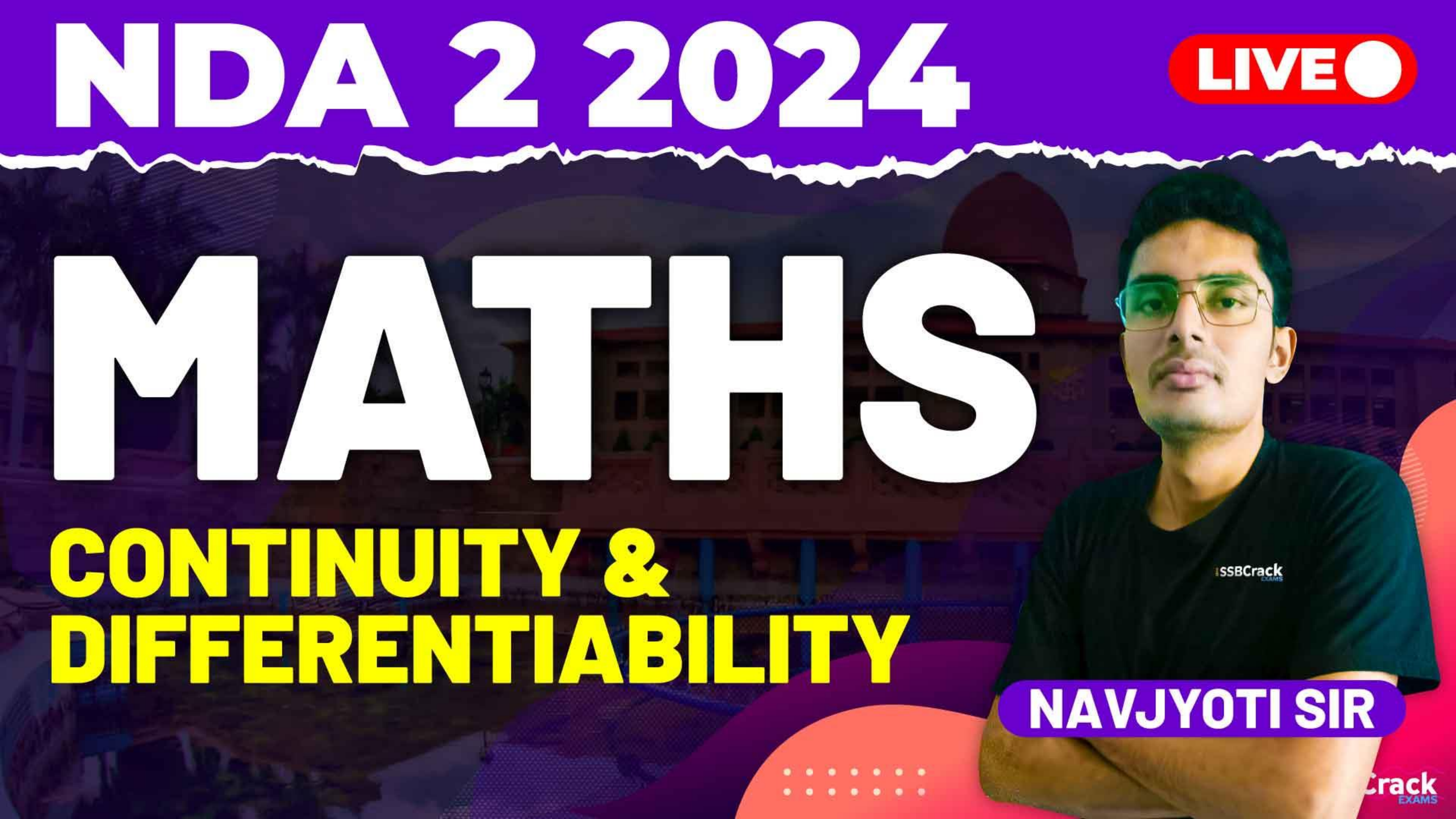

NDA 2 2024
LIVE
MATHS
CONTINUITY &
DIFFERENTIABILITY
NAVJYOTI SIR

20 June 2024 Live Classes Schedule

8:00AM	20 JUNE 2024 DAILY CURRENT AFFAIRS	RUBY MA'AM
9:00AM	20 JUNE 2024 DAILY DEFENCE UPDATES	DIVYANSHU SIR

SSB INTERVIEW LIVE CLASSES

9:00AM	COMPLETE SCREENING TEST	ANURADHA MA'AM

AFCAT 2 2024 LIVE CLASSES

2:30PM	STATIC GK - HISTORY - CLASS 2	DIVYANSHU SIR
4:00PM	MATHS - STATISTICS - CLASS 1	NAVJYOTI SIR
5:30PM	ENGLISH - COMPREHENSION - CLASS 2	ANURADHA MA'AM

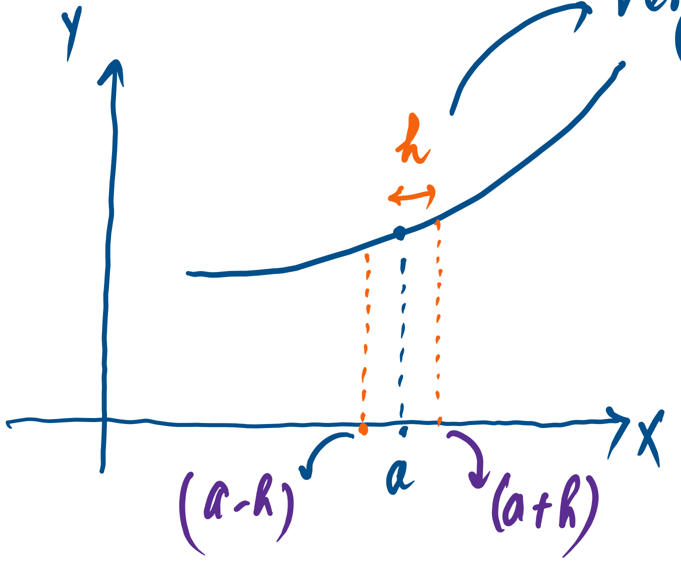
NDA 2 2024 LIVE CLASSES

11:30AM	GK - ANCIENT HISTORY - CLASS 1	RUBY MA'AM
5:30PM	ENGLISH - COMPREHENSION - CLASS 2	ANURADHA MA'AM
6:30PM	MATHS - CONTINUITY & DIFFERENTIABILITY	NAVJYOTI SIR

CDS 2 2024 LIVE CLASSES

11:30AM	GK - ANCIENT HISTORY - CLASS 1	RUBY MA'AM
4:00PM	MATHS - STATISTICS - CLASS 1	NAVJYOTI SIR
5:30PM	ENGLISH - COMPREHENSION - CLASS 2	ANURADHA MA'AM

LIMIT



very small positive quantity (very close to 0)

$f(a+h)$

$f(a-h)$

$x \to a \;,\; f(x) = ?$

$\text{Limit}_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) \Rightarrow \lim_{x \to a} f(x)$ exists.

$f(x) = x + 3$

$\lim_{x \to 3} f(x) = 6$

neighbourhood $\to (a-h, a+h)$

When $x \to 3$; $f(x) \to 6$

LIMIT

$$g(x) = \left(\frac{x^2 - 4}{x - 2}\right)$$

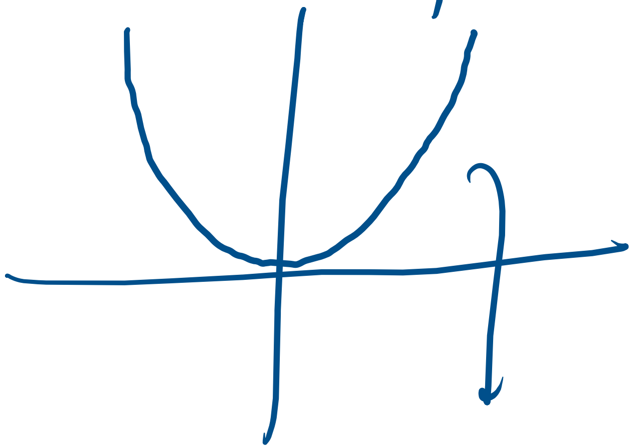
$$\lim_{x \to 2} g(x) = \frac{(x+2)(x-2)}{(x-2)} = x + 2 = 4$$

CONTINUITY

The word 'continuous' means without any break or gap. If the graph of a function has no break or gap or jump, then it is said to be contiuous.

$f(x) = x^2$

$f(x) = x^2$



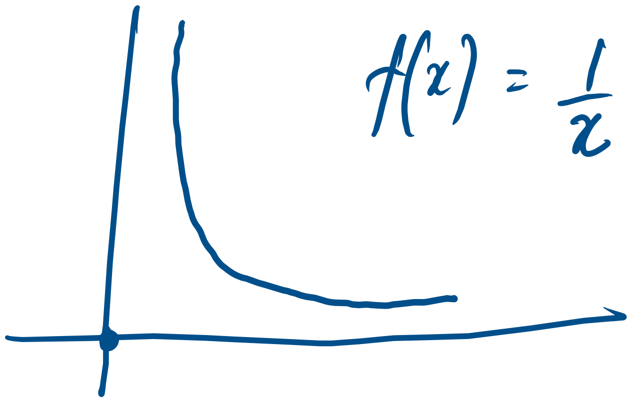
For all $x \in R$, $f(x)$ is defined.

$f(x)$ is a continous function,

& $x = 0$, is a point of continuity.

CONTINUITY

A function which is not continuous is called a discountiuous function. While studying graphs of functions, we see that graphs of functions $\sin x$, $x \cos x$, e^x etc., are continuous but reciprocal function $\frac{1}{x}$ has break at $x = 0$, so it is not contiuous. Similarly, $\tan x$, $\cot x$, $\sec x$ etc., are also discontinous function.



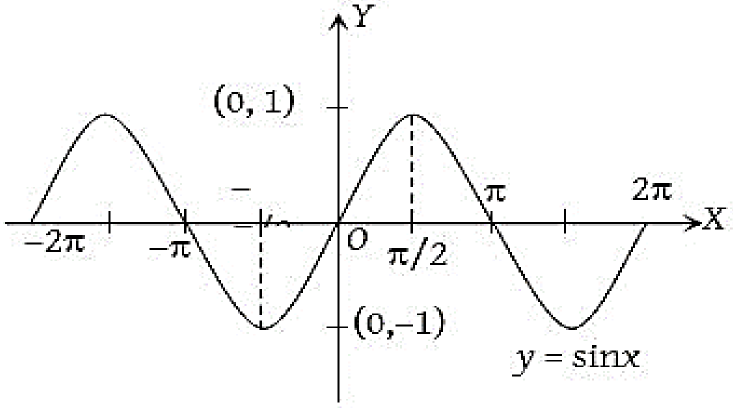
$f(x) = \frac{1}{x}$; at $x = 0$,

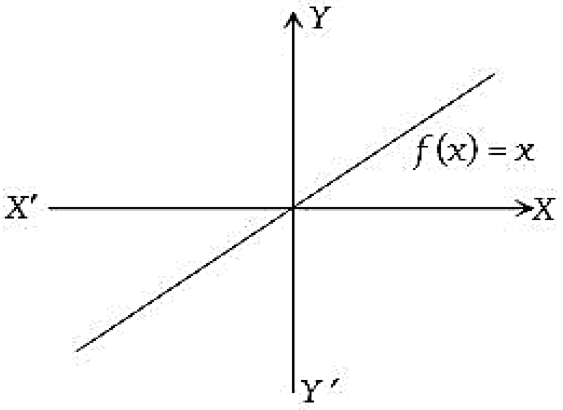
$f(x)$ is not defined.

So, $x = 0$ is a point of discontinuity

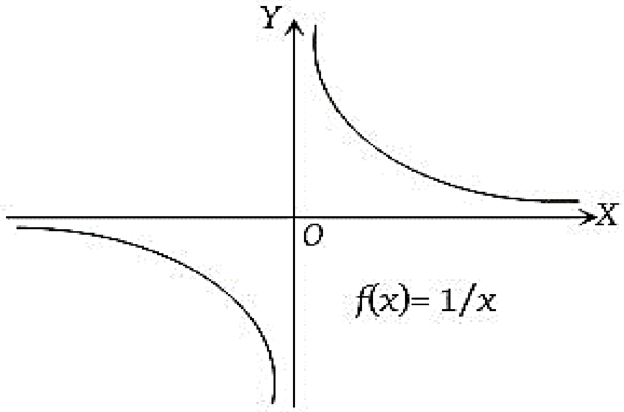
GRAPH OF CONTINOUS FUNCTION

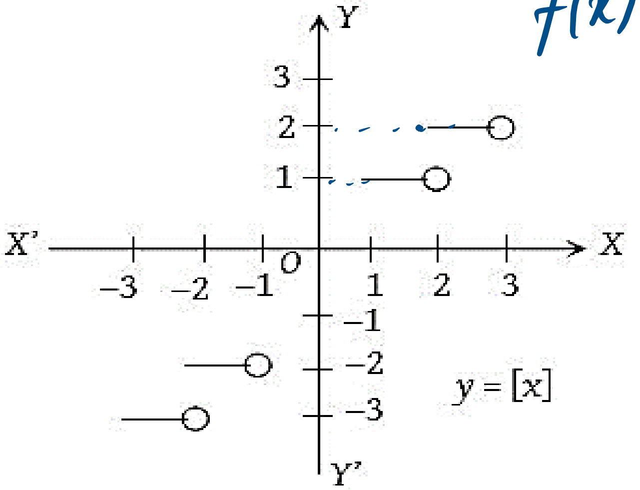
Continuous function





Discontinuous function





$f(x) = [x]$

3.4

3 4

3

$[3.4] = 3$

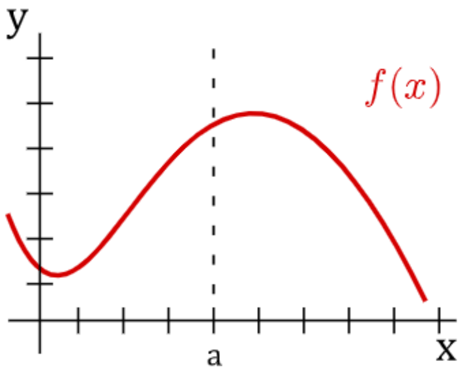
CONTINUITY AT A POINT

A function f is continuous at c if the following three conditions are met.

1. *f(c)* is defined. (a point exists)
2. Lim *f(x)* exists. (no gap or jump in the graph)
3. Lim *f(x)* = *f(c)*. (no hole in the graph)

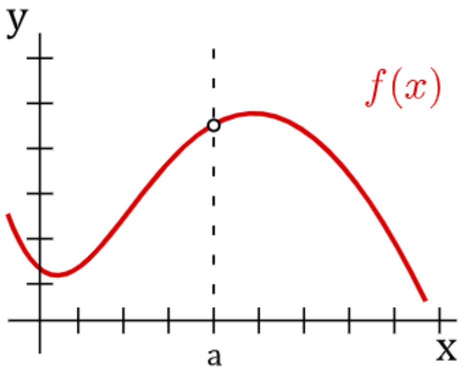
Continuity on an Open Interval:

A function is continuous on an open interval (a,b) if it is continuous at each point in the interval. A function that is continuous an the entire real line (- ∞ , ∞) is everywhere continuous.



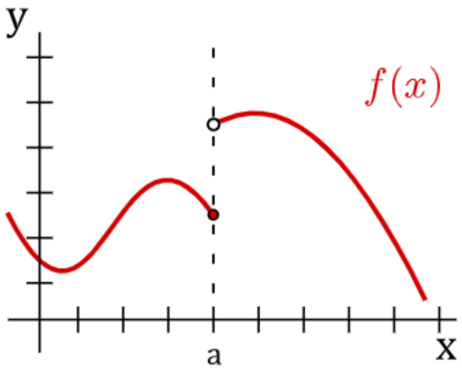
continuous at $x = a$

$\left(\lim_{x \to a} f(x) = f(a)\right)$



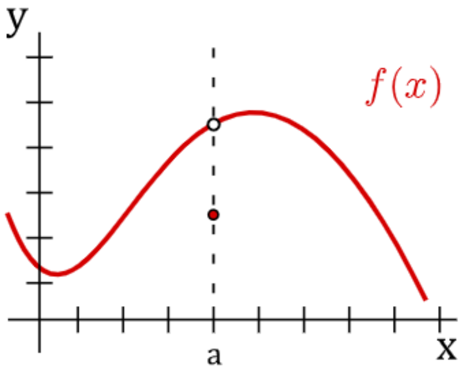
f(a) not defined

(i) fails to hold



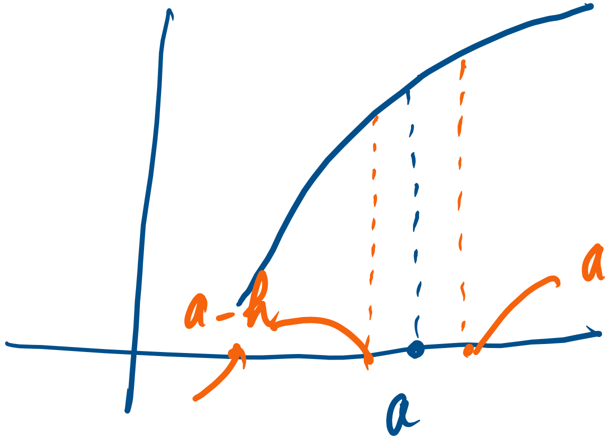
$\lim_{x \to a} f(x)$ does not exist

(ii) fails to hold



$\lim_{x \to a} f(x) \neq f(a)$

(iii) fails to hold



a+h (h is a positive & very small quantity)

$\{ \underline{h \sim 0.0001} \}$

$$(LHL) \underbrace{\lim_{x \to a^-} f(x)}_{} = \underbrace{\lim_{x \to a^+} f(x)}_{} \ (RHL) \Rightarrow \lim_{x \to a} f(x) \text{ exists}, = f(a)$$

$\rightarrow LHL \rightarrow \lim_{h \to 0} f(a-h)$

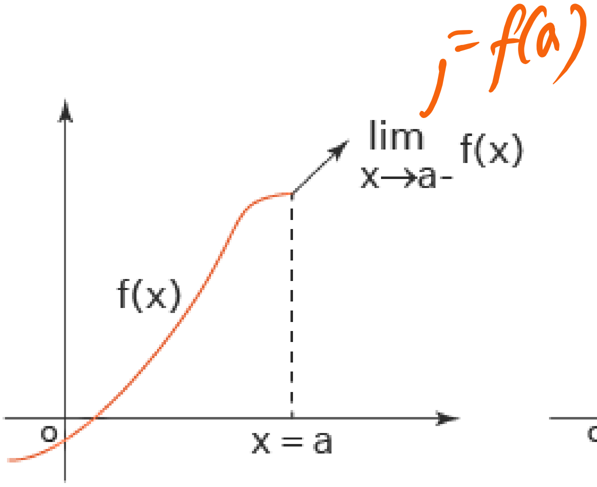
$\rightarrow RHL \rightarrow \lim_{h \to 0} f(a+h)$

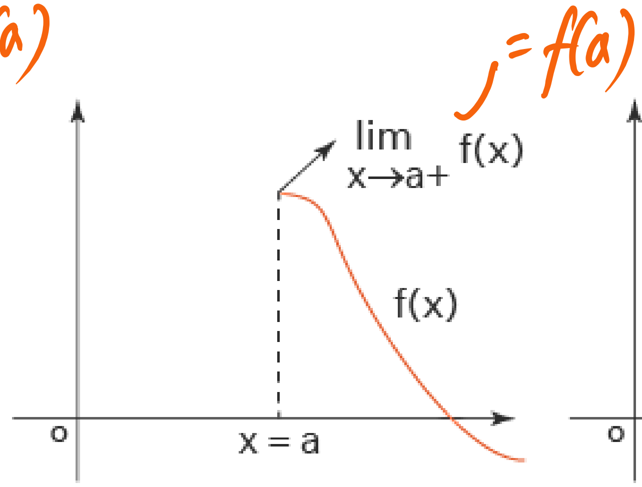
$\rightarrow f(a)$

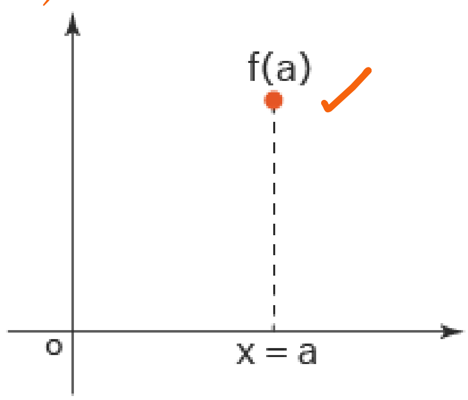
All three should be equal.

Then, $f(x)$ is continuous at $\underline{x = a}$.

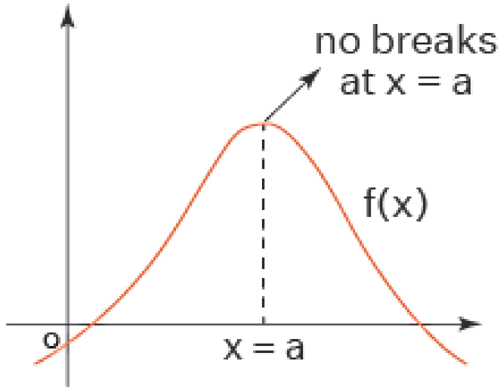
GRAPHICAL APPROACH TO DEFINITION







These three together will make the function f(x) continous at x = a



$\therefore \boxed{\lim_{x \to a} f(x) = f(a)} \Rightarrow$ f(x) is continuous at x = a

EXAMPLE

If $f(x) = \begin{cases} 2x+1, & x > 1 \\ k, & x = 1 \\ 5x-2, & x < 1 \end{cases}$, is continuous at $x = 1$, then the value of k is

(a) 1 (b) 2 (c) 3 (d) 4

RHL

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1} (2x+1) = 3$

$f(1) = k$

LHL

$\lim_{x \to 1^-} f(x) = \lim_{h \to 0} (5(1-h) - 2) = \lim_{h \to 0} 5 - 5h - 2 = 5 - 2 = 3$

$(x = 1-h)$

$\boxed{k = 3}$

EXAMPLE

If $f(x) = \begin{cases} 2x + 1, & x > 1 \\ k, & x = 1 \\ 5x - 2, & x < 1 \end{cases}$, is continuous at $x = 1$, then the value of k is

(a) 1 (b) 2 (c) 3 (d) 4

Ans: (c)

IMPORTANT PROPERTIES

If $y = f(x)$ and $y = g(x)$ are continuous functions at $x = a$, then functions $f(x) \begin{matrix} + \\ - \\ \times \\ \div \end{matrix} g(x)$ are also continuous at $x = a$, only in case of

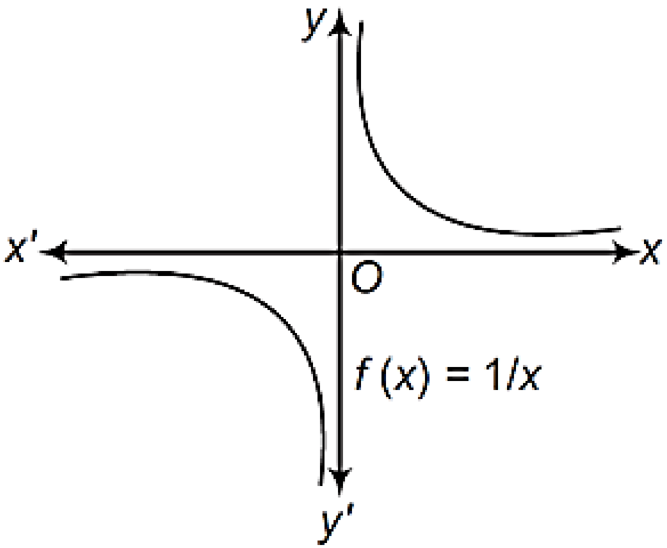
$f(x) \div g(x)$, $g(a) \neq 0$

If $y = f(x)$ and $y = g(x)$ are discontinuous functions at $x = a$, then

$f(x) \begin{matrix} + \\ - \\ \times \\ \div \end{matrix} g(x)$ may be continuous function at $x = a$

DISCONTINOUS FUNCTION

A function f which is not continuous at a point $x = a$ in its domain is said to be discontinuous. The point a is called a point of discontinuity of the function.



CONTINUITY OF COMPOSITE FUNCTION

If the function $u = f(x)$ is continuous at the point $x = a$ and the function $y = g(u)$ is continuous at the point $u = f(a)$, then the composite function $y = gof\ (x) = g\,(f(x))$ is continuous at the point $x = a$.

$u = f(x) \longrightarrow$ at $x = a$

$y = g(u) = g(f(x))$ at $u = f(a)$

gof is continous at $x = a$.

gof

fog

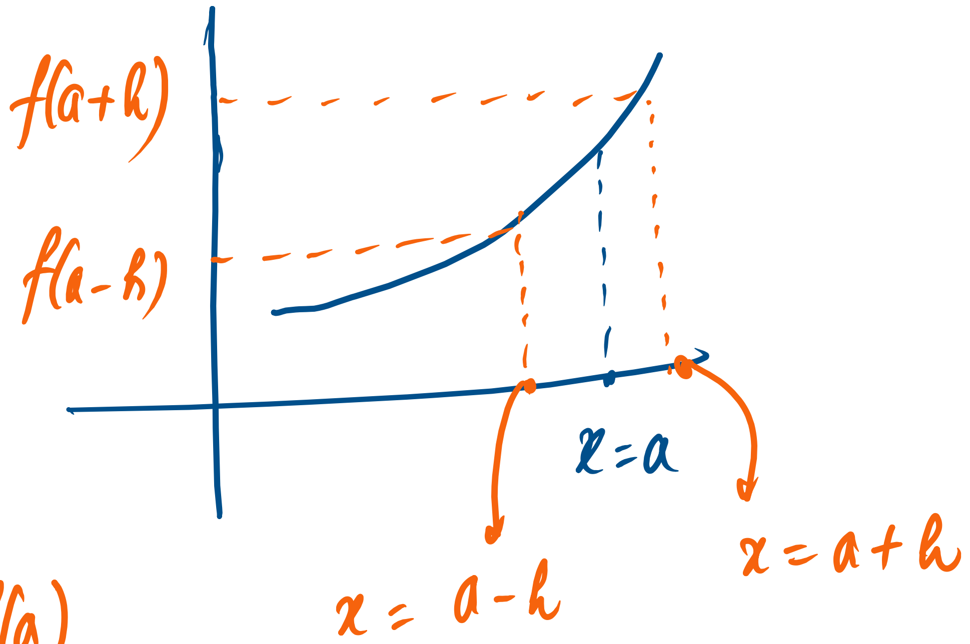
IMPORTANT RESULTS

All polynomials, logarithmic functions, exponential functions, trigonometric functions, modulus function are continuous in their domains. The greatest integer function is discontinuous at integers.

DIFFERENTIABILITY OF FUNCTION

Right hand derivative Right hand derivative of $f(x)$ at $x = a$, denoted by $f'(a+0)$ of $f'(a^+)$, is the $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$.

Left hand derivative Left hand derivative of $f(x)$ at $x = a$, denoted by $f'(a-0)$ or $f'(a^-)$, is the $\lim_{h \to 0} \frac{f(a-h) - f(a)}{-h}$.



$$\frac{\text{change in } y}{\text{change in } x} \quad \frac{f(a+h) - f(a)}{a+h-a} = \frac{f(a+h) - f(a)}{a+h-a}$$

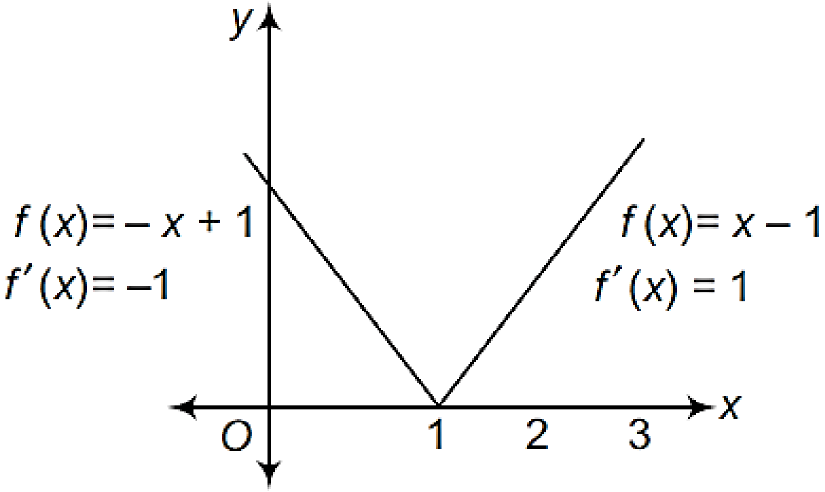
(RHD) (LHD)

As h is very small,

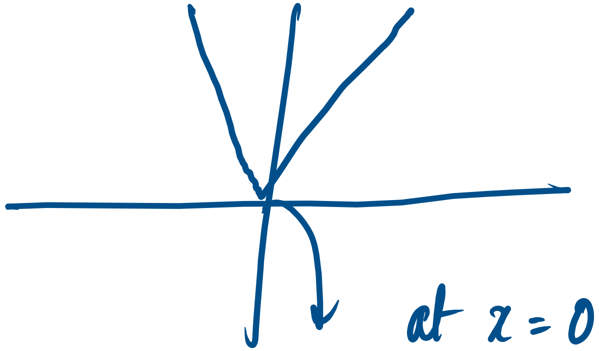
$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \qquad \lim_{h \to 0} \frac{f(a) - f(a-h)}{h}$$

DIFFERENTIABILITY OF FUNCTION

Let us consider the function $f(x) = |x - 1|$, which can be graphically shown,



Which shows $f(x)$ is not differentiable at $x = 1$. Since, $f(x)$ gas sharp edge at $x = 1$.



at $x = 0$,

$$|x| \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

(continous)

Q) If $f(x)=\begin{cases}\dfrac{1-\cos x}{x}, & x\neq 0\\ k, & x=0\end{cases}$ is continuous at $x=0$, then the value of k is

(a) 0

(b) $\frac{1}{2}$

(c) $\frac{1}{4}$

(d) $-\frac{1}{2}$

$$\left\{ k = \lim_{x\to 0} \frac{1-\cos x}{x} \right\}$$

Q) If $f(x)=\begin{cases}\dfrac{1-\cos x}{x}, & x\neq 0\\ k, & x=0\end{cases}$ is continuous at $x=0$, then the value of k is

(a) 0

(b) $\dfrac{1}{2}$

(c) $\dfrac{1}{4}$

(d) $-\dfrac{1}{2}$

Ans: (a)

Q) If the function

$$f(x) = \begin{cases} a + bx, & x < 1 \\ 5, & x = 1 \\ b - ax, & x > 1 \end{cases}$$

is continuous, then what is the value of $(a + b)$?

(a) 5 (b) 10

(c) 15 (d) 20

$$5 = \lim_{x \to 1^-} (a + bx) = \lim_{x \to 1^+} (b - ax)$$

$$\underbrace{a + b(1) = 5}$$

$$b - a(1) = 5$$

Q) If the function

$$f(x) = \begin{cases} a + bx, & x < 1 \\ 5, & x = 1 \\ b - ax, & x > 1 \end{cases}$$

is continuous, then what is the value of $(a + b)$?

(a) 5 (b) 10

(c) 15 (d) 20

Ans: (a)

Q) Let $f(x) = \begin{cases} 1 + \dfrac{x}{2k}, & 0 < x < 2 \\ kx, & 2 \leq x < 4 \end{cases}$

If $\lim_{x \to 2} f(x)$ exists, then what is the value of k?

(a) – 2 (b) – 1

(c) 0 (d) 1

LHL & RHL exists, at $x = a$,

$\lim_{x \to a} f(x)$ exists.

LHL

$\lim_{x \to 2^-} 1 + \frac{x}{2k} = 1 + \frac{1}{k}$

RHL

$\lim_{x \to 2^+} kx = 2k$

LHL (at $x=2$) = RHL (at $x=2$)

$1 + \frac{1}{k} = 2k$

$k + 1 = 2k^2$

$2k^2 - k - 1 = 0$

$$2k^2 - k - 1 = 0$$

$$k = \frac{1 \pm \sqrt{1+8}}{2 \times 2}$$

$$k = \frac{4}{4}, \text{ or } \frac{-2}{4} = \frac{-1}{2}$$

$$k = 1 \; ; \; k = \frac{-1}{2}$$

Q) Let $f(x) = \begin{cases} 1 + \dfrac{x}{2k}, & 0 < x < 2 \\ kx, & 2 \leq x < 4 \end{cases}$

If $\lim\limits_{x \to 2} f(x)$ exists, then what is the value of k?

(a) – 2 (b) – 1

(c) 0 (d) 1

Ans: (d)

$x = -2$ ✓

$x = 0$

LHL $= -1$

RHL $= 2$

discontinous

Q) Let $f(x)$ be defined as follows

$$f(x) = \begin{cases} 2x+1, & -3 < x < -2 \\ x-1, & -2 \le x < 0 \\ x+2, & 0 \le x < 1 \end{cases}$$

Which one of the following statements is correct in respect of the above function?

(a) It is discontinuous at $x = -2$ but continuous at every other point. ✗

(b) It is continuous only in the interval $(-3, -2)$. ✗

✓ (c) It is discontinuous at $x = 0$ but continuous at every other point.

✗

(d) It is discontinuous at every point. ✗

$x = -2$

continous

LHL $\rightarrow$ -3

RHL $\rightarrow$ -3

$f(-2) = -3$

Q) Let $f(x)$ be defined as follows

$$f(x) = \begin{cases} 2x+1, & -3 < x < -2 \\ x-1, & -2 \le x < 0 \\ x+2, & 0 \le x < 1 \end{cases}$$

Which one of the following statements is correct in respect of the above function?

(a) It is discontinuous at $x = -2$ but continuous at every other point.

(b) It is continuous only in the interval $(-3, -2)$.

(c) It is discontinuous at $x = 0$ but continuous at every other point.

(d) It is discontinuous at every point.

Ans: (c)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