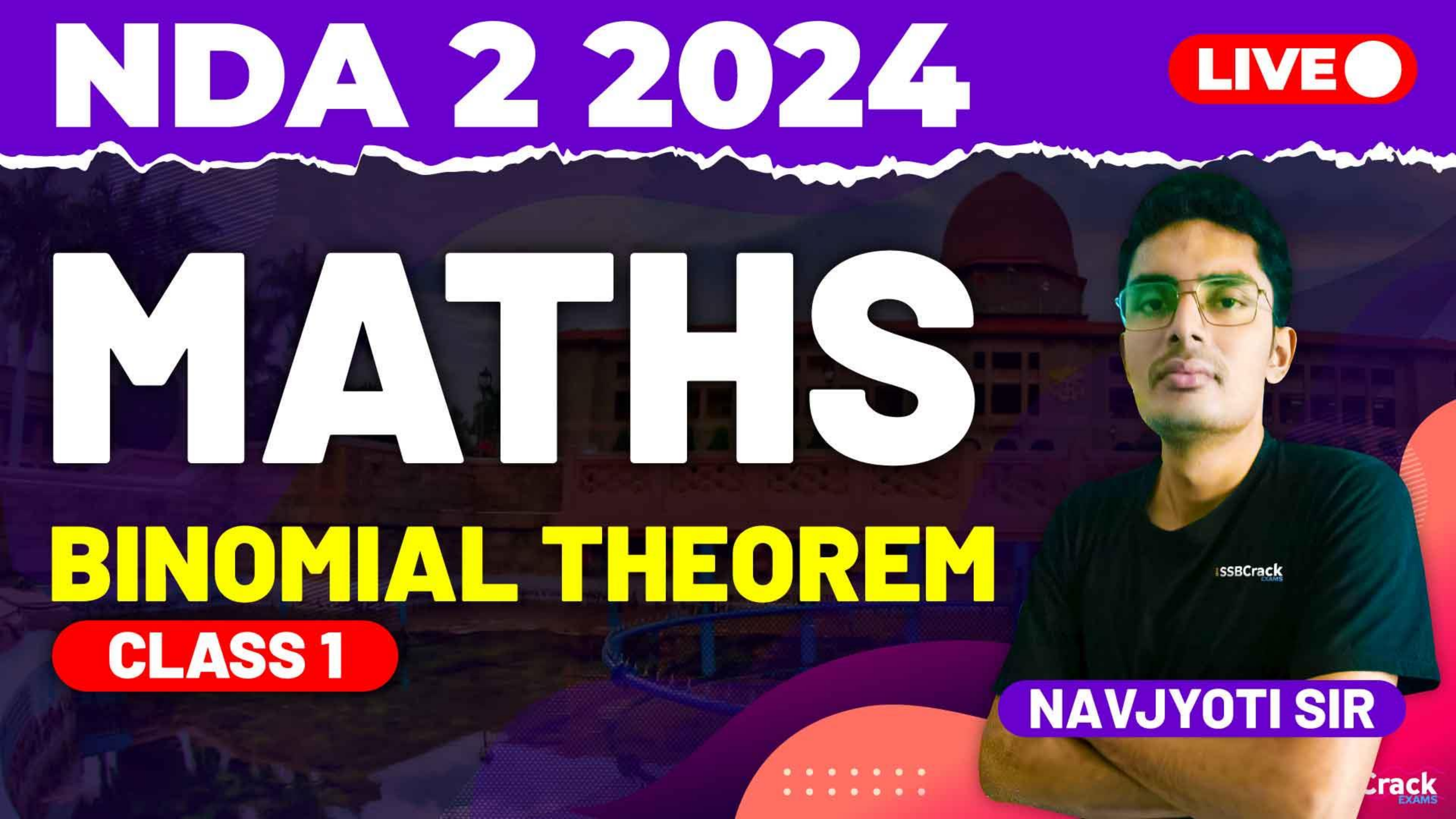

NDA 2 2024
LIVE
MATHS
BINOMIAL THEOREM
CLASS 1
NAVJYOTI SIR
SSBCrack EXAMS
Crack EXAMS

10 June 2024 Live Classes Schedule

8:00AM	10 JUNE 2024 DAILY CURRENT AFFAIRS	RUBY MA'AM

SSB INTERVIEW LIVE CLASSES

9:00AM	OVERVIEW OF TAT & WAT	ANURADHA MA'AM

AFCAT 2 2024 LIVE CLASSES

4:00PM	MATHS - ALGEBRA - CLASS 3	NAVJYOTI SIR
5:30PM	ENGLISH - FILL IN THE BLANKS - CLASS 1	ANURADHA MA'AM

NDA 2 2024 LIVE CLASSES

11:30AM	GK - BIOGEOGRAPHY	RUBY MA'AM
2:30PM	GS - CHEMISTRY - CLASS 1	SHIVANGI MA'AM
5:30PM	ENGLISH - FILL IN THE BLANKS - CLASS 1	ANURADHA MA'AM
6:30PM	MATHS - BINOMIAL THEOREM - CLASS 1	NAVJYOTI SIR

CDS 2 2024 LIVE CLASSES

11:30AM	GK - BIOGEOGRAPHY	RUBY MA'AM
2:30PM	GS - CHEMISTRY - CLASS 1	SHIVANGI MA'AM
4:00PM	MATHS - ALGEBRA - CLASS 3	NAVJYOTI SIR
5:30PM	ENGLISH - FILL IN THE BLANKS - CLASS 1	ANURADHA MA'AM

BINOMIAL THEOREM

$(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + {}^nC_3a^{n-3}b^3 + \ldots\ldots + {}^nC_nb^n$

i.e., $(a+b)^n = \sum_{r=0}^{n} {}^nC_r\, a^{n-r}b^r$; where

$${}^nC_r = \frac{n!}{r!(n-r)!},\ 0 \le r \le n.$$

The coefficients ${}^nC_r$ occuring in the binomial theorem are known as binomial coefficients.

$(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a+b)^n =$

$(a+b)^4$

$(a+b)^5$

$(a+b)^6$

$(a+b)^n = {}^nC_0\, a^{n-0}\, b^0 + {}^nC_1\, a^{n-1}b^1 + {}^nC_2\, a^{n-2}b^2 + \ldots {}^nC_n\, a^{n-n}b^n$

$= \sum_{k=0}^{n} {}^nC_k\, a^{n-k}\, b^k$

${}^nC_0 = 1 = {}^nC_n$

${}^nC_1 = {}^nC_{n-1} = n$

$$(a+b)^4 = {}^4C_0\, a^{4-0} b^0 + {}^4C_1\, a^{4-1} b^1 + {}^4C_2\, a^{4-2} b^2 + {}^4C_3\, a^{4-3} b^3$$

$$+ {}^4C_4\, a^{4-4} b^4$$

$$= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + a^0b^4$$

$$= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

$${}^4C_2 = \frac{4\times3}{2!} = \frac{\cancel{4}^{2}\times3}{\cancel{2}} = 6$$

1. sum of powers of a & b in each term is n.
2. no. of terms in expansion $=(n+1)$.

$(a+b)^2 = a^2 + 2ab + b^2 \rightarrow 3$

$(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2 \rightarrow 4$

${}^4C_0,\ {}^4C_1,\ {}^4C_2,\ {}^4C_3,\ {}^4C_4$ – binomial coefficients

PROPERTIES

(i) There are $(n+1)$ terms in the expansion of $(a+b)^n$

(ii) The sum of powers of a and b in each term of expansion is n.

(iii) The first and last term being a^n and b^n respectively.

(iv) The binomial coefficients in the binomial expansion equidistant from the beginning and the end are equal.

$(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

(first term $= a \rightarrow$ powers decrease

second term $= b \rightarrow$ power increase

$(a+b)^n = a^n + {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 + \ldots b^n$

coefficients

1 2 1 — $(a+b)^2$

1 3 3 1 — $(a+b)^3$

1 4 6 4 1 — $(a+b)^4$

GREATEST BINOMIAL COEFFICIENTS

In a binomial expansion, binomial coefficients of the middle terms are greatest binomial coefficients.

(i) If n is even : ${}^nC_{n/2}$ takes maximum value.

(ii) If n is odd : Both ${}^nC_{\frac{n-1}{2}}$ and ${}^nC_{\frac{n+1}{2}}$ take maximum value.

$\left\{ {}^nC_r = {}^nC_{n-r} \right\}$ ${}^nC_0 = {}^nC_n$

Index of Binomial	Coefficient of various terms
0	1
1	1 1
2	1 2 1
3	1 3 3 1
4	1 4 6 4 1
5	1 5 10 10 5 1

pascal triangle

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

${}^1C_0$ ${}^1C_1$

${}^2C_0$ ${}^2C_1$ ${}^2C_2$

${}^3C_0$ ${}^3C_1$ ${}^3C_2$ ${}^3C_3$

${}^4C_0$ ${}^4C_1$ ${}^4C_2$ ${}^4C_3$ ${}^4C_4$

GENERAL TERM

Let $(r+1)$th term be the general term in the expansion of $(x+a)^n$.

$$\therefore \qquad T_{r+1} = {}^nC_r \, x^{n-r} \, a^r$$

$$\underbrace{{}^nC_0 \, a^n b^0}_{T_1} \quad \underbrace{{}^nC_1 \, a^{n-1} b^1}_{T_2} \quad \underbrace{{}^nC_2 \, a^{n-2} b^2}_{T_3} \quad \underbrace{{}^nC_3 \, a^{n-3} b^3}_{T_4} \quad \cdots \quad \underbrace{{}^nC_n \, a^{n-n} b^n}_{T_{n+1}}$$

$(a+b)^n$ — total terms = $(n+1)$ terms

$$\boxed{T_{k+1} = {}^nC_k \, a^{n-k} \, b^k}$$

4th term in $(a+b)^n \rightarrow {}^nC_3 \, a^{n-3} \, b^3$

19th " " $\rightarrow {}^nC_{18} \, a^{n-18} \, b^{18}$

TERM FROM END

The p^{th} term from the end in the expansion of $(a+b)^n$ is $(n-p+2)^{\text{th}}$ term from the beginning.

pth term from start = $(n-p+2)^{th}$ term from end

4th term from start $= {}^nC_3\, a^{n-3}\, b^3$

4th term from end $= (n-4+2)^{th}$

$= (n-2)^{th}$ term from start

$= {}^nC_{n-2-1}$

$= {}^nC_{n-3}\, a^{n-(n-3)}\, b^{n-3} = {}^nC_{n-3}\, a^3\, b^{n-3}$

${}^nC_r = {}^nC_{n-r}$

$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

n

4

2nd term from start

$T_2 = {}^nC_1 a^{n-1} b^1 = {}^4C_1 a^{4-1} b^1 = 4a^3b$

2nd from end $\equiv (n-p+2)^{th}$ term from start

4 2

$= 4^{th}$ term

$=$

$T_4 = {}^nC_3 a^{n-3} b^3$

$= {}^4C_3 a^{4-3} b^3 = 4a^1b^3$

QUESTION

The approximate value of $(1.0002)^{3000}$ is

A. 1.6

B. 1.4

C. 1.8

D. 1.2

$$(1+x)^n = 1 + nx + {}^nC_2 x^2 + {}^nC_3 x^3 + {}^nC_4 x^4 + \cdots$$

$$(1+0.0002)^{3000}$$

$$= {}^{3000}C_0 (1)^{3000} (0.002)^0 + {}^{3000}C_1 (1)^{2999} (0.0002)^1 + {}^{3000}C_2 (1)^{2998} (0.0002)^2 + \cdots$$

$$= 1\cdot 1\cdot 1 + (3000)(0.0002) + \underbrace{\left(\frac{3000 \times 2999}{2!}\right) \times 1 \times (0.0002)^2 + \cdots}_{\text{(very small terms)}}$$

$$= 1 + 0.6 + \cdots$$

$$\approx (1.6)$$

QUESTION

The approximate value of $(1.0002)^{3000}$ is

A. 1.6

B. 1.4

C. 1.8

D. 1.2

$$(a-b)^n = (a+(-b))^n$$

$$= a^n + {}^nC_1 a^{n-1}(-b)^1 + {}^nC_2 a^{n-2}(-b)^2 + {}^nC_3 a^{n-3}(-b)^3 + \cdots$$

$$= a^n - n a^{n-1} b + {}^nC_2 a^{n-2} b^2 - {}^nC_3 a^{n-3} b^3 + \cdots$$

$$(-1)^n b^n$$

power of b $\begin{cases} \text{odd} - \text{-ve term} \\ \text{even} - \text{+ve term} \end{cases}$

$$(1-x)^n = 1 - nx + {}^nC_2 x^2 - {}^nC_3 x^3 + {}^nC_4 x^4 + \cdots$$

QUESTION

$(a+b)$

The middle term of $\left(2x - \dfrac{1}{3x}\right)^{10}$ is

(a) ${}^{10}C_4 \dfrac{2^4}{3^4}$ (b) $-{}^{10}C_5 \dfrac{2^5}{3^5}$

(c) $-{}^{10}C_4 \dfrac{2^4}{3^5}$ (d) ${}^{10}C_5 \dfrac{2^5}{3^5}$

$n = 10$

no. of terms $= \overbrace{(10+1)}^{(N)} = 11$ (odd) even

middle term $= \left(\dfrac{N+1}{2}\right)^{th}$ term

$= \left(\dfrac{11+1}{2}\right)^{th}$ term $= 6^{th}$ term

$T_6 = {}^{10}C_5 (2x)^{10-5} \left(-\dfrac{1}{3x}\right)^5$

$= -{}^{10}C_5 \, 2^5 \cdot \cancel{x^5} \cdot \dfrac{1}{3^5 \cdot \cancel{x^5}}$

$= -{}^{10}C_5 \dfrac{2^5}{3^5}$

a b c d e

odd

$\left(\dfrac{5+1}{2}\right)^{th}$ term

a b c d

4

$\dfrac{4}{2}$ $\dfrac{4}{2}+1$

2 3

$(a+b)^3$ — $N = 4$

$\left(\dfrac{N}{2}\right)^{th}$ term and $\left(\dfrac{N}{2}+1\right)^{th}$ term

QUESTION

The middle term of $\left(2x - \frac{1}{3x}\right)^{10}$ is

(a) ${}^{10}C_4 \frac{2^4}{3^4}$

(b) $-{}^{10}C_5 \frac{2^5}{3^5}$

(c) $-{}^{10}C_4 \frac{2^4}{3^5}$

(d) ${}^{10}C_5 \frac{2^5}{3^5}$

ANSWER : (b)

QUESTION

What are the values of k, if the term independent of x in the expansion of $\left(\sqrt{x}+\frac{k}{x^2}\right)^{10}$ is 405?

(a) ± 3 (b) ± 6 (c) ± 5 (d) ± 4

General term of $\left(\sqrt{x}+\frac{k}{x^2}\right)^{10}$

$$T_{r+1} = {}^{10}C_r (\sqrt{x})^{10-r}\left(\frac{k}{x^2}\right)^r$$

$$= {}^{10}C_r \; x^{\frac{1}{2}(10-r)} k^r (x^{-2})^r$$

$$= {}^{10}C_r \; k^r \; x^{5-\frac{r}{2}-2r}$$

power of x

$5-\frac{r}{2}-2r$

For independent term, power of $x = 0$

$$5-\frac{r}{2}-2r=0$$

$$5-\frac{5r}{2}=0$$

$$r=2$$

$$405 = {}^{10}C_2 k^2$$

$$k^2 = \frac{405}{{}^{10}C_2} = \frac{\cancel{405}^{\cancel{81}\,9} \times \cancel{2}}{\cancel{10}\times \cancel{9}_{5}}$$

$$k^2=9$$

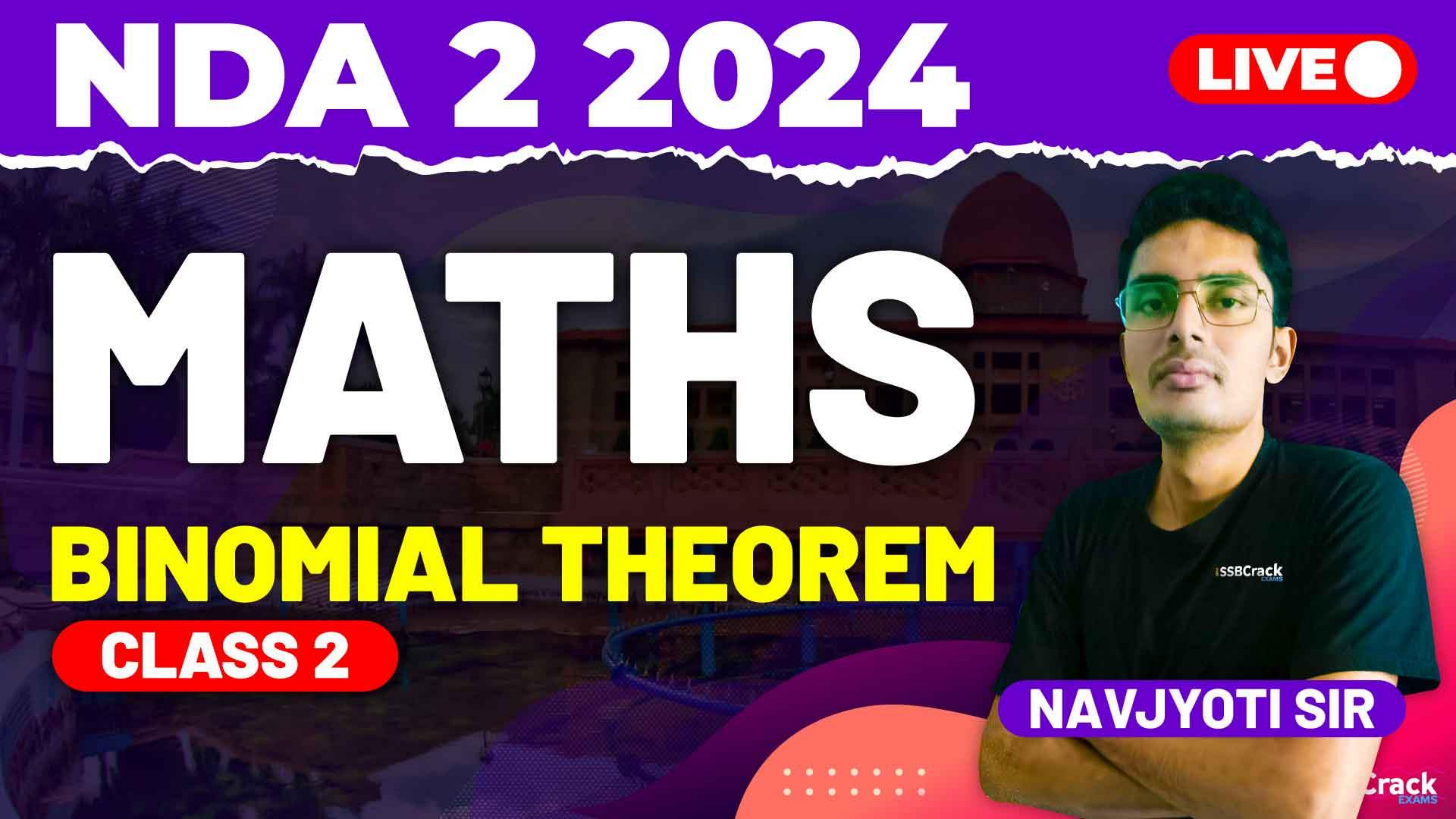
$$k=\pm 3$$

QUESTION

What are the values of k, if the term independent of x in the expansion of $\left(\sqrt{x} + \frac{k}{x^2}\right)^{10}$ is 405?

(a) ± 3 (b) ± 6 (c) ± 5 (d) ± 4

ANSWER : (a)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