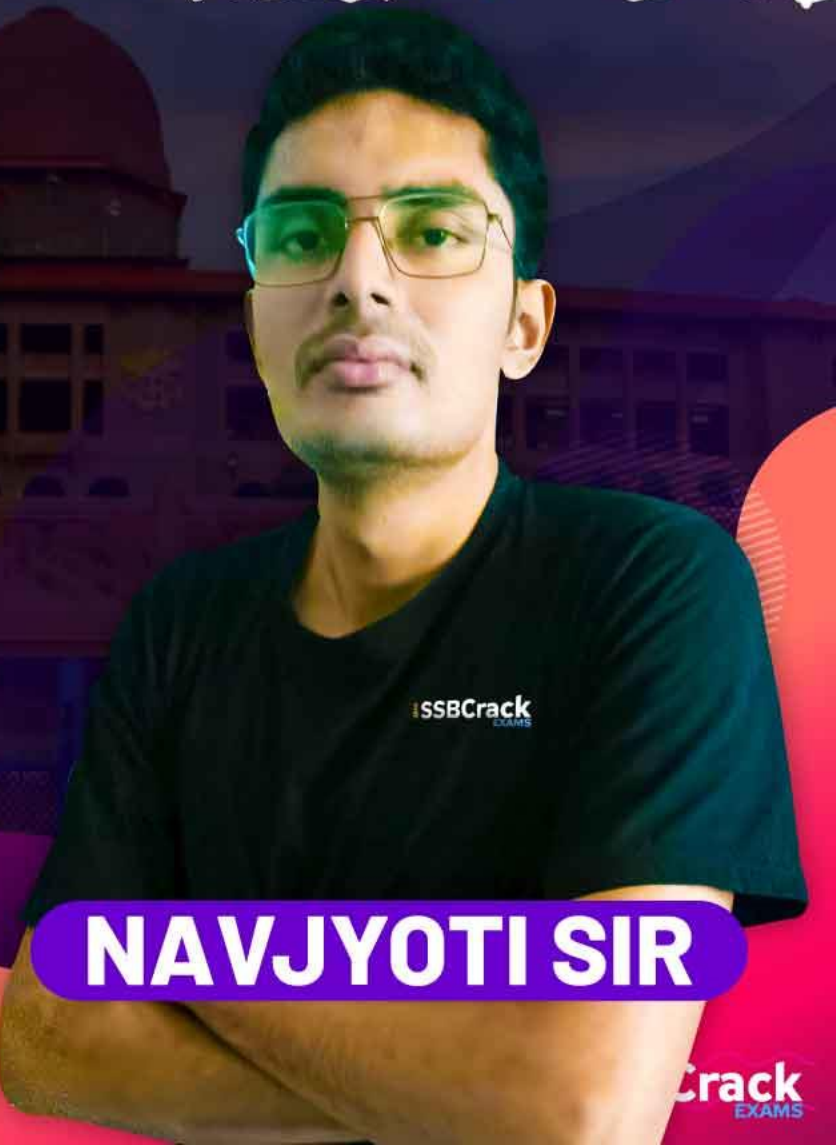
NDA 2 2024

LIVE

MATHS

MATRICES & DETERMINANTS

CLASS 1



NAVJYOTI SIR

14 June 2024 Live Classes Schedule

8:00AM	14 JUNE 2024 DAILY CURRENT AFFAIRS	RUBY MA'AM

SSB INTERVIEW LIVE CLASSES

9:00AM	OVERVIEW OF GPE & PRACTICE	ANURADHA MA'AM

AFCAT 2 2024 LIVE CLASSES

4:00PM	MATHS - GEOMETRY - CLASS 1	NAVJYOTI SIR
5:30PM	ENGLISH - CLOZE TEST - CLASS 3	ANURADHA MA'AM

NDA 2 2024 LIVE CLASSES

11:30AM	GK - INDIAN GEOGRAPHY - CLASS 3	RUBY MA'AM
2:30PM	GS - CHEMISTRY - CLASS 5	SHIVANGI MA'AM
5:30PM	ENGLISH - CLOZE TEST - CLASS 3	ANURADHA MA'AM
6:30PM	MATHS - GEOMETRY - CLASS 1	NAVJYOTI SIR

CDS 2 2024 LIVE CLASSES

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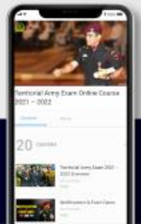
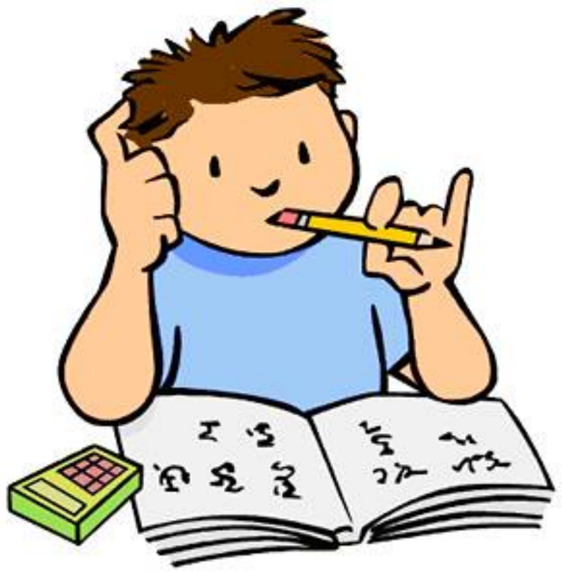


WHAT WILL WE STUDY ?

- Matrix and its Order
- Types of Matrices
- Operations
- Transpose
- Symmetric and Skew Symmetric Matrix
- Determinant
- Properties of Determinant



SSBCrack EXAMS

MATRIX AND ITS ORDER

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

(Matrix)

Rows = 2

Columns = 3

Order of a matrix = $\underbrace{2 \times 3}$

(no. of rows) × (no. of columns)

(Horizontal) (vertical)

$$\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} = A$$

3×1

Rows = 3

C = 1

$D = \begin{bmatrix} 6 & 7 & 8 & 9 \end{bmatrix}$

collection of numbers / functions arranged in rows & columns.

TYPES

1. ROW MATRIX $\begin{bmatrix} 4 & 3 & 2 \end{bmatrix}$ — only 1 row. order = $1 \times m$

2. COLUMN MATRIX $\begin{bmatrix} 10 \\ 9 \\ 8 \\ 7 \end{bmatrix}$ — only 1 column. order = $n \times 1$

3. SQUARE MATRIX

equal rows + equal columns

$[a]_{1\times 1}$ $\quad \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times 2}$ $\quad \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}_{3\times 3}$

Order = $m \times m$

SSBCrack EXAMS

TYPES

4. ZERO / NULL MATRIX – if all elements are zero, $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

5. DIAGONAL MATRIX – square matrix with diagonal elements as non-zero, (row no. = column no.)

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \to \begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & -7 \end{bmatrix}$$

6. SCALAR MATRIX

Diagonal elements are equal,

$$\begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

SSBCrack EXAMS

TYPES

7. IDENTITY / UNIT MATRIX

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3} = \underline{I}$$

8. UPPER AND LOWER TRIANGULAR MATRIX

$$\begin{bmatrix} 4 & 3 & 2 \\ 0 & 7 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$ upper tria. matrix

$$\begin{bmatrix} 4 & 0 & 0 \\ 3 & 7 & 0 \\ 2 & 1 & 6 \end{bmatrix}$$ Lower tria. matrix.

EQUALITY OF MATRICES

i) order has to be equal.

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \neq \begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix}$$

ii) all corresponding elements should be equal.

$$\begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$\Rightarrow a = 2,\ b = 3,\ c = 5,\ d = 6$

SSBCrack EXAMS

ADDITION AND SUBTRACTION

i) Two matrix A & B can be added/ subtracted only if they have the same order.

$$\begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 7 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 5 & 7 \\ 12 & 7 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} =$$

(cannot add)

QUESTION

If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 & 2 \\ 4 & 3 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $D = \begin{bmatrix} 4 & 6 & 8 \\ 5 & 7 & 9 \end{bmatrix}$, then

which of the sums A + B, B + C, C + D and B + D is defined?

$A + B$ — ✗

$B + C$ — ✗

$C + D$ — ✗

$B + D$ — (same - order) — ✓

MULTIPLCATION - SCALAR

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$3A = 3\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3\times1 & 3\times2 \\ 3\times3 & 3\times4 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 9 & 12 \end{bmatrix}$$

MATRIX MULTIPLICATION

no. of columns of first matrix = no. of rows of second matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

(2×3) (3×1)

$$= \begin{bmatrix} 1\times 2 + 2\times 3 + 3\times 4 \\ 4\times 2 + 5\times 3 + 6\times 4 \end{bmatrix}_{2\times 1} = \begin{bmatrix} 2+5+12 \\ 8+15+24 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & g & i \\ f & h & j \end{bmatrix}$$

2×2 2×3

$$= \begin{bmatrix} a\times e + b\times f & a\times g + b\times h & a\times i + b\times j \\ c\times e + d\times f & c\times g + d\times h & c\times i + d\times j \end{bmatrix}$$

2×3

$$\begin{bmatrix} 2 & a \\ 3 & b \end{bmatrix} \begin{bmatrix} 6 & c \\ 7 & d \end{bmatrix}$$

$2 \times 2 \qquad 2 \times 2$

$$= \begin{bmatrix} 2 \times 6 + a \times 7 & 2 \times c + a \times d \\ 3 \times 6 + b \times 7 & 3 \times c + b \times d \end{bmatrix}$$

SSBCrack EXAMS

QUESTION

If $\begin{bmatrix} 2x & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} x \\ 8 \end{bmatrix} = O$, find the value of x.

zero matrix

$$\underset{1\times 2}{\begin{bmatrix} 2x & 3 \end{bmatrix}} \underset{2\times 2}{\begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}} = \underset{1\times 2}{\begin{bmatrix} 2x\times 1 + 3\times -3 & 2x\times 2 + 3\times 0 \end{bmatrix}}$$

$$\underset{1\times 2}{\begin{bmatrix} 2x-9 & 4x \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} x \\ 8 \end{bmatrix}} = \begin{bmatrix} 2x^2 - 9x + 32x \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$$2x^2 + 23x = 0$$

$$2x\left(x + \frac{23}{2}\right) = 0$$

$$x = 0 \qquad x = -\frac{23}{2}$$

SSBCrack EXAMS

PROPERTIES OF MATRIX MULTIPLICATION

Associative Law for Multiplication : If A, B and C be three matrices of order $m \times n$ and $n \times p$ and $p \times q$, respectively, then $(AB)\,C = A\,(BC)$.

A B C

$m \times n$ $n \times p$ $p \times q$

$(AB) \rightarrow m \times p$

$(BC) \rightarrow n \times q$

$(AB)C \rightarrow m \times q$

$A(BC) \rightarrow m \times q$

Distributive Law : If A, B, C be three matrices of order $m \times n$, $n \times p$ and $n \times q$ respectively.

then $A\,(B + C) = AB + AC$

PROPERTIES OF MATRIX MULTIPLICATION

Matrix multiplication is not commutative.

i.e. $AB \neq BA$ (in general)

$AB = BA$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 3 & 7 \end{bmatrix} \quad (AB \neq BA)$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 3 & 4 \end{bmatrix}$$

$IA = AI = A.$

If A, B are, respectively $m \times n$, $k \times l$ matrices, then both AB and BA are defined if and only if $n = k$ and $l = m$.

If AB is defined, then BA need not be defined.

If AB and BA are both defined, it is not necessary that AB = BA.

If the product of two matrices is a zero matrix, it is not necessary that one of the matrices is a zero matrix.

For three matrices A, B and C of the same order, if $A = B$, then $AC = BC$, but converse is not true.

$A. A = A^2$, $A. A. A = A^3$, so on

TRANSPOSE

$$A = \begin{bmatrix} 1 & 2 & 3 \\ a & b & c \\ 4 & 5 & 6 \end{bmatrix}_{3\times 3} \longrightarrow A^T / A' = \begin{bmatrix} 1 & a & 4 \\ 2 & b & 5 \\ 3 & c & 6 \end{bmatrix}$$

→ only defined for square matrix, rows values $\rightleftharpoons$ column values (interchange)

$$\begin{bmatrix} 2 & 3 \\ a & b \end{bmatrix} \longrightarrow \begin{bmatrix} 2 & a \\ 3 & b \end{bmatrix}$$

SSBCrack EXAMS

PROPERTIES OF TRANSPOSE

$(A^T)^T = A,$

$(kA)^T = kA^T$ (where k is any constant)

$(A + B)^T = A^T + B^T$

$(AB)^T = B^T A^T$

SYMMETRIC MATRIX

$$A = A^T$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

SKEW - SYMMETRIC MATRIX

$A^T = -A$

diagonal elements $= 0$,

$$\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} \xrightarrow{\text{Transpose}} \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix} = (-1)\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

$$= (-1)A = \underline{-A}$$

PROPERTIES

$\underline{A}$

$A + A^T$ — symmetric

$A - A^T$ — skew symmetric

$$\boxed{A = \frac{1}{2}(A+A^T) + \frac{1}{2}(A-A^T)}$$

DETERMINANT

only for square matrix.

$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$

$|A| = \Delta = \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} = (2\times 6) - (3\times 5) = 12 - 15 = -3$

$\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = a\times d - b\times c$

3×3

$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$

Δ = 1st row $= a(ei - fh) - b(di - fg) + c(dh - eg)$

Δ = 1st column $= a(ei - fh) - d(bi - ch) + g(bf - ce)$ ✓

SSBCrack EXAMS

PROPERTIES

$|A'| = |A|$, where A' = transpose of matrix A.

If we interchange any two rows (or columns), then sign of the determinant changes.

If any two rows or any two columns in a determinant are identical (or proportional), then the value of the determinant is zero.

$$\begin{array}{c|ccc|} & C_1 & C_2 & C_3 \\ R_1 & a & b & c \\ R_2 & d & e & f \\ R_3 & g & h & i \end{array} = \Delta$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} = 0$$

any one row, or any one column $= 0$

$(\Delta = 0)$

SSBCrack EXAMS

PROPERTIES

Multiplying a determinant by k means multiplying the elements of only one row (or one column) by k.

If we multiply each element of a row (or a column) of a determinant by constant k, then value of the determinant is multiplied by k.

If elements of a row (or a column) in a determinant can be expressed as the sum of two or more elements, then the given determinant can be expressed as the sum of two or more determinants.

$$\begin{vmatrix} 1+3 & 2+3 & 4+3 \\ a & b & c \\ d & e & f \end{vmatrix}$$

$$\begin{vmatrix} 4 & 5 & 7 \\ a & b & c \\ d & e & f \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ a & b & c \\ d & e & f \end{vmatrix} + \begin{vmatrix} 3 & 3 & 4 \\ a & b & c \\ d & e & f \end{vmatrix}$$

$$k\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = \begin{vmatrix} ka & kb & kc \\ d & e & f \\ g & h & i \end{vmatrix}$$

any row, or

any column

PROPERTIES

If to each element of a row (or a column) of a determinant the equimultiples of corresponding elements of other rows (columns) are added, then value of determinant remains same.

SSBCrack EXAMS

PROPERTIES

If all the elements of a row (or column) are zeros, then the value of the determinant is zero.

If value of determinant 'Δ' becomes zero by substituting $x = \alpha$, then $x - \alpha$ is a factor of 'Δ'.

If all the elements of a determinant above or below the main diagonal consists of zeros, then the value of the determinant is equal to the product of diagonal elements.

$\begin{vmatrix} - & - & - \\ - & - & - \end{vmatrix} = f(x)$ polynomial,

in variable x,

$f(\alpha) = 0$

$\Rightarrow x - \alpha$ is factor of $f(x)$.

SSBCrack EXAMS

QUESTION

Let $\Delta = \begin{vmatrix} Ax & x^2 & 1 \\ By & y^2 & 1 \\ Cz & z^2 & 1 \end{vmatrix}$ and $\Delta_1 = \begin{vmatrix} A & B & C \\ x & y & z \\ zy & zx & xy \end{vmatrix}$, then

(A) $\Delta_1 = -\Delta$

(B) $\Delta \neq \Delta_1$

(C) $\Delta - \Delta_1 = 0$

(D) None of these

SSBCrack EXAMS

QUESTION

Let $\Delta = \begin{vmatrix} Ax & x^2 & 1 \\ By & y^2 & 1 \\ Cz & z^2 & 1 \end{vmatrix}$ and $\Delta_1 = \begin{vmatrix} A & B & C \\ x & y & z \\ zy & zx & xy \end{vmatrix}$, then

(A) $\Delta_1 = -\Delta$ (B) $\Delta \neq \Delta_1$

(C) $\Delta - \Delta_1 = 0$ (D) None of these

$R_1 \to xR_1$ $R_2 \to yR_2$ $R_3 \to zR_3$

$$\Delta_1 = \begin{vmatrix} A & x & zy \\ B & y & zx \\ C & z & xy \end{vmatrix} = \left(\frac{1}{x}\right)\left(\frac{1}{y}\right)\left(\frac{1}{z}\right)\begin{vmatrix} Ax & x^2 & xyz \\ By & y^2 & xyz \\ Cz & z^2 & xyz \end{vmatrix} = \frac{xyz}{xyz}\begin{vmatrix} Ax & x^2 & 1 \\ By & y^2 & 1 \\ Cz & z^2 & 1 \end{vmatrix}$$

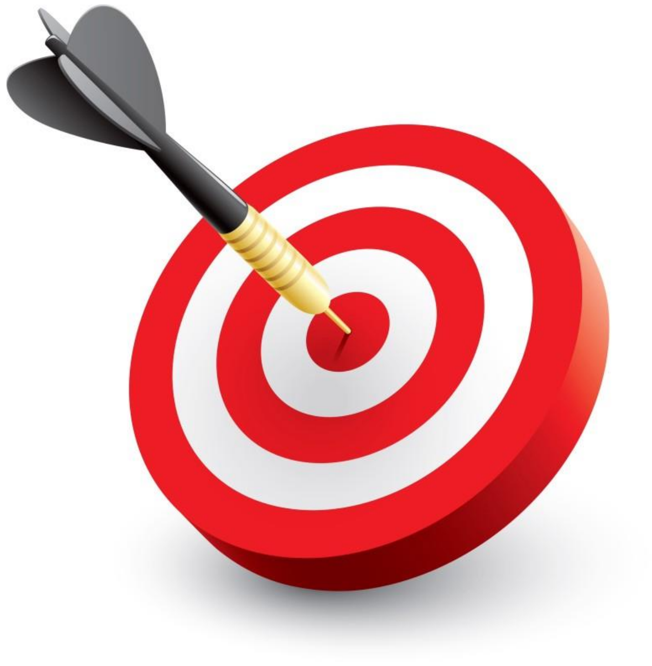
(transpose)

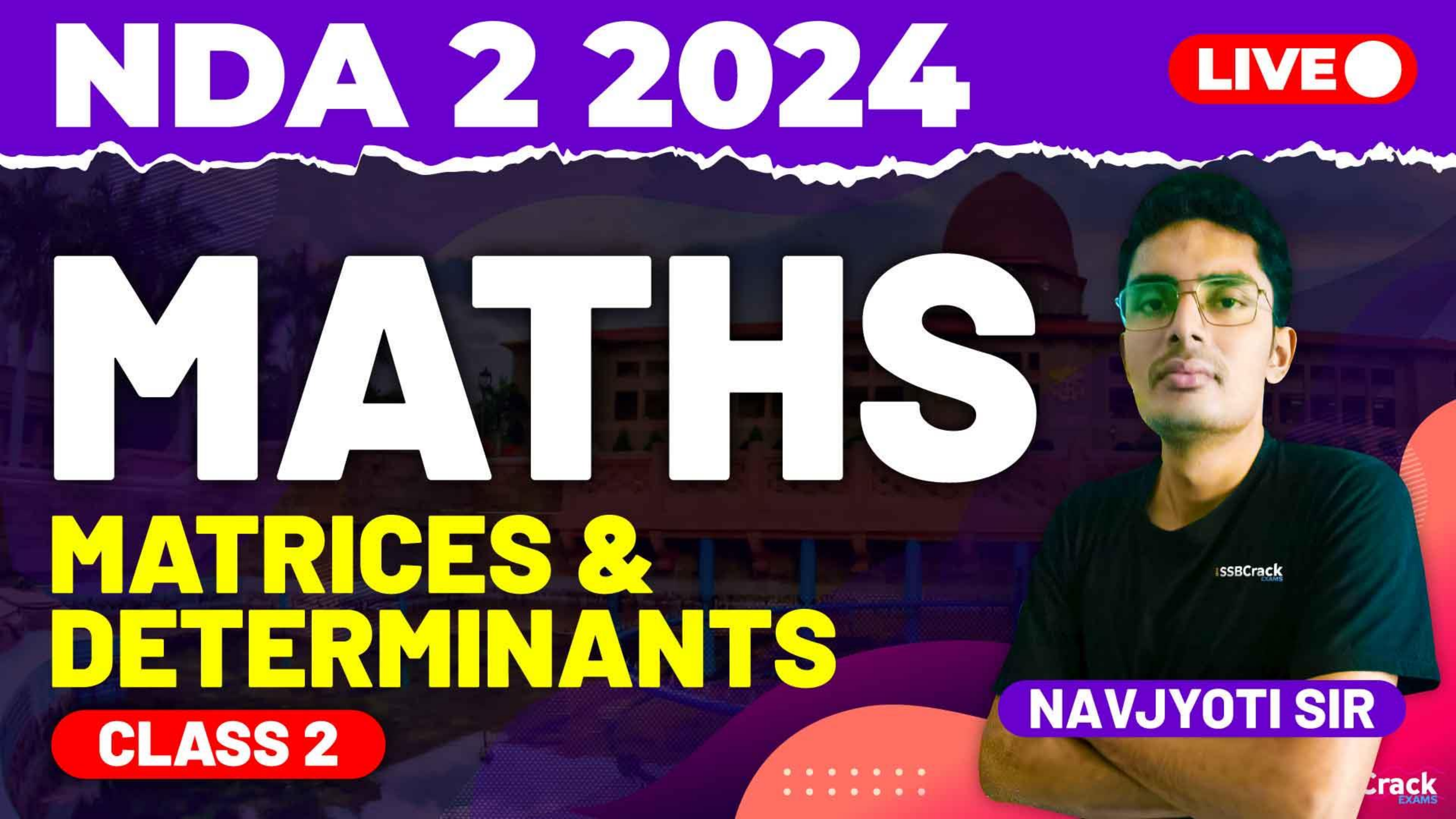
$$= 1\begin{vmatrix} Ax & x^2 & 1 \\ By & y^2 & 1 \\ Cz & z^2 & 1 \end{vmatrix} = \Delta$$

ANSWER : C

Summary

- Matrix and its Order
- Types of Matrices
- Operations
- Transpose
- Symmetric and Skew Symmetric Matrix
- Determinant
- Properties of Determinant
- Practise Questions

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