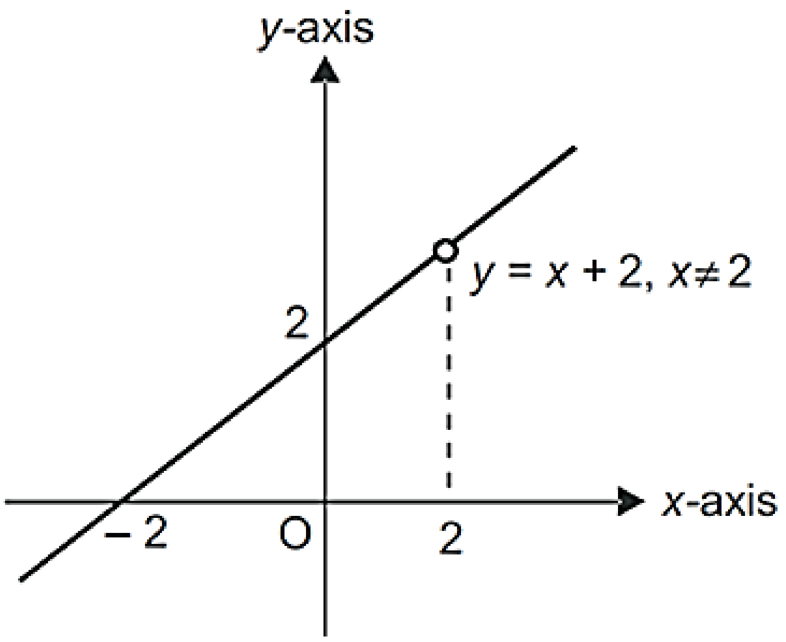
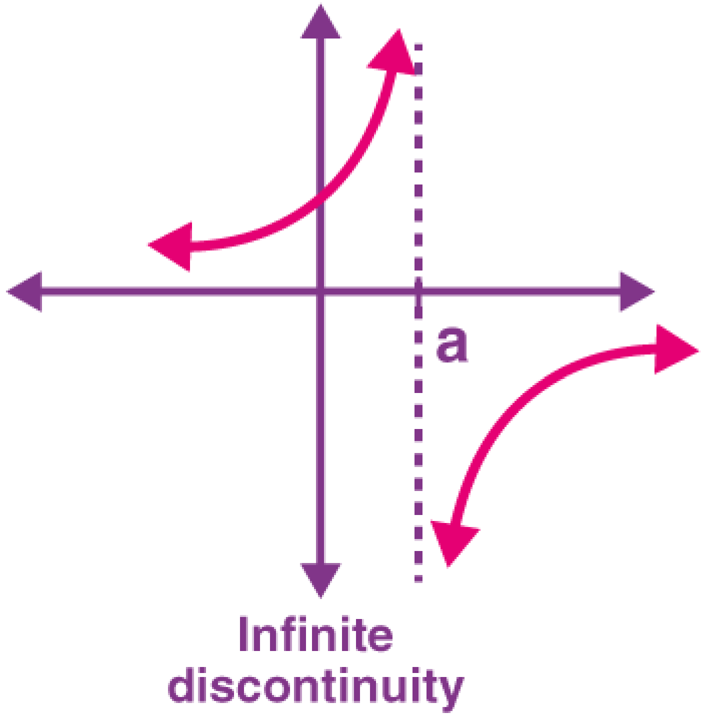

NDA 2 2024
LIVE
MATHS
LIMITS
SSBCrack EXAMS
NAVJYOTI SIR
Crack EXAMS

LIMIT AT A POINT

Limit of a function $y = f(x)$ at a point $x = a$ exists, if as x approaches a from left or from right, $y = f(x)$ approaches to same value.





EXAMPLE

$$f(x) = \frac{x^2 - 4}{x - 2}$$

(we have to check limit at $x = 2$)

$f(x) = x + 2$ (if $x \neq 2$)

- Function is not defined at $x = 2$.

Now, if x= 1.9, 1.999, 1.99999 — $f(x) = 3.9$, 3.999, 3.99999

and x= 2.1, 2.001, 2.0001 — $f(x) = 4.1$, 4.001, 4.0001

$\lim_{x \to 2} f(x) = 4$

$f(2)$ not defined.

INDETERMINATE FORM

An **indeterminate form** is an expression involving two functions whose limit cannot be determined solely from the limits of the individual functions.

If a function $f(x)$ takes any of the following forms at $x = a, \frac{0}{0}, \frac{\infty}{\infty}, \infty - \infty, 0 \times \infty, 0^0, \infty^0, 1^\infty$, then $f(x)$ is said to be indeterminate at $x = a$.

IMPORTANT RESULTS

① $\lim_{x \to a} \frac{\sin f(x)}{f(x)} = 1$ ✓

② $\lim_{x \to a} \cos f(x) = 1$ ✓

③ $\lim_{x \to a} \frac{\tan f(x)}{f(x)} = 1$ ✓

④ $\lim_{x \to a} (1 + f(x))^{1/f(x)} = e$ ✓

⑤ $\lim_{x \to a} \frac{e^{f(x)} - 1}{f(x)} = 1$ ✓

⑥ $\lim_{x \to a} \frac{b^{f(x)} - 1}{f(x)} = \log b \ (b > 0)$

⑦ $\lim_{x \to a} \frac{x^m - a^m}{x^n - a^n} = \frac{m}{n} a^{m-n}$ | $\frac{ma^{m-1}}{na^{n-1}}$

$f(a) = 0$ ✓

① $\lim_{h \to 0} \frac{\sin h}{h} = 1$

$\lim_{x \to a} \frac{x^m - a^m}{x - a} = ma^{m-1} = \frac{m}{n} a^{m-n}$

④ $(1 + (0))^{1/0} \to e$ $\quad (1)^{1/0}$

L'HOSPITAL RULE

Let $f(x)$ and $g(x)$ be two functions such that $f(a) = 0$ and $g(a) = 0$, then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

For other indeterminate forms we have to convert them into either $\frac{0}{0}$ or $\frac{\infty}{\infty}$ form and then we may apply L' Hospital's rule.

L'HOSPITAL RULE

Determinate-Indeterminate Forms Table

Indeterminate Forms	Determinate Forms
$0/0$	$\infty + \infty = \infty$
$\pm\infty/\pm\infty$	$-\infty - \infty = -\infty$
$\infty - \infty$	$0^{\infty} = 0$
$0(\infty)$	$0^{-\infty} = \infty$
0^0	$(\infty) \cdot (\infty) = \infty$
1^{∞}	
∞^0	
Use L'Hôpital's Rule	Do *Not* Use L'Hôpital's Rule

IMPORTANT EXPANSIONS

(xi) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots\ldots\ldots\ldots$

(xii) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots\ldots\ldots\ldots$

(xiii) $\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 + \frac{17}{315}x^7 \ldots\ldots\ldots$

(xiv) $\sin hx = x + \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots\ldots\ldots\ldots$

(xv) $\cos hx = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots\ldots\ldots\ldots$

(xvi) $\tan hx = x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17}{315}x^7 \ldots\ldots\ldots$

(xvii) $\sin^{-1} x = x + \frac{x^3}{3!} + \frac{9x^5}{5!} + \ldots\ldots\ldots$

(xviii) $\cos^{-1} x = \frac{\pi}{2} - \left(x + \frac{x^3}{3!} + \frac{9x^5}{5!} + \ldots\ldots\ldots \right)$

(xix) $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots\ldots\ldots$

Q)What is the value of $\lim_{x \to \infty} \frac{\sin x}{x}$?

(a) 1 (b) 0

(c) ∞ (d) -1

$x = \frac{1}{h}$

$x \to \infty \Rightarrow \frac{1}{h} \to 0$

$\lim_{\frac{1}{h} \to 0} \frac{\sin\left(\frac{1}{h}\right)}{\left(\frac{1}{h}\right)} = h \sin\left(\frac{1}{h}\right) \to 0$

$= 0$ ✓

$\lim_{x \to \infty} \frac{\sin x}{x} = 0$

Q) What is the value of $\lim\limits_{x \to \infty} \dfrac{\sin x}{x}$?

(a) 1
(b) 0
(c) ∞
(d) -1

Ans: (b)

Q) What is the value of $\lim_{x \to \infty} \left\{ x \sin\left(\frac{2}{x}\right) \right\}$?

(a) 2 (b) 1

(c) 1/2 (d) ∞

$$\lim_{x \to \infty} \frac{\sin\left(\frac{2}{x}\right) \times 2}{\frac{1}{x} \times 2} = \left(\frac{\sin\left(\frac{2}{x}\right)}{\left(\frac{2}{x}\right)} \right) \times 2$$

$x \to 0$

$2x \to 0$

$x^2 \to 0$

$$\lim_{\frac{1}{x} \to 0} \quad \lim_{\frac{2}{x} \to 0} \left(\frac{\sin\left(\frac{2}{x}\right)}{\left(\frac{2}{x}\right)} \right) \times 2 = 1 \times 2 = 2$$

Q) What is the value of $\lim_{x \to \infty} \left\{ x \sin\left(\frac{2}{x}\right) \right\}$?

(a) 2 (b) 1

(c) 1/2 (d) ∞

Ans: (a)

Q) What is the value of $\lim_{x \to 0} \frac{\cos(ax) - \cos(bx)}{x^2}$? $= \frac{f(x)}{g(x)}$

(a) $a - b$ (b) $a + b$

(c) $\frac{b^2 - a^2}{2}$ ✓ (d) $\frac{b^2 + a^2}{2}$

L-Hospital rule,

$$\frac{f'(x)}{g'(x)}$$

$$\lim_{x \to 0} \frac{-\sin(ax)a + \sin(bx)b}{2x}$$

$$\lim_{x \to 0} \frac{-a^2\cos(ax) + b^2\cos(bx)}{2} = \frac{-a^2 + b^2}{2}$$

SSBCrack EXAMS

Q) What is the value of $\lim_{x \to 0} \frac{\cos(ax) - \cos(bx)}{x^2}$?

(a) $a - b$

(b) $a + b$

(c) $\frac{b^2 - a^2}{2}$

(d) $\frac{b^2 + a^2}{2}$

Ans: (c)

Q) $\lim_{x \to \frac{\pi}{6}} \dfrac{2\sin^2 x + \sin x - 1}{2\sin^2 x - 3\sin x + 1}$ is equal to

(a) 3 (b) −3 (c) 6 (d) 0

$$\frac{2 \times \frac{1}{4} + \frac{1}{2} - 1}{2 \times \frac{1}{4} - \frac{3}{2} + 1}$$

$$\lim_{x \to \frac{\pi}{6}} \frac{4\sin x\cos x + \cos x}{4\sin x\cos x - 3\cos x}$$

$$= \frac{4 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}}{\frac{2\sqrt{3}}{2} - \frac{3\sqrt{3}}{2}} = \frac{\frac{3\sqrt{3}}{2}}{-\frac{\sqrt{3}}{2}} = -3$$

Q) $\lim\limits_{x \to \frac{\pi}{6}} \dfrac{2\sin^2 x + \sin x - 1}{2\sin^2 x - 3\sin x + 1}$ is equal to

(a) 3 (b) –3 (c) 6 (d) 0

Ans: (b)

Q) If $\lim_{x \to 1} \frac{x^4 - 1}{x - 1} = \lim_{x \to k} \frac{x^3 - k^3}{x^2 - k^2}$, then k is

(a) $\frac{4}{3}$ (b) $\frac{3}{8}$ (c) $\frac{3}{2}$ (d) $\frac{8}{3}$ ✓

$\left\{ \lim_{x \to a} \frac{x^n - a^n}{x - a} = n a^{n-1} \right.$

$\left\{ \lim_{x \to a} \frac{x^m - a^m}{x^n - a^n} = \frac{m}{n} a^{m-n} \right.$

$$\lim_{x \to 1} \frac{x^4 - 1^4}{x - 1} = \frac{\lim_{x \to k} \frac{x^3 - k^3}{x - k}}{\lim_{x \to k} \frac{x^2 - k^2}{x - k}}$$

$$4(1)^{4-1} = \frac{3k^2}{2k^1} \Rightarrow 4 = \frac{3k}{2} \Rightarrow k = \frac{8}{3} ✓$$

Q) If $\lim\limits_{x \to 1} \frac{x^4 - 1}{x - 1} = \lim\limits_{x \to k} \frac{x^3 - k^3}{x^2 - k^2}$, then k is

(a) $\frac{4}{3}$ (b) $\frac{3}{8}$ (c) $\frac{3}{2}$ (d) $\frac{8}{3}$

Ans: (d)

Q) If $\lim_{x \to 1} \dfrac{x^2 - ax + b}{x - 1} = 5$, then $a + b$ is equal to

(a) -4 (b) 1 (c) -7 (d) 5

$$5 = \lim_{x \to 1} \frac{2x - a}{1}$$

$$5 = \lim_{x \to 1} 2x - a$$

$$2 - a = 5$$

$$a = -3$$

$$\lim_{x \to 1} x^2 - ax + b = 5(x-1)$$

$$1 + 3 + b = 5x - 5$$

$$4 + b = 0$$

$$b = -4$$

Q) If $\lim_{x \to 1} \dfrac{x^2 - ax + b}{x - 1} = 5$, then $a + b$ is equal to

(a) -4 (b) 1 (c) -7 (d) 5

Ans: (c)

$$\lim_{x \to 0} \frac{x \cot(4x)}{\sin^2 x \cot^2(2x)} \text{ is equal to}$$

(a) 0 (b) 1 (c) 4 (d) 2

$$\lim_{x\to 0} \frac{x \tan^2 2x}{\tan 4x \sin^2 x}$$

$$\lim_{x\to 0} \frac{1}{4}\left(\frac{4x}{\tan 4x}\right) \frac{\tan^2 2x}{4x^2} \times \frac{4x^2}{\sin^2 x}$$

$$\lim_{x\to 0} \frac{1}{4}\left(\frac{4x}{\tan 4x}\right)\left(\frac{\tan 2x}{2x}\right)^2 \times 4 \times \left(\frac{x}{\sin x}\right)^2 = 1 \times 1^2 \times 1^2$$

$$= 1$$

$$\lim_{x\to 0} \frac{\tan x}{x} = 1$$

$$\lim_{x\to 0} \frac{x}{\tan x} = 1$$

$$\frac{1}{\frac{\tan x}{x}} = 1$$

$2x \to 0$

$4x \to 0$

What is the set of all points, where the function $f(x) = \frac{x}{1+|x|}$ is differentiable?

(a) $(-\infty, \infty)$ only ✓

(b) $(0, \infty)$ only

(c) $(-\infty, 0) \cup (0, \infty)$ only

(d) $(-\infty, 0)$ only

$$f(x) = \begin{cases} \frac{x}{1+x} ; & x \geq 0 \\ \frac{x}{1-x} ; & x < 0 \end{cases}$$

LHD

$$\lim_{h \to 0} \frac{f(0-h) - f(0)}{-h}$$

$$\lim_{h \to 0} \left(\frac{-h}{1+h} - 0\right) / -h = 1$$

RHD

$$\lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$\lim_{h \to 0} \left(\frac{h}{1+h} - 0\right) / h = 1$$

The function $f(x)$ is given by

$$f(x) = \begin{cases} x^2, & \text{if } x \text{ is rational} \\ -x^2, & \text{if } x \text{ is irrational} \end{cases}$$ then, it is

(a) continuous at $x = 0$

(b) continuous at $x = \frac{1}{2}$

(c) discontinuous at $x = 0$

(d) None of the above

If the function $f(x) = \dfrac{2x - \sin^{-1} x}{2x + \tan^{-1} x}, (x \neq 0)$ is continuous at each point of its domain, then the value of $f(0)$ is

(a) 2 (b) 1/3 (c) 2/3 (d) –1/3

$$\lim_{n\to\infty}\left(\frac{1}{1-n^2}+\frac{2}{1-n^2}+\ldots+\frac{n}{1-n^2}\right) \text{ is equal to}$$

(a) 0

(b) $-\frac{1}{2}$

(c) $\frac{1}{2}$

(d) None of these

If $f(x) = \begin{cases} \dfrac{3x + 4\tan x}{x}; x \neq 0 \\ k; \quad x = 0 \end{cases}$ is continuous at $x = 0$,

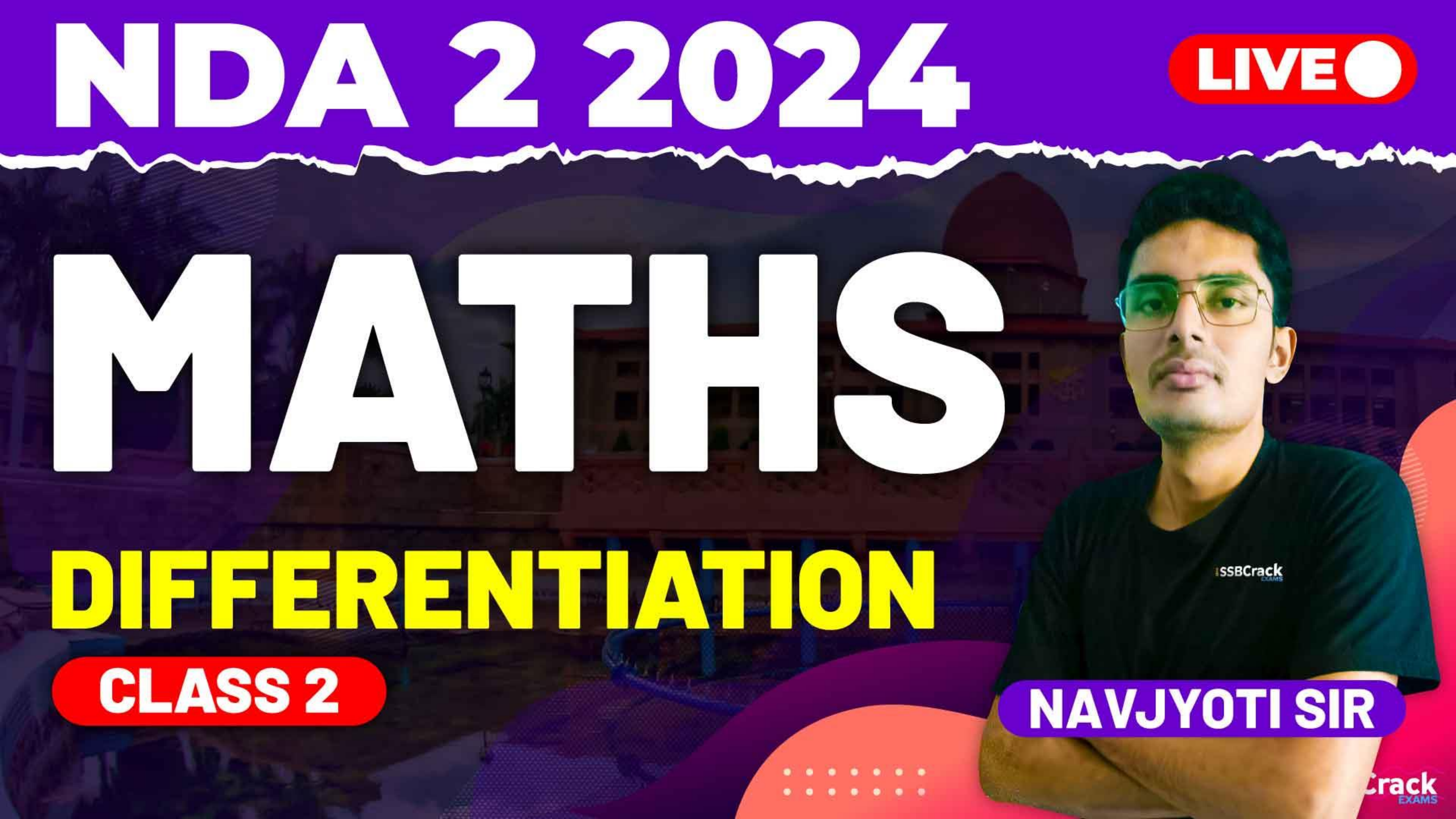
then the value of k is

(a) 7

(b) 6

(c) – 5

(d) – 1


NDA 2 2024
LIVE
MATHS
DIFFERENTIATION
CLASS 2
NAVJYOTI SIR