NDA 2 2024

LIVE

MATHS

MATRICES & DETERMINANTS

CLASS 2

NAVJYOTI SIR

17 June 2024 Live Classes Schedule

8:00AM	17 JUNE 2024 DAILY CURRENT AFFAIRS	RUBY MA'AM
9:00AM	17 JUNE 2024 DAILY DEFENCE UPDATES	DIVYANSHU SIR

AFCAT 2 2024 LIVE CLASSES

2:30PM	STATIC GK - INTERNATIONAL ORGANIZATION & HQ	DIVYANSHU SIR
4:00PM	MATHS - GEOMETRY - CLASS 2	NAVJYOTI SIR

NDA 2 2024 LIVE CLASSES

11:30AM	GK - MINERAL & RESOURCES	RUBY MA'AM
2:30PM	GS - CHEMISTRY - CLASS 6	SHIVANGI MA'AM
6:30PM	MATHS - MATRICES & DETERMINANTS - CLASS 2	NAVJYOTI SIR

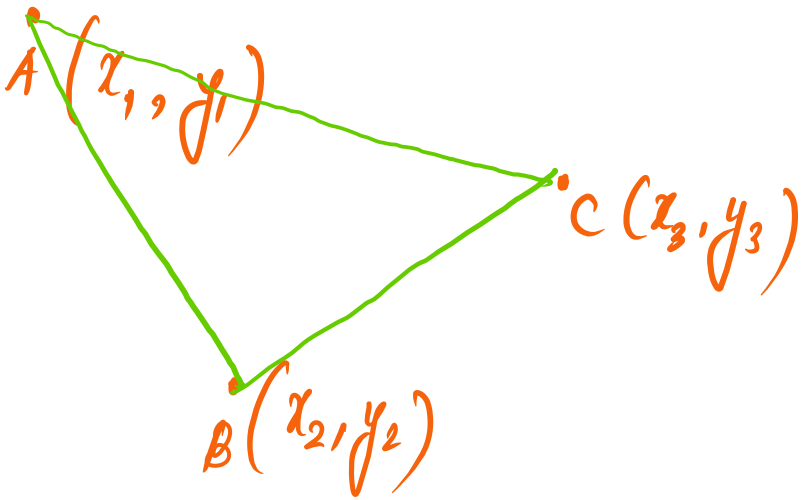
CDS 2 2024 LIVE CLASSES

11:30AM	GK - MINERAL & RESOURCES	RUBY MA'AM
2:30PM	GS - CHEMISTRY - CLASS 6	SHIVANGI MA'AM
4:00PM	MATHS - GEOMETRY - CLASS 2	NAVJYOTI SIR

WHAT WILL WE STUDY ?

- Area of Triangle
- Minors and Cofactors
- Adjoint of a Matrix
- Inverse of a Matrix
- Solution of system of Simultaneous Linear Equations
- Consistent and Inconsistent Solutions

AREA OF TRIANGLE



$$\text{Area} = \frac{1}{2}\left[\underline{x_1(y_2-y_3)+x_2}(y_3-y_1)\underline{+x_3}(y_1-y_2)\right]$$

$$= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

MINORS AND COFACTORS

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times 3} = [a_{ij}]_{3\times 3}$$

$$(-1)^{i+j} \rightarrow \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

$$\text{minor of } a_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}_{2\times 2} =$$

$$\text{minor of } a_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}_{2\times 2} =$$

$$A_{11} = \text{cofactor of } a_{11} = (-1)^{1+1}(\text{minor at } a_{11})$$

$$= (-1)^{1+1} \underbrace{\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}}$$

(order of minor – determinant is one less than that of matrix)

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$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \longrightarrow \text{minor for each element} \longrightarrow (-1)^{i+j}$$

$\downarrow$

cofactor matrix

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

ADJOINT

→ Transpose of cofactor matrix.

minors

If $A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$, then $adj\ A = \begin{vmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{vmatrix}$, where A_{ij} is co-factor of a_{ij}.

cofactor matrix → $\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$

PROPERTIES

$$A(\text{adj. } A) = |A|\, I_n = (\text{adj } A)\, A$$

$$|\text{adj } A| = |A|^{n-1}$$

$$\text{adj}\,(\text{adj } A) = |A|^{n-2} A$$

$$|\text{adj}\,(\text{adj } A)| = |A|^{(n-1)^2}$$

I_n — identity matrix of n^{th} order

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{— order 3 } (n = 3)$$

$|A| \rightarrow$ determinant of A.

PROPERTIES

$\text{adj}\,(AB) = (\text{adj}\,B)\,(\text{adj}\,A)$

$\text{adj}\,(A^m) = (\text{adj}\,A)^m,\ m \in N$

$\text{adj}\,(kA) = k^{n-1}\,(\text{adj.}\,A),\ k \in R$

$\text{adj}\,(I_n) = I_n$

$\text{adj}\,O = O$

PROPERTIES

A is symmetric $\Rightarrow$ adj A is also symmetric

A is diagonal matrix $\Rightarrow$ adj A is also diagonal matrix

A is triangular matrix $\Rightarrow$ adj A is also triangular matrix

A is singular matrix $\Rightarrow |\text{adj } A| = 0$

$|A| = 0$ not that adj A will be a zero matrix.

SSBCrack EXAMS

INVERSE OF A MATRIX

(A)

$(A)(B) = I$

inverse of A.

$B = A^{-1}$

$A^{-1} = \frac{1}{|A|} \operatorname{adj} A$

if $|A| = 0$, A^{-1} does not exist.

→ A is invertible (its inverse exists) only if $|A| \neq 0$, or A is a non-singular matrix.

SSBCrack EXAMS

PROPERTIES

$\rightarrow (AB)^{-1} = B^{-1} . A^{-1}$

$|A^{-1}| = |A|^{-1}$ $\longrightarrow$ reciprocal of det (A), or $\frac{1}{|A|}$.

$(A^{-1})^{-1} = A$

SSBCrack EXAMS

SOLUTION OF SYSTEM OF LINEAR EQUATIONS

$$a_1x + b_1y + c_1z + d_1 = 0$$

$$a_2x + b_2y + c_2z + d_2 = 0$$

$$a_3x + b_3y + c_3z + d_3 = 0$$

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -d_1 \\ -d_2 \\ -d_3 \end{bmatrix}$$

(A) coefficient matrix

(X)

(B) constant term matrix

$$AX = B$$

$$\underline{X = A^{-1}B}$$

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NATURE OF SOLUTION

If $|A| \neq 0$, then $AX = B$ has a unique solution.

x, y, z will have one unique value

$$A^{-1} = \frac{1}{|A|} \operatorname{adj} A$$

If $|A| = 0$, and $(\operatorname{adj} A)\,B \neq 0$ then the system of equation is inconsistent.

(no - solution)

consistent

(one or more than one)

If $|A| = 0$ and $(\operatorname{adj} A)\,B = 0$, then the system of equation has infinitely many solutions.

In this case, we put one of the variables equal to k.
Let $z = k$, then we find the value of x and y in terms of k.

SSBCrack EXAMS

SPECIAL CASES OF MATRIX

Orthogonal Matrix

A square matrix A is called orthogonal, if

$AA^T = I = A^TA$ $\rightarrow$ $\boxed{A^T = A^{-1}}$

Idempotent Matrix

A square matrix A is called an idempotent matrix if $A^2 = A$

Ex. $A = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$ is a idempotent matrix because here $A^2 = A$

$$\begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}\begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix} = A \cdot A$$

$$= \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix} = A$$

SPECIAL CASES OF MATRIX

Involutory Matrix

A square matrix A is called an involutory matrix if
$A^2 = I$ or $A^{-1} = A$

Ex. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is an involutory matrix.

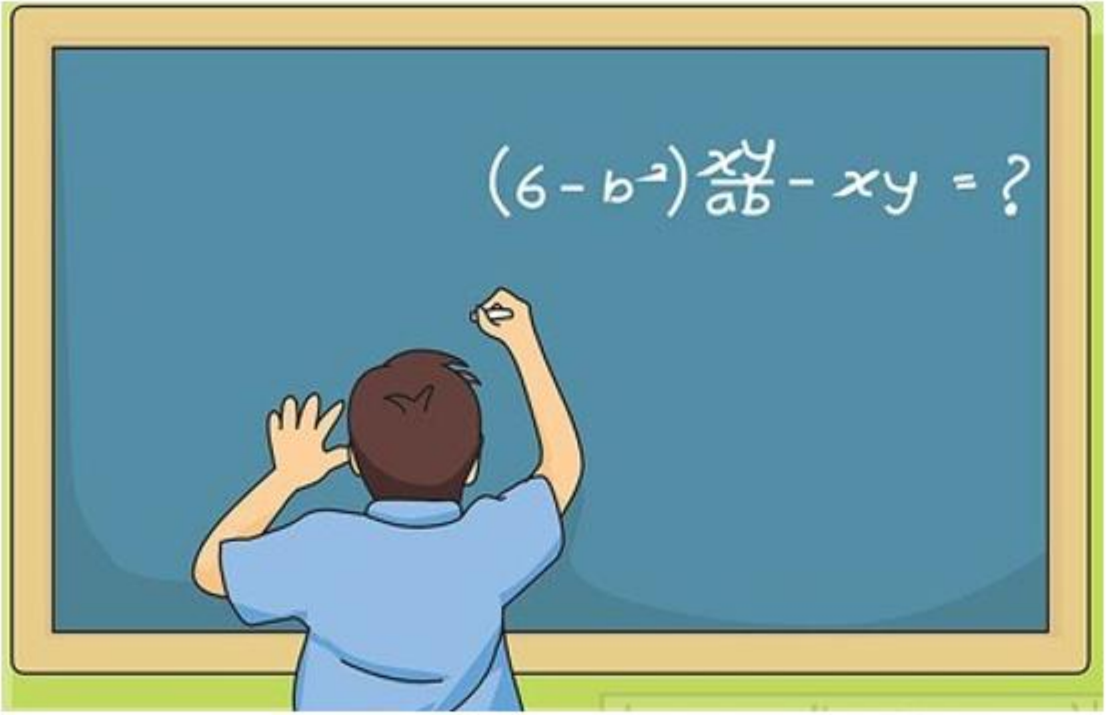
Nilpotent Matrix

A square matrix A is called a nilpotent matrix if there exist $p \in N$ such that $A^p = 0$

Ex. $A = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}$ is a nilpotent matrix

$A^2 = A \cdot A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

PRACTISE TIME !



SSBCrack EXAMS

QUESTION

If the matrix $\begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \alpha \end{bmatrix}$ (A) is orthogonal, then the value of α is

(a) $\pm\frac{1}{\sqrt{2}}$ (b) ± 2

(c) $\pm\frac{1}{\sqrt{3}}$ (d) None of these

$$A^T = \begin{bmatrix} 0 & \alpha & \alpha \\ 2\beta & \beta & -\beta \\ \gamma & -\gamma & \alpha \end{bmatrix}$$

$$|A| = -2\beta(\alpha^2+\gamma\alpha) + \gamma(-\alpha\beta - \alpha\beta)$$

$$= -2\beta\alpha^2 - 2\alpha\beta\gamma - 2\alpha\beta\gamma$$

$$= -2\beta\alpha^2 - 4\alpha\beta\gamma$$

$$\text{Cofactors}(A) = \begin{bmatrix} \alpha\beta-\gamma\beta & -(\alpha^2+\gamma\alpha) & -2\alpha\beta \\ -(-3\beta\gamma) & -\gamma\alpha & -2\beta\alpha \\ -3\beta\gamma & +\gamma\alpha & -2\alpha\beta \end{bmatrix}$$

adj A

$$\frac{1}{(-2\beta\alpha^2-4\alpha\beta\gamma)}\begin{bmatrix} -2\alpha\beta & 3\beta\gamma & -3\beta\gamma \\ -(\alpha^2+\gamma\alpha) & -\gamma\alpha & \gamma\alpha \\ -2\alpha\beta & -2\alpha\beta & 2\alpha\beta \end{bmatrix}$$

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If the matrix $\begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \alpha \end{bmatrix}$ is orthogonal, then the value of α is

(a) $\pm\frac{1}{\sqrt{2}}$

(b) ± 2

✓ (c) $\pm\frac{1}{\sqrt{3}}$

(d) None of these

$A \cdot A^T = I$

$$\begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \alpha \end{bmatrix}\begin{bmatrix} 0 & \alpha & \alpha \\ 2\beta & \beta & -\beta \\ \gamma & -\gamma & \alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\underbrace{4\beta^2 + \gamma^2}= 1$

$\underbrace{2\beta^2 - \gamma^2} = 0$

$\underline{6\beta^2 = 1} \Rightarrow \beta^2 = \frac{1}{6}$

$\left(\gamma^2 = \frac{1}{3}\right)$

$\gamma = \pm\frac{1}{\sqrt{3}}$

$-2\beta^2 + \alpha\gamma = 0$

$-2\left(\frac{1}{6}\right) + \alpha\left(\frac{1}{\sqrt{3}}\right) = 0$

$\alpha = \frac{1}{3} \Big/ \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

SSBCrack EXAMS

QUESTION

The determinant $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ is

(a) independent of θ only ✓
(b) independent of x only
(c) independent of both θ and x
(d) None of the above

$x(-x^2-1) - \sin\theta(-x\sin\theta - \cos\theta) + \cos\theta(-\sin\theta + x\cos\theta)$

$= -x^3 - x + x\sin^2\theta + x\cos^2\theta$

$= -x^3$

NDA 2 2024 LIVE CLASS - MATHS - PART 2

QUESTION

$$\begin{vmatrix} {}^xC_1 & {}^xC_2 & {}^xC_3 \\ {}^yC_1 & {}^yC_2 & {}^yC_3 \\ {}^zC_1 & {}^zC_2 & {}^zC_3 \end{vmatrix} =$$

$$\begin{vmatrix} x & \frac{x(x-1)}{2} & \frac{x(x-1)(x-2)}{6} \\ y & \frac{y(y-1)}{2} & \frac{y(y-1)(y-2)}{6} \\ z & \frac{z(z-1)}{2} & \frac{z(z-1)(z-2)}{6} \end{vmatrix}$$

$x^2 - y^2 - 3(x-y)$

$x^2 - 3x + \cancel{2}$

$-y^2 + 3y - \cancel{2}$

(a) $xyz(x-y)(y-z)(z-x)$ (b) $\frac{xyz}{6}(x-y)(y-z)(z-x)$

(c) $\frac{xyz}{12}(x-y)(y-z)(z-x)$ (d) None of these

$$xyz\left(\frac{1}{2}\right)\left(\frac{1}{6}\right)\begin{vmatrix} 1 & x-1 & (x-1)(x-2) \\ 1 & y-1 & (y-1)(y-2) \\ 1 & z-1 & (z-1)(z-2) \end{vmatrix} = \begin{vmatrix} 1 & x-1 & (x-1)(x-2) \\ 0 & x-y & (x-y)(x+y-3) \\ 0 & y-z & (y-z)(y+z-3 \end{vmatrix}$$

$R_2 \to R_1 - R_2$ $R_3 \to R_2 - R_3$

$$x^2 - y^2 - 3(x-y)$$

$$(x+y)(x-y) - 3(x-y)$$

$$(x-y)(x+y-3)$$

$$x^2 - 3x + 2$$
$$y^2 - 3y + 2$$
$$(-)\quad(+)\quad(-)$$

$$\begin{vmatrix} 1 & x-1 & (x-1)(x-2) \\ 0 & x-y & (x-y)(x+y-3) \\ 0 & y-z & (y-z)(y+z-3) \end{vmatrix} = (x-y)(y-z)\begin{vmatrix} 1 & x-1 & (x-1)(x-2) \\ 0 & 1 & x+y-3 \\ 0 & 1 & y+z-3 \end{vmatrix}$$

$$(x-y)(y-z)(\cancel{y}+z-\cancel{3}-x-\cancel{y}+\cancel{3})$$

$$= (x-y)(y-z)(z-x)$$

QUESTION

For any 2×2 matrix A, if $A(\text{adj. } A) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, then $|A|$ is equal to :

(a) 0 (b) 10 (c) 20 (d) 100

QUESTION

If 1, ω, ω^2 the cube roots of unity, then the value of

$$\begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^{2n} & 1 & \omega^n \\ \omega^n & \omega^{2n} & 1 \end{vmatrix}$$ is equal to

(a) 1 (b) ω (c) ω^2 (d) 0

SSBCrack EXAMS

QUESTION

If the equations $x + ay - z = 0, 2x - y + az = 0, ax + y + 2z = 0$ have non-trivial solutions, then $a =$

(a) 2 (b) -2 (c) $\sqrt{3}$ (d) $-\sqrt{3}$

SSBCrack EXAMS

QUESTION

The system $ax - 3y + 5z = 4, x - ay + 3z = 2, 9x - 7y + 8az = 0$ has

(a) a unique solution for all a

(b) no solution for all a

(c) unique solution if $4a^3 - 45a + 58 = 0$

(d) no solution if $4a^3 - 45a + 58 = 0$

SSBCrack EXAMS

QUESTION

If $\begin{vmatrix} 1+a & 1 & 1 \\ 1+b & 1+2b & 1 \\ 1+c & 1+c & 1+3c \end{vmatrix} = 0$ where

$a \neq 0, b \neq 0, c \neq 0$, then $a^{-1} + b^{-1} + c^{-1}$ is

(a) 4

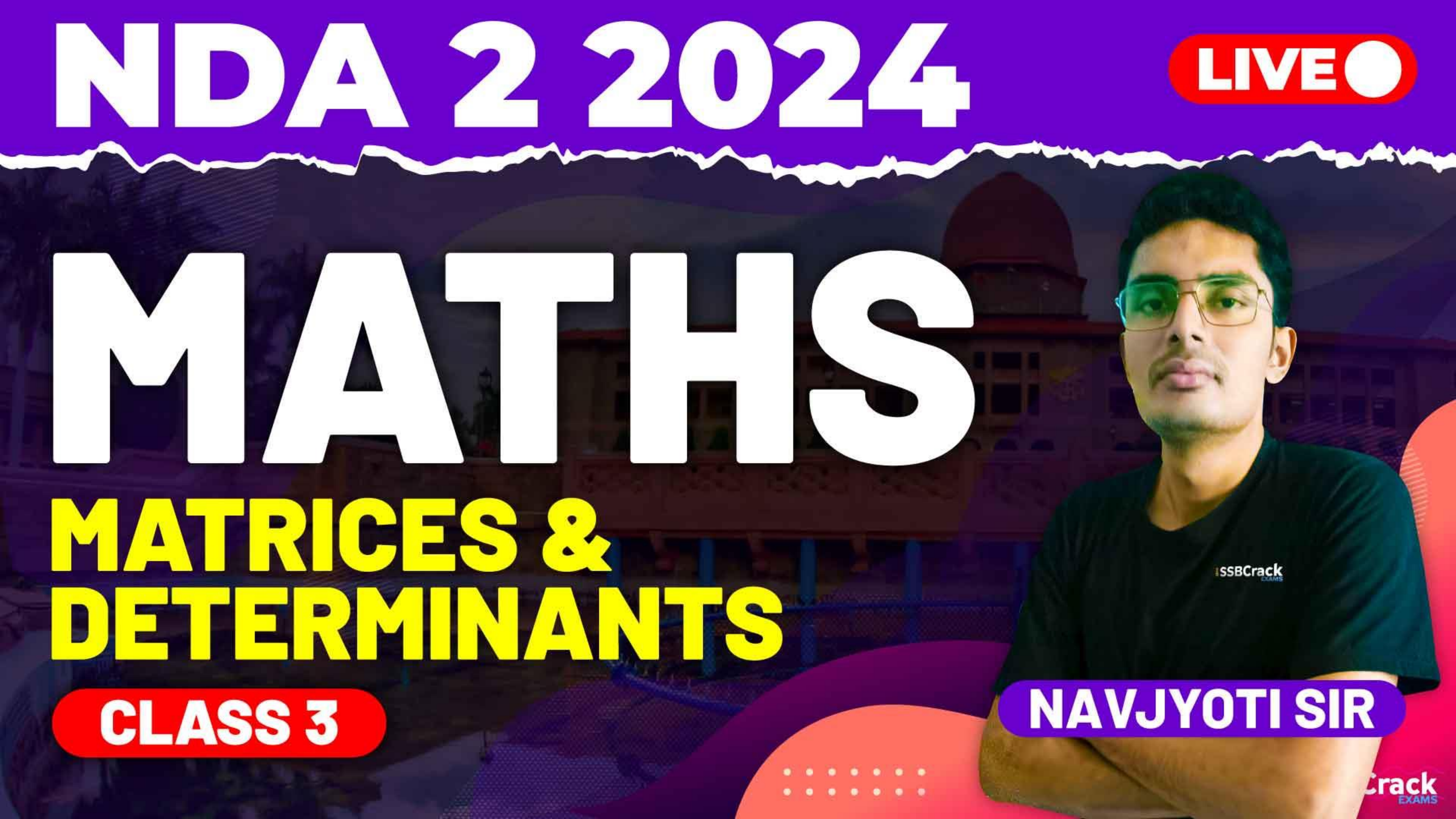
(b) –3

(c) –2

(d) –1

Summary

- Area of Triangle
- Minors and Cofactors
- Adjoint of a Matrix
- Inverse of a Matrix
- Solution of system of Simultaneous Linear Equations
- Consistent and Inconsistent Solutions
- Practise Questions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