NDA 2 2024

LIVE

MATHS

INDEFINITE & DEFINITE INTEGRATION

CLASS 1

NAVJYOTI SIR

27 June 2024 Live Classes Schedule

8:00AM	27 JUNE 2024 DAILY CURRENT AFFAIRS	RUBY MA'AM
9:00AM	27 JUNE 2024 DAILY DEFENCE UPDATES	DIVYANSHU SIR

SSB INTERVIEW LIVE CLASSES

9:30AM	MOCK PERSONAL INTERVIEW	ANURADHA MA'AM

AFCAT 2 2024 LIVE CLASSES

2:30PM	STATIC GK - IMPORTANT INTERNATIONAL GROUPS	DIVYANSHU SIR

NDA 2 2024 LIVE CLASSES

11:30AM	GK - MODERN HISTORY - CLASS 1	RUBY MA'AM
2:30PM	GS - CHEMISTRY MCQS - CLASS 4	SHIVANGI MA'AM
5:30PM	ENGLISH - ORDERING OF WORDS - CLASS 2	ANURADHA MA'AM
6:30PM	MATHS - INDEFINITE & DEFINITE INTEGRATION - CLASS 1	NAVJYOTI SIR

CDS 2 2024 LIVE CLASSES

11:30AM	GK - MODERN HISTORY - CLASS 1	RUBY MA'AM
2:30PM	GS - CHEMISTRY MCQS - CLASS 4	SHIVANGI MA'AM
5:30PM	ENGLISH - ORDERING OF WORDS - CLASS 2	ANURADHA MA'AM

INTRODUCTION TO INTEGRATION

$f(x) \longrightarrow f'(x)$

differentiation

integration

reverse of differentiation,

$\int f'(x)\, dx = f(x) + c$

(symbol)

function to be integrated

constant of integration

$\sin x \longrightarrow \cos x$

$\int \cos x\, dx = \sin x + C$

Indefinite

$\int f(x)\, dx$

$= F(x) + C$

Definite

$\int_0^{2\pi} \sin x\, dx$

$= [\cos x]_0^{2\pi}$

SSBCrack EXAMS

FORMULAE

$$\int e^x \, dx = e^x + C$$

$$\int \frac{dx}{x} = \log_e |x| + C$$

$$\int \frac{1}{x} dx = \log_e |x| + C$$

$$\int a^x \, dx = \frac{a^x}{\log_e a} + C$$

$$(a^x)' = \int a^x . \log a$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

FORMULAE

$$\int \operatorname{cosec}^2 x \, dx = -\cot x + C$$

$$\int \sec x \tan x \, dx = \sec x + C$$

$$\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + C$$

SSBCrack EXAMS

FORMULAE

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + c \quad \text{or} \quad -\cos^{-1}x + c$$

$$\int \frac{1}{x\sqrt{x^2-1}}\,dx = \sec^{-1}x + c \quad \text{or} \quad -\operatorname{cosec}^{-1}x + c$$

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1}x + c \quad \text{or} \quad -\cot^{-1}x + c$$

PROPERTIES

$$\frac{d}{dx} f(x) = f'(x) \text{ and } \int f'(x)dx = f(x) + C$$

$$\int \{f(x) \pm g(x)\}\, dx = \int f(x)dx \pm \int g(x)dx$$

$$\int (\sin x + x^3)\, dx = \int \sin x\, dx + \int x^3\, dx$$

$$\int k.f(x)dx = k.\int f(x)\, dx$$

$$\int 4\cos^2 x\, dx = 4\int \cos^2 x\, dx =$$

$$\int (k_1 f_1(x) + k_2 f_2(x) + \ldots +, k_n f_n(x))\, dx = k_1 \int f_1(x)dx + k_2 \int f_2(x)dx + \ldots + k_n \int f_n(x)dx$$

SSBCrack EXAMS

FORMULAE

$\log(x^m) = m\log x$

$$\int \tan x\, dx = -\log|\cos x| + C = \log(|\cos x|)^{-1}$$

$= \log|\sec x| + c$

diff. $\rightarrow \frac{1}{\sec x} \cdot (\sec x \tan x) = \tan x$

$$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C,$$

$(x^n)' \rightarrow nx^{n-1}$

$\int x^4 dx = \frac{x^5}{5} + c$

$x^4 \rightarrow 4x^3 \rightarrow \int x^3 dx = \frac{1}{4}(x^4) + c$ ($+1$)

$\int dx = \int 1 \cdot dx = \int x^0 \cdot dx = \frac{x^1}{1} + c \rightarrow x + c$

FORMULAE

SSBCrack EXAMS

$$\int \sec x \, dx = \log|\sec x + \tan x| + C$$

$$\frac{1}{\sec x + \tan x} \times \left(\sec x \tan x + \sec^2 x\right) = \sec x$$

$$\int \operatorname{cosec} x \, dx = \log|\operatorname{cosec} x - \cot x| + C$$

$$\int \cot x \, dx = \log|\sin x| + c = -\log|\operatorname{cosec} x| + c$$

FORMULAE

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C = -\cos^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$$

$$\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\frac{x}{a} + C$$

$1 \longrightarrow a^2$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}(x) + c$$

(a^2)

$$\left[\sin^{-1}\left(\frac{x}{a}\right)\right]' = \frac{1}{\sqrt{1 - \frac{x^2}{a^2}}} \times \frac{1}{a}$$

$$= \frac{a}{\sqrt{a^2 - x^2}} \times \frac{1}{a} = \frac{1}{\sqrt{a^2 - x^2}}$$

EXAMPLE

The value of $\int \frac{dx}{x^2 - x + 1}$ is

(a) $\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + C$

(b) $-\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + C$

(c) $\frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + C$

(d) None of the above

$$\int \frac{dx}{x^2 - 2 \times \frac{1}{2} \times x + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 1}$$

$$= \int \frac{dx}{\left(x - \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

of the form $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c$

$$= \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{x - \frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + c$$

$$= \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + c$$

EXAMPLE

The value of $\int \frac{dx}{x^2 - x + 1}$ is

(a) $\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + C$

(b) $-\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + C$

(c) $\frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + C$

(d) None of the above

Ans: (a)

SSBCrack EXAMS

FORMULAE

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log\left|\frac{x-a}{x+a}\right| + C, x > a$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + C, a > x$$

$$\int \frac{dx}{\sqrt{a^2 + x^2}} = \log|x + \sqrt{a^2 + x^2}| + C$$

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \log|x + \sqrt{x^2 - a^2}| + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c$$

FORMULAE

$$\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2} x \sqrt{a^2 - x^2} + \frac{1}{2} a^2 \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \sqrt{a^2 + x^2}\, dx = \frac{1}{2} x \sqrt{x^2 + a^2} + \frac{1}{2} a^2 \log\left| x + \sqrt{a^2 + x^2} \right| + C$$

↕ (similar structure)

$$\int \sqrt{x^2 - a^2}\, dx = \frac{1}{2} x \sqrt{x^2 - a^2} - \frac{1}{2} a^2 \log\left| x + \sqrt{x^2 - a^2} \right| + C$$

SUBSTITUTION METHOD

$$\int f(g(x)).\, g'(x)dx = \int f(t)dt \quad \text{(changing variable)}$$

$$\int \frac{1}{x}\cos(\log x)\, dx = \int \cos t\, dt$$

$\log x = t$

$$= \sin t + c$$

$\left\{ \frac{1}{x} = \frac{dt}{dx} \right\}$

$$= \sin(\log x) + c$$

$\left\{ \frac{1}{x}dx = dt \right\}$

SSBCrack EXAMS

EXAMPLE

The value of $\int \frac{f'(x)}{\sqrt{f(x)}} dx$ is

(a) $\frac{1}{2}\sqrt{f(x)} + C$

(b) $2\sqrt{f(x)} + C$

(c) $\frac{1}{2} f(x) + C$

(d) $2 f(x) + C$

Let $t = f(x)$

$dt = f'(x)\, dx$

$I = \int \frac{dt}{\sqrt{t}} = \int \frac{1}{\sqrt{t}} dt$

$= 2\sqrt{t} + c$

$= 2\sqrt{f(x)} + c$

$(\sqrt{t})' \rightarrow \frac{1}{2\sqrt{t}}$

$2\sqrt{t} \leftarrow \int \frac{1}{\sqrt{t}} dt$

$\int t^{-1/2} dt$

$= \frac{t^{-1/2+1}}{1/2} = 2\sqrt{t} + c$

SSBCrack EXAMS

EXAMPLE

The value of $\int \frac{f'(x)}{\sqrt{f(x)}} dx$ is

(a) $\frac{1}{2}\sqrt{f(x)} + C$

(b) $2\sqrt{f(x)} + C$

(c) $\frac{1}{2} f(x) + C$

(d) $2f(x) + C$

Ans: (b)

EXAMPLE

$\int \frac{x^2-1}{x^4+x^2+1}\,dx$ is equal to

(a) $\log(x^4+x^2+1)+C$

(b) $\log\frac{x^2-x+1}{x^2+x+1}+C$

(c) $\frac{1}{2}\log\frac{x^2-x+1}{x^2+x+1}+C$ ✓

(d) $\frac{1}{2}\log\frac{x^2+x+1}{x^2-x+1}+C$

(I) $\int \frac{x^2\left(1-\frac{1}{x^2}\right)}{x^2\left(x^2+1+\frac{1}{x^2}\right)}dx = \int \frac{1-\frac{1}{x^2}}{x^2+\frac{1}{x^2}+1}dx = \int \frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)^2-1}dx$

$x^2+\frac{1}{x^2}+1 \rightarrow \left(x+\frac{1}{x}\right)^2-2+1$

(II) Let $t = x+\frac{1}{x}$

$dt = 1-\frac{1}{x^2}$

$I = \int \frac{dt}{t^2-1}$

$\left\{ \int \frac{f'(t)\,dt}{f(t)} = \log f(t)+c \right\}$

(of form $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+c$

$= \frac{1}{2}\log\left|\frac{t-1}{t+1}\right|+c$

$= \frac{1}{2}\log\left(\frac{x^2+1-x\,/\,x}{x^2+1+x\,/\,x}\right)$

SSBCrack EXAMS

EXAMPLE

$\int \frac{x^2 - 1}{x^4 + x^2 + 1} dx$ is equal to

(a) $\log(x^4 + x^2 + 1) + C$

(b) $\log \frac{x^2 - x + 1}{x^2 + x + 1} + C$

(c) $\frac{1}{2} \log \frac{x^2 - x + 1}{x^2 + x + 1} + C$

(d) $\frac{1}{2} \log \frac{x^2 + x + 1}{x^2 - x + 1} + C$

Ans: (c)

INTEGRATION BY PARTS

SSBCrack EXAMS

$$\int \mathbf{1} \times \mathbf{2}\, dx = \mathbf{1} \int \mathbf{2}\, dx - \int [\frac{d}{dx}(\mathbf{1}) \int \mathbf{2}\, dx]\, dx$$

Here, **1** denotes first function and **2** denotes second function.

I L A T E

(Inverse trig. function) $\sin^{-1}x$, $\cos^{-1}x$

Logarithmic $(\log x)$

Algebraic $(x, x^2, x^3, \ldots)$

Trignometric $(\sin x, \cos x \ldots)$

Exponential (e^x)

$\int \sin^{-1}x \cdot x^3\, dx$ — (1) (2)

$\int \log x \cdot e^x\, dx$ — (1) (2)

EXAMPLE

The value of $\int x \log x \, dx$ is

(a) $\frac{x^2}{2}\log x + \frac{1}{4}x^2 + C$

(b) $\frac{x^2}{2}\log x - \frac{1}{4}x^2 + C$ ✓

(c) $\frac{x^2}{2}\log x + x^2 + C$

(d) None of the above

$\int x \log x \, dx$

Algeb. (A) (2) — Logarithmic (L) (1)

I, L, A, T, E

$1 \int 2\, dx - \int \left[\frac{d}{dx}(1) \int 2\, dx \right] dx$

$= \log x \int x\, dx - \int \left[\frac{1}{x} \int x\, dx \right] dx$

$= \frac{x^2}{2}\log x - \int \frac{1}{x} \times \frac{x^2}{2} dx = \frac{x^2}{2}\log x - \frac{1}{2}\int x\, dx$

$\frac{x^2}{2}\log x - \frac{x^2}{4} + C$

EXAMPLE

The value of $\int x \log x \, dx$ is

(a) $\frac{x^2}{2}\log x + \frac{1}{4}x^2 + C$

(b) $\frac{x^2}{2}\log x - \frac{1}{4}x^2 + C$

(c) $\frac{x^2}{2}\log x + x^2 + C$

(d) None of the above

Ans: (b)

EXPONENTIAL TYPE

- In this Lecture, we evaluate integral of form

$$\int e^x[f(x) + f'(x)]$$

- In such form of integral, we need to remember direct result

$$\int e^x[f(x) + f'(x)] = e^x f(x) + C$$

$$\int e^x(\sin x + \cos x)dx = e^x \sin x + c$$

SSBCrack EXAMS

EXAMPLE

The value of $\int \frac{e^x (1+\sin x)}{(1+\cos x)} dx$ is

(a) $e^x \tan\frac{x}{2}+C$ ✓

(b) $e^x \tan x+C$

(c) $2e^x \tan\frac{x}{2}+C$

(d) None of these

$$\int e^x \left[\frac{1}{1+\cos x} + \frac{\sin x}{1+\cos x}\right] dx$$

$$e^x \int [f(x) + g(x)] dx$$

$$f(x): \quad \frac{1}{1+\cos x} = \frac{1}{2\cos^2 x/2} = \frac{1}{2}\sec^2\frac{x}{2}$$

$$g(x): \quad \frac{\sin x}{1+\cos x} = \frac{2\sin x/2 \cos x/2}{2\cos^2 x/2} = \tan\frac{x}{2}$$

$$\int e^x \left(\frac{1}{2}\sec^2\frac{x}{2} + \tan\frac{x}{2}\right) dx$$

($p'(x)$, $p(x)$)

$$= e^x \tan\frac{x}{2} + c$$

SSBCrack EXAMS

EXAMPLE

The value of $\int \frac{e^x (1+\sin x)}{(1+\cos x)} dx$ is

(a) $e^x \tan \frac{x}{2} + C$

(b) $e^x \tan x + C$

(c) $2e^x \tan \frac{x}{2} + C$

(d) None of these

Ans: (a)

EXAMPLE

The value of $\int \frac{dx}{1+2\sin x+\cos x}$ is

(a) $\log(2+4\tan\frac{x}{2})+C$

(b) $\frac{1}{2}\log(2+4\tan\frac{x}{2})+C$

(c) $\frac{1}{2}\log(2-4\tan\frac{x}{2})+C$

(d) None of the above

$$\int \frac{dx}{1+2\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)+\left(2\cos^2\frac{x}{2}-1\right)}$$

$$I=\int \frac{dx}{4\sin\frac{x}{2}\cos\frac{x}{2}+2\cos^2\frac{x}{2}}$$

$$=\int \frac{dx}{2\cos^2\frac{x}{2}\left(\frac{2\sin x/2}{\cos x/2}+1\right)} = \frac{1}{2}\int \frac{\sec^2 x/2 \, dx}{\left(1+2\tan\frac{x}{2}\right)}$$

$$=\int \frac{\frac{1}{2}\sec^2 x/2}{1+2\tan\frac{x}{2}}dx$$

Let $1+2\tan\frac{x}{2}=t$

$\left(0+2\times\frac{1}{2}\sec^2\frac{x}{2}dx\right)=dt$

$\sec^2\frac{x}{2}dx=dt$

$$=\int\frac{dt}{2(t)} = \frac{1}{2}\int\frac{1}{t}dt$$

NDA 2 2024 LIVE CLASS - MATHS - PART 1

$$\frac{1}{2}\int \frac{1}{t}\,dt = \frac{1}{2}\log t + c$$

$$= \frac{1}{2}\log\left|1 + 2\tan\frac{x}{2}\right| + c$$

EXAMPLE

The value of $\int \frac{dx}{1 + 2\sin x + \cos x}$ is

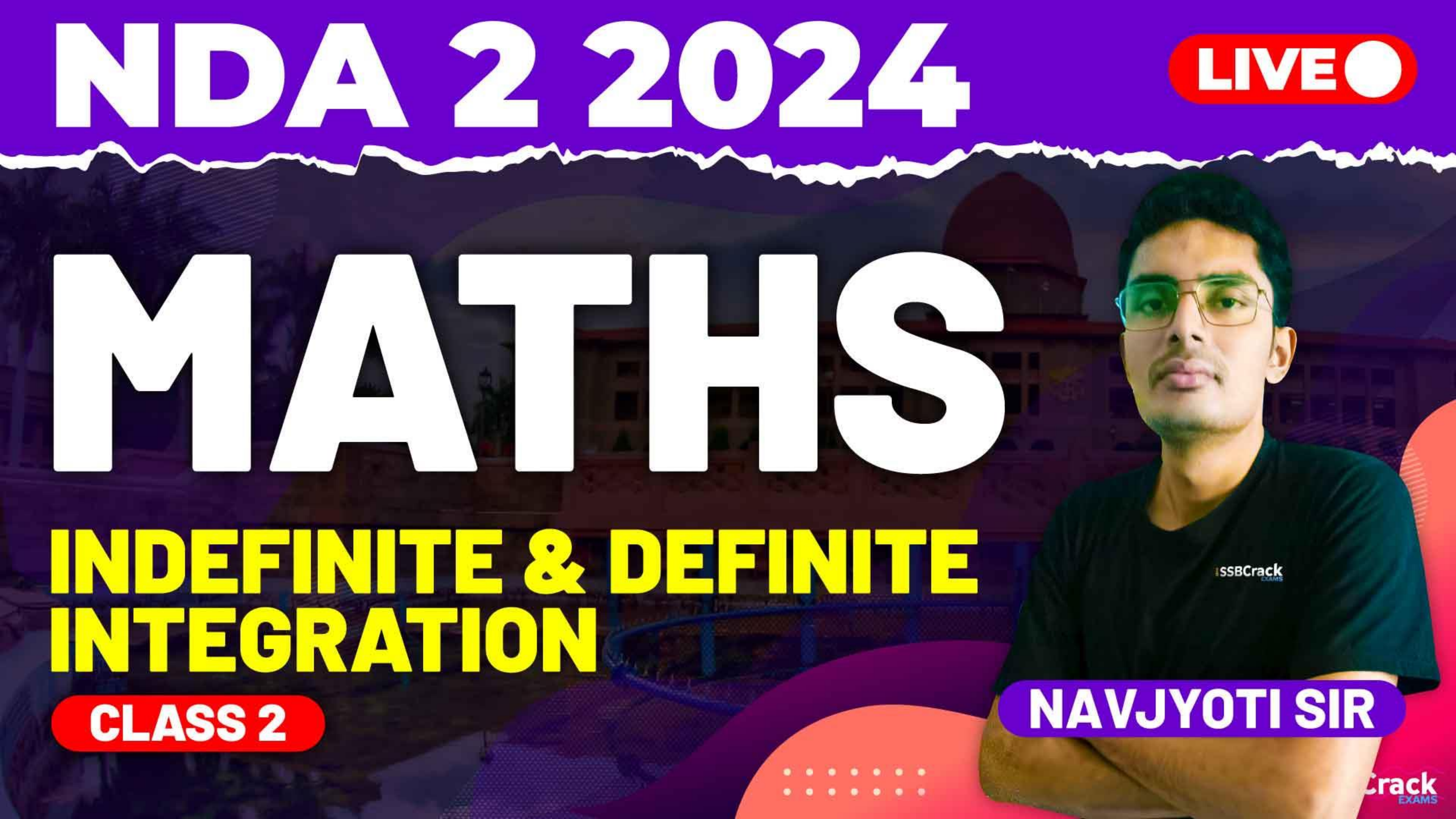
(a) $\log(2 + 4\tan\frac{x}{2}) + C$

(b) $\frac{1}{2}\log(2 + 4\tan\frac{x}{2}) + C$

(c) $\frac{1}{2}\log(2 - 4\tan\frac{x}{2}) + C$

(d) None of the above

Ans: (b)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