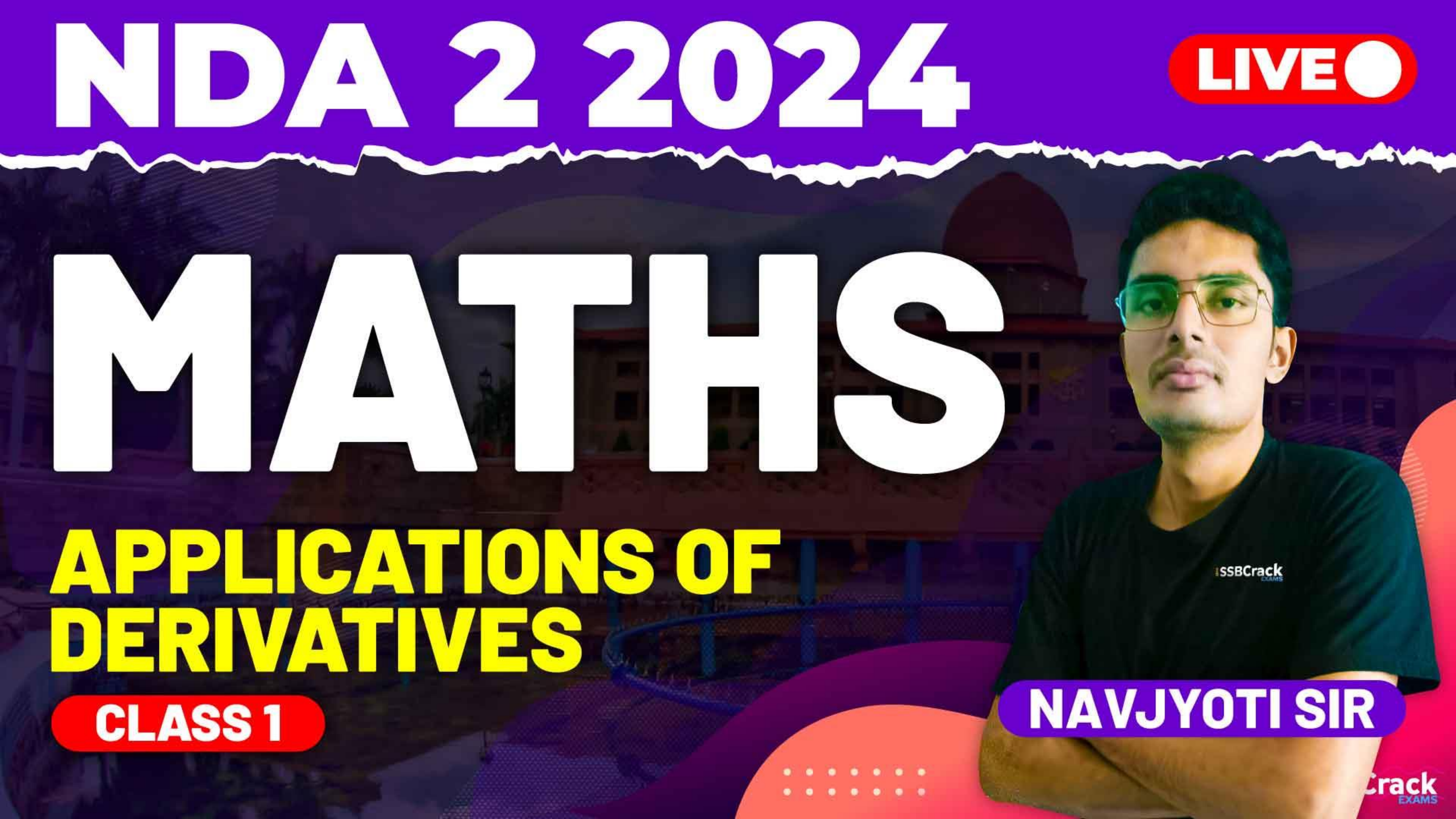

NDA 2 2024
LIVE
MATHS
APPLICATIONS OF DERIVATIVES
CLASS 1
SSBCrack EXAMS
NAVJYOTI SIR

25 June 2024 Live Classes Schedule

8:00AM	25 JUNE 2024 DAILY CURRENT AFFAIRS	RUBY MA'AM
9:00AM	25 JUNE 2024 DAILY DEFENCE UPDATES	DIVYANSHU SIR

SSB INTERVIEW LIVE CLASSES

9:30AM	MOCK PERSONAL INTERVIEW	ANURADHA MA'AM

AFCAT 2 2024 LIVE CLASSES

2:30PM	STATIC GK - INTERNATIONAL DAYS, SUMMITS & MEETINGS 2023-24	DIVYANSHU SIR
4:00PM	MATHS - PROBABILITY	NAVJYOTI SIR
5:30PM	ENGLISH - WORD SUBSTITUTION - CLASS 3	ANURADHA MA'AM

NDA 2 2024 LIVE CLASSES

11:30AM	GK - MEDIEVAL HISTORY - CLASS 1	RUBY MA'AM
2:30PM	GS - CHEMISTRY MCQS - CLASS 2	SHIVANGI MA'AM
6:30PM	MATHS - APPLICATIONS OF DERIVATIVES - CLASS 1	NAVJYOTI SIR

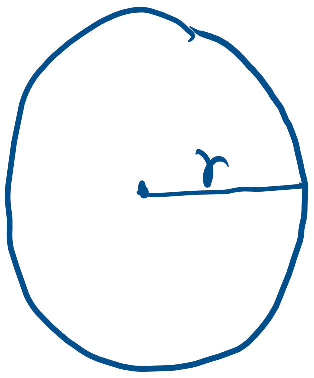
CDS 2 2024 LIVE CLASSES

11:30AM	GK - MEDIEVAL HISTORY - CLASS 1	RUBY MA'AM
2:30PM	GS - CHEMISTRY MCQS - CLASS 2	SHIVANGI MA'AM
4:00PM	MATHS - PROBABILITY	NAVJYOTI SIR

SSBCrack EXAMS

RATE OF CHANGE OF QUANTITIES

$\frac{dy}{dx}$ - rate of change of y w.r.t. x,
$=$



$\frac{d}{dt}(r) = \frac{dr}{dt}$ (rate of change of radius w.r.t. time)

MOTION IN STRAIGHT LINE

If x and v denotes the displacement and velocity of a particle at any instant t, then velocity and acceleration is given by

$$v = \frac{dx}{dt}$$

$$v = \frac{\text{change in displacement}}{\text{Time taken for change}} = \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

and
$$a = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d^2x}{dt^2}$$

Where, a is acceleration of particle. If the sign of acceleration is opposite to that of velocity, then the acceleration is called retardation which means decrease in magnitude of the velocity.

EXAMPLE

A particle moves in a straight line in such a way that its velocity at any point is given by $v^2 = 2 - 3x$, where x is measured from a fixed point. The acceleration is

(a) -4
(b) $-\frac{3}{2}$
(c) 3
(d) None of these

$$2v \frac{dv}{dt} = -3 \frac{dx}{dt}$$

$$a = \frac{-3}{2v}(v) = -\frac{3}{2}$$

EXAMPLE

A particle moves in a straight line in such a way that its velocity at any point is given by $v^2 = 2 - 3x$, where x is measured from a fixed point. The acceleration is

(a) -4 (b) $-\frac{3}{2}$

(c) 3 (d) None of these

Ans: (b)

EXAMPLE

A point moves in a straight line during the time $t=0$ to $t=3$ according to the laws $s=15t-2t^2$. The average velocity of the point is

(a) 4 (b) 9

(c) 3 (d) 2

(OR)

$v = \frac{ds}{dt} = 15-4t$

$v(\text{at } t=0) = 15$

$v(\text{at } t=3) = 3$

$\text{avg.} = \frac{15+3}{2}$

$= 9$

$$\text{avg. velocity} = \frac{\text{Total distance / displacement}}{\text{Total time}}$$

$$= \frac{(15t-2t^2)_{t=3} - (15t-2t^2)_{t=0}}{3-0} = \frac{(45-18)-0}{3} = \frac{27}{3} = 9$$

EXAMPLE

A point moves in a straight line during the time $t = 0$ to $t = 3$ according to the laws $s = 15t - 2t^2$. The average velocity of the point is

(a) 4 (b) 9

(c) 3 (d) 2

Ans: (b)

SSBCrack EXAMS

TANGENT AND NORMAL

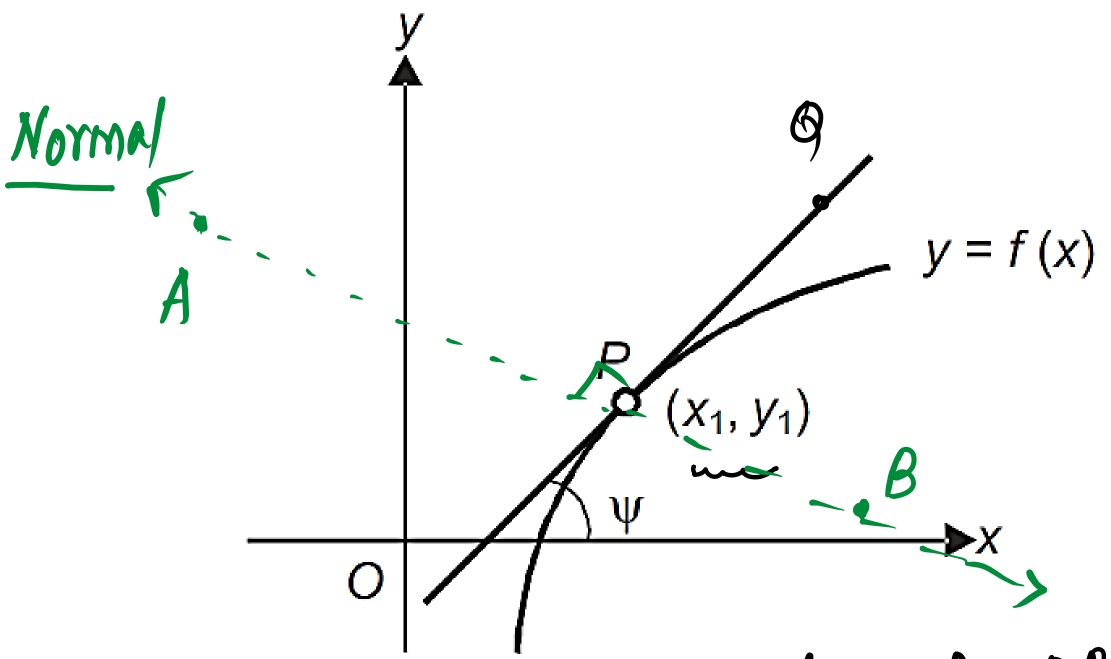
$$\text{Slope of tangent at } P = \left(\frac{dy}{dx}\right)_{(x_1,\, y_1)} = \tan\psi = m$$

Normal – line at 90° to tangent,

Slope (APB)

$$= \frac{-1}{m} = \frac{-1}{\left(\frac{dy}{dx}\right) \text{at } (x_1, y_1)}$$

$$= -\left(\frac{dx}{dy}\right)_{(x_1, y_1)}$$



OPQ is tangent.

slope of OPQ $= \left.\frac{dy}{dx}\right)_{x = x_1;\, y = y_1} = m$

TANGENT AND NORMAL

Equation of tangent at (x_1, y_1) is

$$(y - y_1) = \left(\frac{dy}{dx}\right)_{(x_1,\, y_1)} (x - x_1)$$

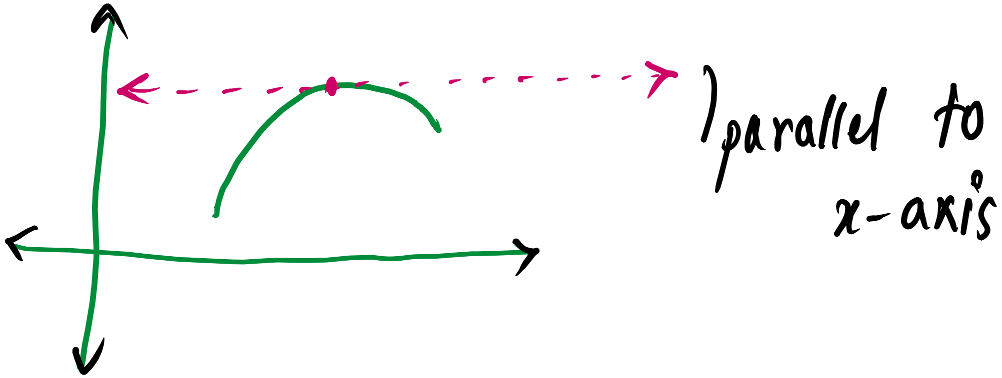
and equation of normal at (x_1, y_1) is

$$(y - y_1) = -\left(\frac{dx}{dy}\right)_{(x_1,\, y_1)} (x - x_1)$$

IMPORTANT RESULTS

If tangent is parallel to x-axis, then

$$\left(\frac{dy}{dx}\right)_{(x_1\,,\ y_1)} = 0$$



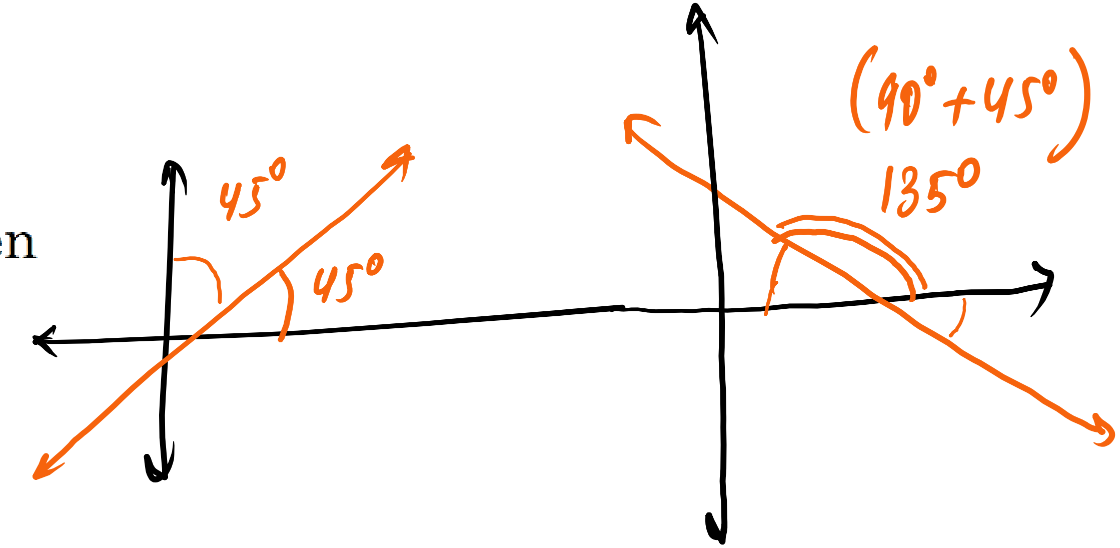
If tangent is parallel to y-axis or perpendicular to x-axis, then

$$\left(\frac{dx}{dy}\right)_{(x_1\,,\ y_1)} = 0$$

If the tangent is equally inclined to the axes, then

$$\frac{dy}{dx} = \tan 45° \text{ or } \tan 135°$$

$$= \pm 1$$



SSBCrack EXAMS

EXAMPLE

The point on the curve $3y = 6x - 5x^3$ in which the normal is passing through the origin is

(0,0)

(a) $\left(1, \frac{1}{3}\right)$ (b) $(2, 3)$

(c) $(1, 2)$ (d) $(-3, 3)$

eqn of normal,

$y - 0 = -\frac{1}{2}(x - 0)$

$2y = -x$

slope of normal $= -\frac{1}{\left(\frac{dy}{dx}\right)_{(0,0)}} = -\frac{1}{\frac{1}{3}(6-0)} = -\frac{1}{2}$

(a) $\left(1, \frac{1}{3}\right)$ satisfies,

curve $3y = 6x - 5x^3$

$3\frac{dy}{dx} = 6 - 15x^2 \Rightarrow \frac{dy}{dx} = \frac{1}{3}(6 - 15x^2)$

SSBCrack EXAMS

EXAMPLE

The point on the curve $3y = 6x - 5x^3$ in which the normal is passing through the origin is

(a) $\left(1, \frac{1}{3}\right)$

(b) $(2, 3)$

(c) $(1, 2)$

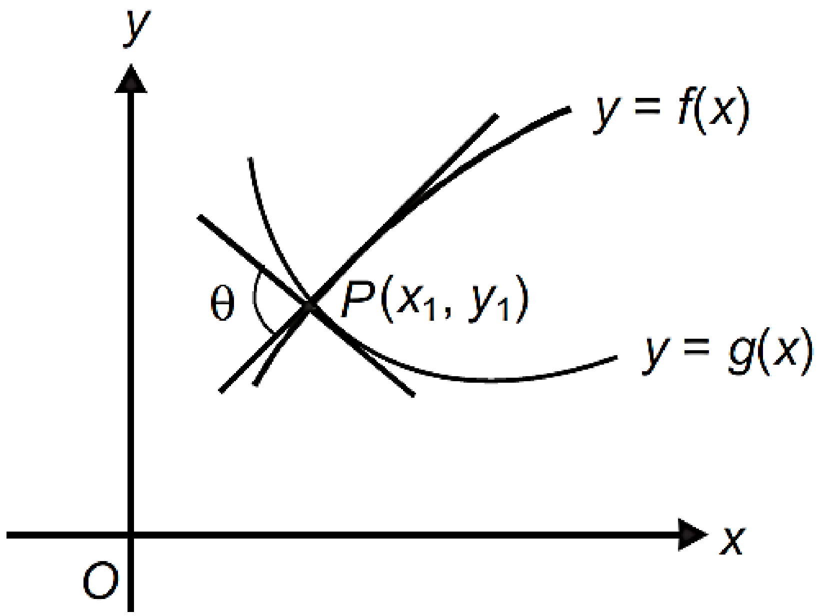
(d) $(-3, 3)$

Ans: (a)

ANGLE OF INTERSECTION

Angle of intersection is angle between tangents at intersection points of two curves.
Let they intersect at $P(x_1, y_1)$,
then

$$m_1 = \left[\frac{df(x)}{dx}\right]_{(x_1\,,\,y_1} \qquad m_2 = \left[\frac{dg(x)}{dx}\right]_{(x_1\,,\,y_1)} \qquad \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$



SSBCrack EXAMS

EXAMPLE

The angle of intersection of the curves $y = x^2$, $6y = 7 - x^3$ at point (1, 1) is

(a) π (b) $\frac{\pi}{2}$ (c) 2π (d) 4π

$y = x^2$

$\left.\frac{dy}{dx}\right|_{(1,1)} = 2 \times 1 = 2 = m_1$

$6y = 7 - x^3$

$6\frac{dy}{dx} = -3x^2 = \frac{1}{6}(-3) = \frac{-1}{2} = m_2$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{2 + 1/2}{1 + (-1)}\right| = \left|\frac{5/2}{0}\right|$

$\theta = 90^\circ \left(\frac{\pi}{2}\right)$

SSBCrack EXAMS

EXAMPLE

The angle of intersection of the curves $y = x^2, 6y = 7 - x^3$ at point (1, 1) is

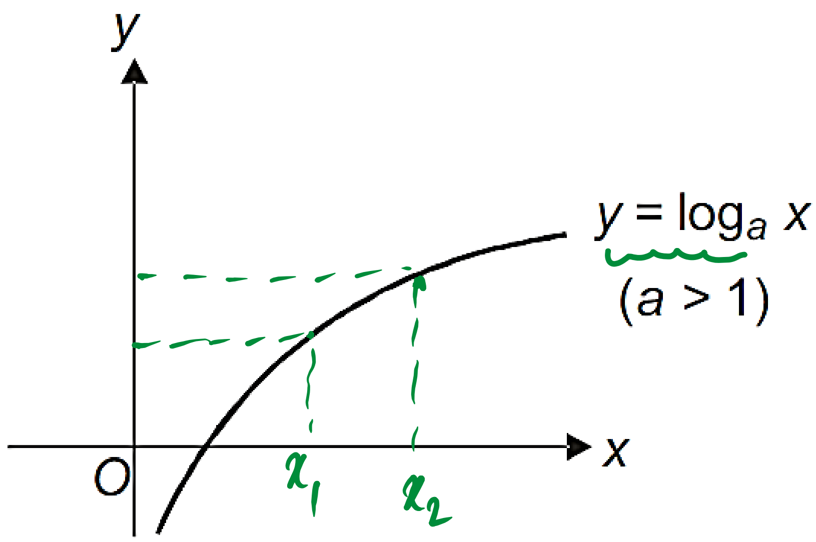
(a) π (b) $\frac{\pi}{2}$ (c) 2π (d) 4π

Ans: (b)

INCREASING FUNCTION

A function $y = f(x)$ is a increasing function, if $f(x)$ increases as x increases *i.e.*, $x_1 > x_2$

$\Rightarrow f(x_1) > f(x_2), (x_1, x_2) \in$ Domain of $f(x)$. Here, $\frac{dy}{dx} > 0$



$\frac{dy}{dx} = +ve$

EXAMPLE

The intervals in which the function $f(x) = 2x^2 - \log|x|,\ x \neq 0$ is increasing in

(a) $(-\infty, +\infty)$

(b) $\left(-\frac{1}{2}, 0\right) \cup \left(\frac{1}{2}, \infty\right)$

(c) $\left[-\frac{1}{2}, 1\right]$

(d) None of the above

$\log|x| = \log x$ (as log is defined only for +ve nos.)

$f'(x) = 4x - \frac{1}{x} = 0$

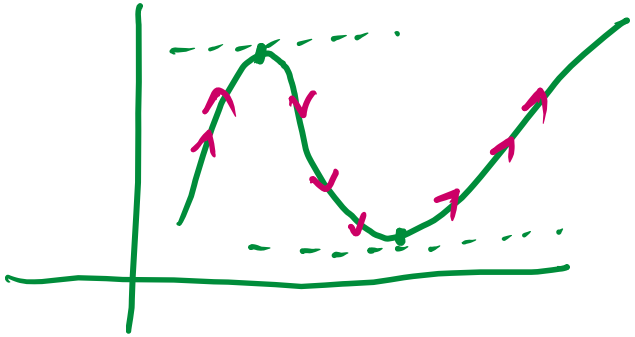
critical points

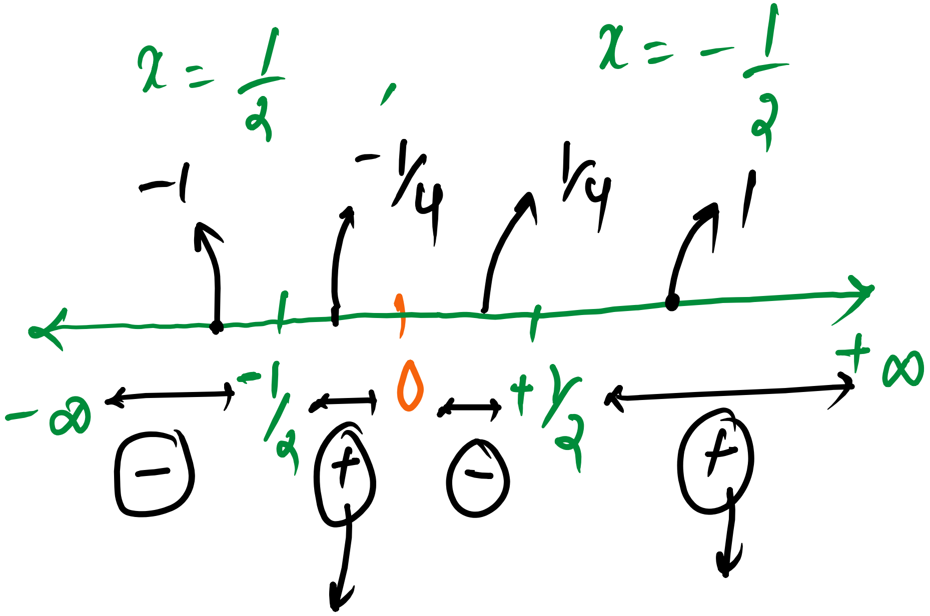
$4x - \frac{1}{x} = 0$

$4x^2 - 1 = 0$

$x^2 = \frac{1}{4} \Rightarrow x = \pm\frac{1}{2}$

$x = \frac{1}{2},\ x = -\frac{1}{2}$





$\left(4x - \frac{1}{x}\right)$

$-1, -\frac{1}{4}, \frac{1}{4}, 1$

$\frac{dy}{dx}$ (sign)

if $x = a$ is critical point,

→ $f'(a) = 0$, (or)/ and

→ $f'(a)$ not defined.

EXAMPLE

The intervals in which the function $f(x) = 2x^2 - \log|x|,\ x \neq 0$ is increasing in

(a) $(-\infty, +\infty)$

(b) $\left(-\frac{1}{2}, 0\right) \cup \left(\frac{1}{2}, \infty\right)$

(c) $\left[-\frac{1}{2}, 1\right]$

(d) None of the above

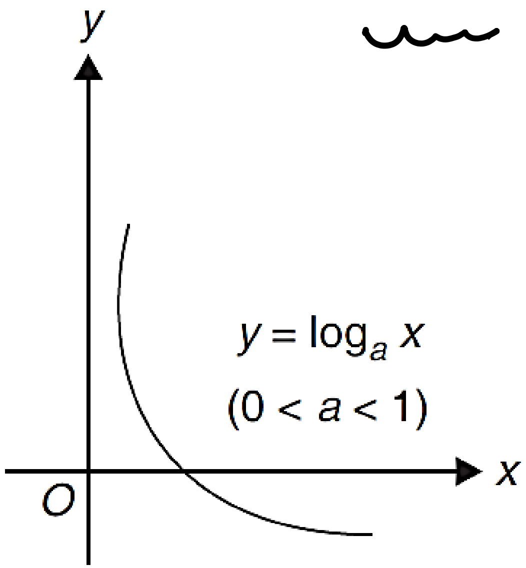
Ans: (b)

SSBCrack EXAMS

DECREASING FUNCTION

A function $y = f(x)$ is a decreasing function, if $f(x)$ decreases as x increases *i.e.*, $x_1 > x_2$

$\Rightarrow \quad f(x_1) < f(x_2)$. Here, $\frac{dy}{dx} < 0$



EXAMPLE

The interval in which the function $f(x) = (x + 2)\, e^{-x}$ is decreasing in

(a) $(-1, \infty)$ (b) $(-1, 1)$

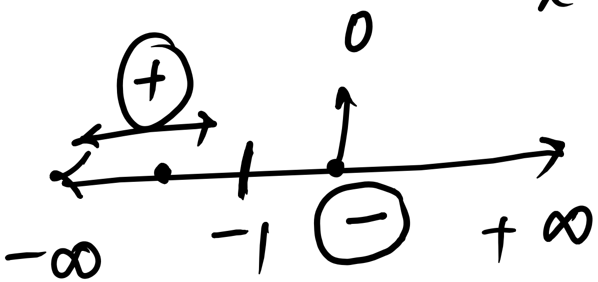
(c) $(-1, 2)$ (d) $(1, 2)$

$$f'(x) = (x+2)(-e^{-x}) + e^{-x}(1)$$

$$= -xe^{-x} - 2e^{-x} + e^{-x}$$

$$= -xe^{-x} - e^{-x}$$

$$= \underbrace{e^{-x}}(-x-1)$$



critical points

$$e^{-x}(-x-1) = 0$$

$$\underbrace{e^{-x} = 0} \qquad -x-1 = 0$$

$$\boxed{x = -1}$$

$$\frac{1}{e^x} = 0$$

$$x \approx \infty$$

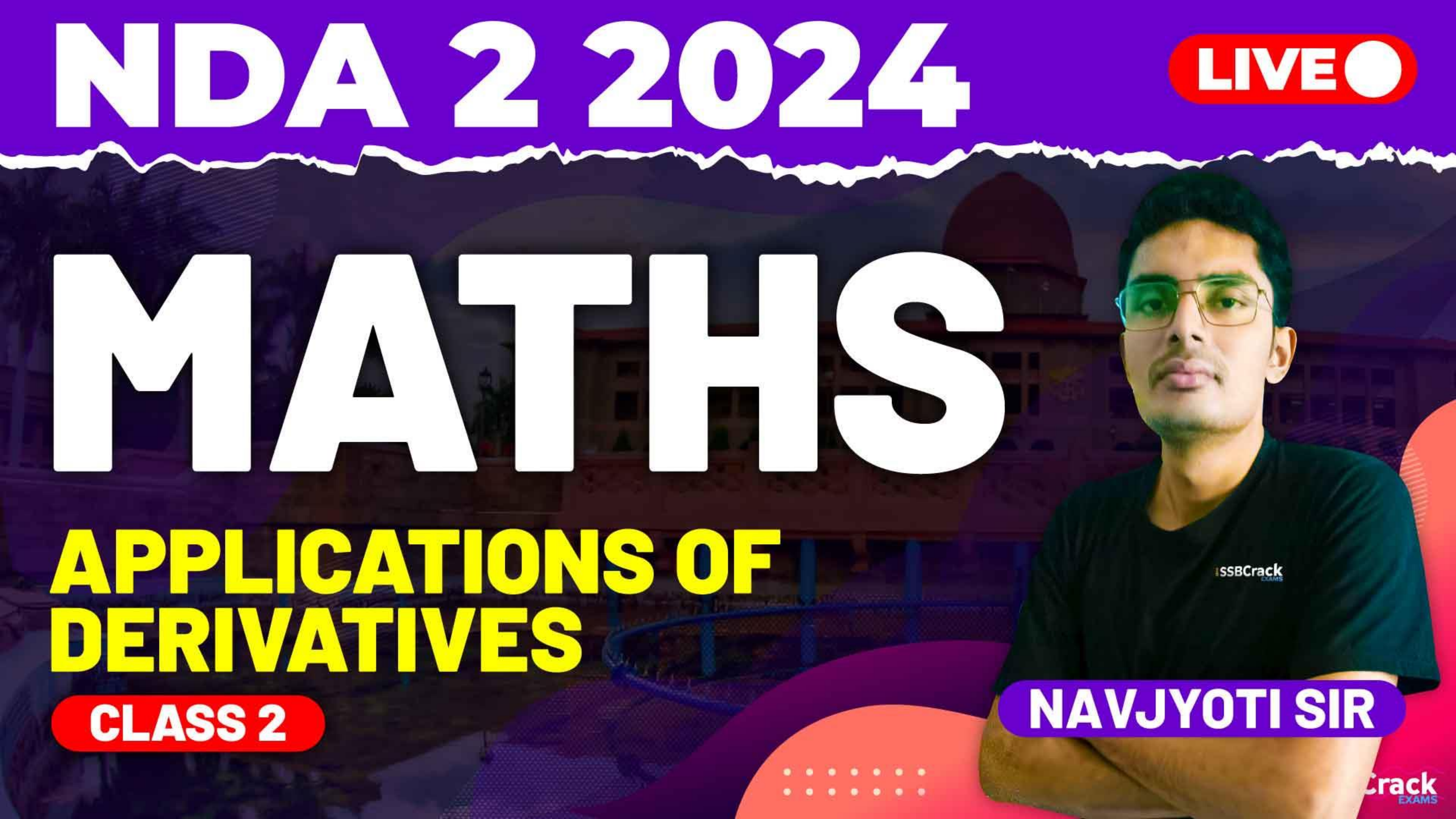
EXAMPLE

The interval in which the function $f(x) = (x + 2)\, e^{-x}$ is decreasing in

(a) $(-1, \infty)$ (b) $(-1, 1)$

(c) $(-1, 2)$ (d) $(1, 2)$

Ans: (a)


NDA 2 2024
LIVE
MATHS
APPLICATIONS OF DERIVATIVES
CLASS 2
NAVJYOTI SIR
SSBCrack EXAMS
Crack EXAMS