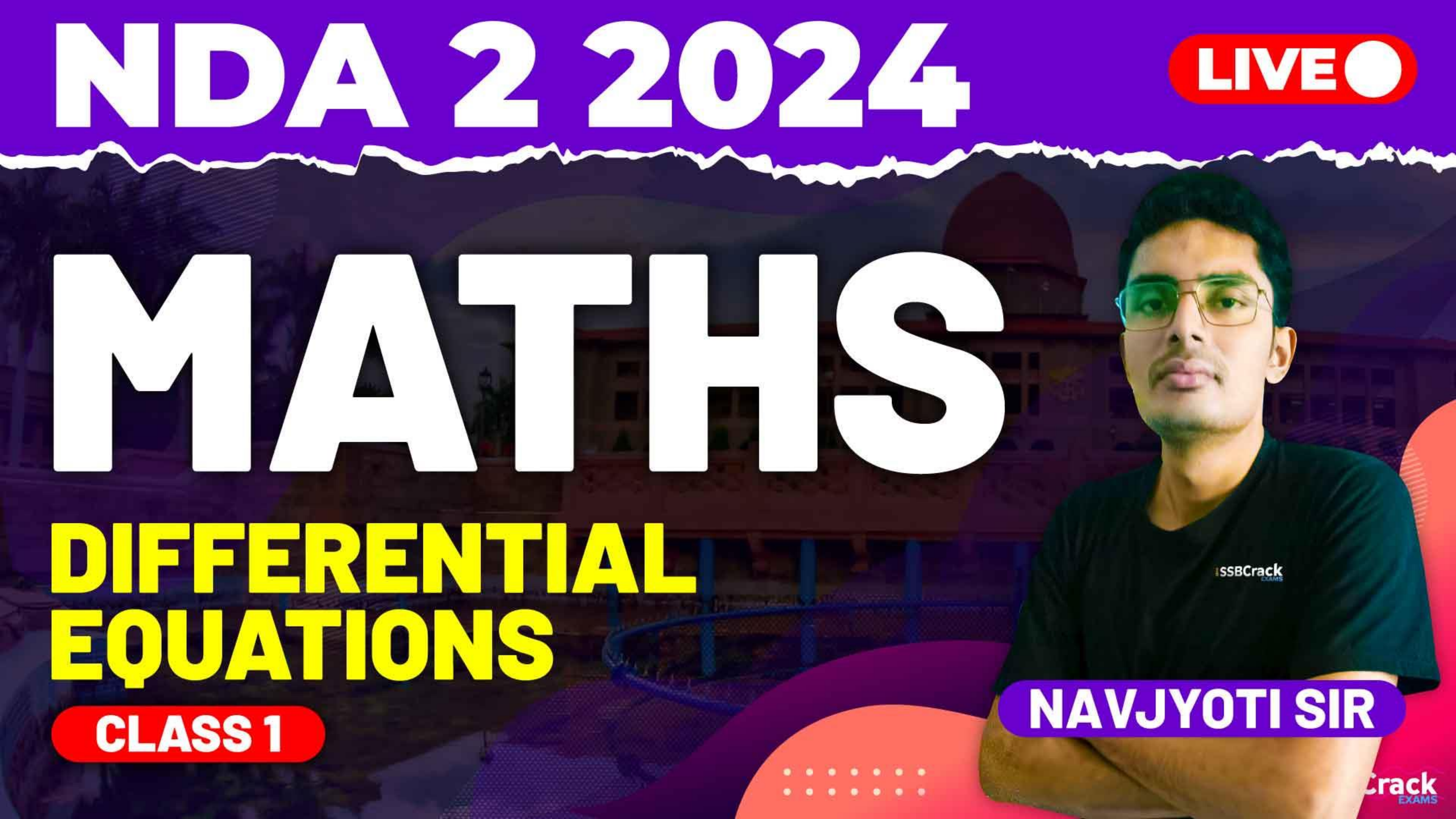
NDA 2 2024
LIVE
MATHS
DIFFERENTIAL EQUATIONS
CLASS 1
SSBCrack EXAMS
NAVJYOTI SIR
Crack EXAMS

03 July 2024 Live Classes Schedule

8:00AM	03 JULY 2024 DAILY CURRENT AFFAIRS	RUBY MA'AM
9:00AM	03 JULY 2024 DAILY DEFENCE UPDATES	DIVYANSHU SIR

SSB INTERVIEW LIVE CLASSES

9:00AM	OVERVIEW OF OIR & PRACTICE	ANURADHA MA'AM

NDA 2 2024 LIVE CLASSES

11:30AM	GK - MODERN HISTORY - CLASS 5	RUBY MA'AM
1:00PM	GS - PHYSICS - CLASS 3	NAVJYOTI SIR
2:30PM	GS - CHEMISTRY MCQS - CLASS 8	SHIVANGI MA'AM
4:00PM	MATHS - DIFFERENTIAL EQUATIONS - CLASS 1	NAVJYOTI SIR
5:30PM	ENGLISH - PARTS OF SPEECH - CLASS 1	ANURADHA MA'AM

CDS 2 2024 LIVE CLASSES

11:30AM	GK - MODERN HISTORY - CLASS 5	RUBY MA'AM
1:00PM	GS - PHYSICS - CLASS 3	NAVJYOTI SIR
2:30PM	GS - CHEMISTRY MCQS - CLASS 8	SHIVANGI MA'AM
5:30PM	ENGLISH - PARTS OF SPEECH - CLASS 1	ANURADHA MA'AM

INTRODUCTION OF DIFFERENTIAL EQUATION

- An expression comprising of dependent and independent variable and derivatives of dependent with respect to independent variable (differential coefficients) that satisfy some relation as a equation.

y – dependent

x – independent

$\left.\begin{matrix} dy \\ dx \end{matrix}\right\}$ differential coefficient

For Example:

(1) $\underbrace{\frac{dy}{dx} = x^2 - 5y + xy}$,

(2) $\underbrace{x\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = x - 1}$

ORDER OF DIFFERENTIAL EQUATION

- The order of differential equation is the highest order derivative involved in its expression.

  Note: It is always a positive Integer.

Example:

$\frac{d^3y}{dx^3} + x\frac{dy}{dx} + 4 = 0$ has order 3.

$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$ has order 2.

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DEGREE OF DIFFERENTIAL EQUATION

- If a differential equation is a polynomial equation in derivatives, then the highest power of the highest order derivative involved is called degree of that differential equation. It is always a positive integer.
- **Note**: If a differential equation is not a polynomial equation in derivatives, then the degree of that differential equation is not defined.

(powers of derivative terms should be whole)

- Example: $\left(\frac{dy}{dx}\right)^2 + logy = x^2$ has degree 2 and $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$ has degree 1

order = 1

Degree = 2

order = 2

degree = 1

EXAMPLE

Find order and Degree of following Differential equation

1) $\frac{d^4y}{dx^4} - sin\left(\frac{d^3y}{dx^3}\right) = 0$

O – 4

D – not defined

2) $\frac{d^3y}{dx^3} - \left(\frac{dy}{dx}\right)^{\frac{2}{3}} = 0$

$\frac{d^3y}{dx^3} = \left(\frac{dy}{dx}\right)^{\frac{2}{3}}$

$\left(\frac{d^3y}{dx^3}\right)^3 = \left(\frac{dy}{dx}\right)^2$

$\left(\frac{d^3y}{dx^3}\right)^3 - \left(\frac{dy}{dx}\right)^2 = 0$

order — 3

degree — 3

FORMULATION OF DIFFERENTIAL EQUATION

In order to formulate differential equation, we have to remove arbitrary constants.

'n' constants — curve's eqn, differentiate n times.

CASE OF ONE ARBITRARY CONSTANT

- **Step 1**: Write the given general solution containing one arbitrary constant.
- **Step 2**: Separate the arbitrary constant and the variables.
- **Step 3**: Differentiate both sides to eliminate the arbitrary constant and get the required differential equation.

Special Case: In some cases, separation of arbitrary constant is not possible, then differentiate both sides of the given solution and evaluate the arbitrary constant. Finally, substitute this value of constant in given solution.

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EXAMPLE

Form the differential equation for $x^2 + y^2 = ax^3$, where a is constant .

$$\frac{x^2+y^2}{x^3} = a$$

$$\frac{1}{x} + \frac{y^2}{x^3} = a$$

$$\left(\frac{-1}{x^2}\right) + \frac{x^3\left(2y\frac{dy}{dx}\right) - y^2\left(3x^2\right)}{\left(x^3\right)^2} = 0$$

$$\frac{-1}{x^2} + \frac{1}{x^3}\, 2y\frac{dy}{dx} - \frac{3y^2}{x^4} = 0$$

$$\frac{2y}{x^3}\frac{dy}{dx} = \frac{3y^2}{x^4} + \frac{1}{x^2}$$

$$\frac{dy}{dx} = \frac{3y^2+x^2}{x^4}\left(\frac{x^3}{2y}\right)$$

$$\frac{dy}{dx} = \frac{3y^2+x^2}{2xy}$$

$$2xy\frac{dy}{dx} - 3y^2 - x^2 = 0$$

CASE OF TWO ARBITRARY CONSTANT

- **Step 1**: Write the given general solution containing two arbitrary constants.
- **Step 2**: Differentiate both sides and remove one arbitrary constant.
- **Step 3**: Differentiate again to remove second arbitrary constant.

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EXAMPLE

Form the differential equation for $y = \underline{A\cos\alpha x + B\sin\alpha x}$, where A and B are constant.

$$\frac{dy}{dx} = A(-\sin\alpha x)\alpha + B(\cos\alpha x)\alpha = \alpha(-A\sin\alpha x + B\cos\alpha x)$$

$$\frac{d^2y}{dx^2} = \alpha\left(-(A\cos\alpha x)(\alpha) - (B\sin\alpha x)(\alpha)\right) = -\alpha^2\left(\underline{A\cos\alpha x + B\sin\alpha x}\right)$$

$$= -\alpha^2 y$$

$$\frac{d^2y}{dx^2} + \alpha^2 y = 0$$

DIFFERENTIAL EQUATION OF FAMILY OF CURVES

- In such cases, we have to use our knowledge of particular curves.
- Curves such as straight line, parabola, hyperbola, circles are considered.
- We have to follow similar steps of differentiation and have to remove arbitrary constants.

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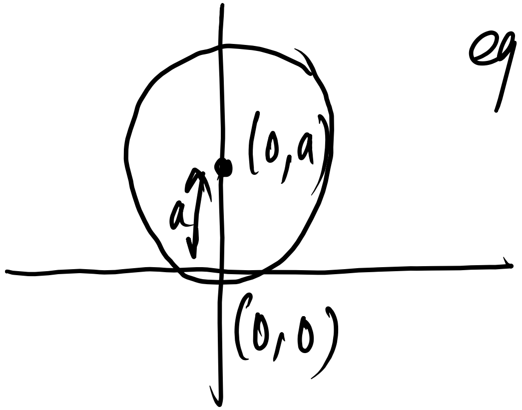
EXAMPLE

Find the differential equation representing the family of circles passing through origin and having centre on $y-$axis.



eqn of circle, $(x-0)^2+(y-a)^2 = a^2$

$x^2+(y-a)^2 = a^2$

$x^2+y^2-2ay+a^2 = a^2$

$x^2+y^2-2ay = 0$

$\frac{x^2+y^2}{2y} = a$

(OR)

Differentiate,

$2x + 2(y-a)\frac{dy}{dx} = 0$

$-2(y-a)\frac{dy}{dx} = 2x$

$a-y = \frac{x}{\left(\frac{dy}{dx}\right)}$

$$\frac{x^2+y^2}{2y} = a$$

Differentiate,

$$\frac{2y\left(2x+2y\frac{dy}{dx}\right)-(x^2+y^2)\left(2\frac{dy}{dx}\right)}{(2y)^2} = 0$$

$$\frac{dy}{dx}\left(4y^2-2x^2-2y^2\right) = -4xy$$

$$\frac{dy}{dx} = \frac{4xy}{2(x^2-y^2)}$$

(OR) $a = \dfrac{x}{\left(\frac{dy}{dx}\right)} + y$

$$\left(\frac{dy}{dx}\right) = p$$

$$a = \frac{x+yp}{p}$$

$$x^2+y^2-2ay = 0$$

$$x^2+y^2-2\left(\frac{x+yp}{p}\right) = 0$$

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VARIABLE SEPARABLE

- A differential equation is said to have separable variables if it is of the form

$$f(x)dx = g(y)dy.$$

soln. of a diff. eqn ≡ eqn. of curve

first order

first degree

Variable Seperable

homogeneous

Linear diff eqn.

METHOD TO SOLVE

- **Step 1**: Express the given differential equation in the form $f(x)dx = g(y)dy$.
- **Step 2**: On integrating both sides, we get $\int f(x)dx = \int g(y)dy$
- **Step 3**: Evaluate the above integrals to get the solution of the form

  $F(x) = G(y) + c$, where c is an arbitrary constant.

Remark: The constants of integration that appear on both the sides are combined together to give just one arbitrary constant C.

EXAMPLE

Solve the differential equation $\frac{dy}{dx} = (1 + x^2)(1 + y^2)$.

$$\int \frac{1}{1+y^2}\, dy = \int (1+x^2)\, dx$$

$$\tan^{-1}(y) = x + \frac{x^3}{3} + c$$

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PARTICULAR SOLUTION

- A solution obtained by giving particular values to the arbitrary constants in the general solution of the differential equation is called a particular solution of that differential equation.
- In other words, a particular solution is a solution that is free from arbitrary constants.

General solution : constant c is kept.

Particular solution : value of c is substituted.

EXAMPLE

Find the particular solution of the differential equation

$\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$, given that $\boxed{y = 1 \text{ when } x = 0.}$

$$\frac{dy}{dx} = (1+x^2) + y^2(1+x^2) = (1+y^2)(1+x^2)$$

$$\int \frac{1}{1+y^2}\,dy = \int (1+x^2)\,dx$$

$$\tan^{-1}(y) = x + \frac{x^3}{3} + c$$

$y = 1$

$x = 0$

$$\frac{\pi}{4} = 0 + 0 + c$$

$$c = \frac{\pi}{4}$$

$$\tan^{-1} y = x + \frac{x^3}{3} + \frac{\pi}{4}$$

HOMOGENEOUS FUNCTION

- A Function $F(x, y)$ is said to be homogeneous function of degree n, if

  $F(\alpha x, \alpha y) = \alpha^n F(x, y)$ for any non-zero constant α.

  Example: $F_1(x, y) = 5x^2 - 6xy + y^2$

  $5(\alpha^2)x^2 - 6(\alpha x)(\alpha y) + (\alpha y)^2$

  $\alpha^2(5x^2 - 6xy + y^2) = \alpha^2 F_1(x, y)$

  degree 2 ✓

- A Differential equation is said to be homogeneous if it is of the form

  $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$ or $\frac{dx}{dy} = \frac{f(x,y)}{g(x,y)}$ where both $f(x, y)$ and $g(x, y)$ are homogeneous function of same degree n.

METHOD TO SOLVE

- **Step 1**: Check the given differential equation in the homogeneous form

$$\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$$

- **Step 2**: Put $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ in given equation to get

$$v + x\frac{dv}{dx} = F(v)$$

variable - seperable form,

i.e., $\frac{1}{F(v)-v}dv = \frac{1}{x}dx.$

METHOD TO SOLVE

- **Step 3**: Integrate both sides to get a solution in terms of v and x of the form

  $\boldsymbol{G(v) = logx + c}$, where c is an arbitrary constant.

- **Step 4**: Replace v by $\frac{y}{x}$ to get the required solution in terms of x and y.

  $y = vx$

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EXAMPLE

Find general solution of the differential equation $x\frac{dy}{dx} - y = \sqrt{x^2 + y^2}$

$\frac{dy}{dx} = \frac{\sqrt{x^2+y^2} + y}{x}$ $\quad \underline{\propto f(x,y)}$

$\underline{y = vx}$

$v + x\frac{dv}{dx} = \frac{\sqrt{x^2(1+v^2)} + vx}{x}$

$v + x\frac{dv}{dx} = \frac{\sqrt{1+v^2} + v}{1}$

$x\frac{dv}{dx} = \sqrt{1+v^2}$

$\int \frac{1}{\sqrt{1+v^2}}\,dv = \int \frac{1}{x}\,dx$

$\log\left|v + \sqrt{1+v^2}\right| = \log x + \log C$

$v + \sqrt{1+v^2} = Cx$

$\frac{y}{x} + \sqrt{1+\frac{y^2}{x^2}} = Cx$

LINEAR DIFFERENTIAL EQUATIONS

- A differential equation is said to be linear differential equation of first order if it can be put in either of the forms :

Type 1: $\frac{dy}{dx} + Py = Q$, where P and Q are constants or functions of x only.

Type 2: $\frac{dx}{dy} + Px = Q$, where P and Q are constants or functions of y only.

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METHOD TO SOLVE TYPE 1

- **Step 1**: Consider the given Linear differential equation in the form $\frac{dy}{dx} + Py = Q$
- **Step 2**: Identify P and Q. ~ $f(x)$
- **Step 3**: Evaluate the Integrating Factor, $I.F. = e^{\int P\,dx}$
- **Step 4**: General solution of the given linear differential equation is

$$\boxed{(I.F.)y = \int (I.F.)Q\,dx + c}$$

- **Step 5**: Evaluate the above integral to get the required solution of the given differential equation.

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EXAMPLE

Find the general solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = e^x$.

$\frac{dy}{dx} + \left(\frac{1}{x}\right)y = e^x$, of form $\frac{dy}{dx} + Py = Q$

$IF = e^{\int P dx} = e^{\int \frac{1}{x} dx} = e^{\log x} = x$

Soln.

$y \times IF = \int (IF) Q \, dx + c$

$yx = \int x(e^x) \, dx + c$

$xy = e^x(x-1) + c$

$\int x e^x dx$ (I, II)

$= xe^x - \int 1 \cdot e^x dx$

$= xe^x - e^x$

METHOD TO SOLVE TYPE 2

- **Step 1**: Consider the given Linear differential equation in the form $\frac{dx}{dy} + Px = Q$
- **Step 2**: Identify P and Q.
- **Step 3**: Evaluate the Integrating Factor, $I.F. = e^{\int P\,dy}$
- **Step 4**: General solution of the given linear differential equation is

  $$(I.F.)x = \int (I.F.)Q\,dy + c$$

- **Step 5**: Evaluate the above integral to get the required solution of the given differential equation.

EXAMPLE

Find the general solution of the differential equation $(x+y)\frac{dy}{dx} = 1.$

$$\frac{dy}{dx} = \frac{1}{x+y}$$

$$x+y = \frac{1}{\left(\frac{dy}{dx}\right)} = \frac{dx}{dy}$$

$$\frac{dx}{dy} - x = y \quad \Big| \quad \frac{dx}{dy} + Px = Q \quad \Big| \quad \left.\begin{array}{l} P = -1 \\ Q = y \end{array}\right\} \text{functions of } y.$$

$$IF = e^{\int P\,dy} = e^{\int(-1)\,dy} = e^{-y}$$

Soln.

$$x(IF) = \int (IF)\,y\,dy + C$$

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REDUCIBLE D.E.

- In this section, we shall discuss the equations of form $\frac{dy}{dx} = f(ax + by + c)$
- These can be reduced to the variable separable form by the substitution,

$$ax + by + c = v \qquad \frac{dv}{dx} = f(v)$$

variable

- Further, derived equation can be solved by the method of separation of variables which we had already discussed in previous lectures.

Seperable.

.

EXAMPLE

Find the general solution of the differential equation $(x + y + 1)\frac{dy}{dx} = 1.$

$$\frac{dy}{dx} = \frac{1}{x+y+1} \quad \text{of form} \quad \frac{dy}{dx} = f(ax+by+c)$$

$$v = x+y+1$$

$$\frac{dv}{dx} = 1 + \frac{dy}{dx}$$

$$\frac{dv}{dx} - 1 = \frac{1}{v}$$

$$\frac{dv}{dx} = \frac{1+v}{v}$$

$$\int \frac{v}{1+v}\,dv = \int dx$$

$$\int \frac{1+v}{1+v}\,dv - \int \frac{1}{1+v}\,dv = \int dx$$

$$\underline{v - \log(1+v) = x + c}$$

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PROBLEMS ON EXPONENTIAL GROWTH

Suppose a quantity y is growing with time t in such a way that its growth rate at any time is proportional to its value at that time,

i.e., $\frac{dy}{dt} \propto y$

$\frac{dy}{dt} = ky$, for some constant k

$\frac{1}{y} dy = kdt$

PROBLEMS ON EXPONENTIAL GROWTH

Integrating both sides, we get

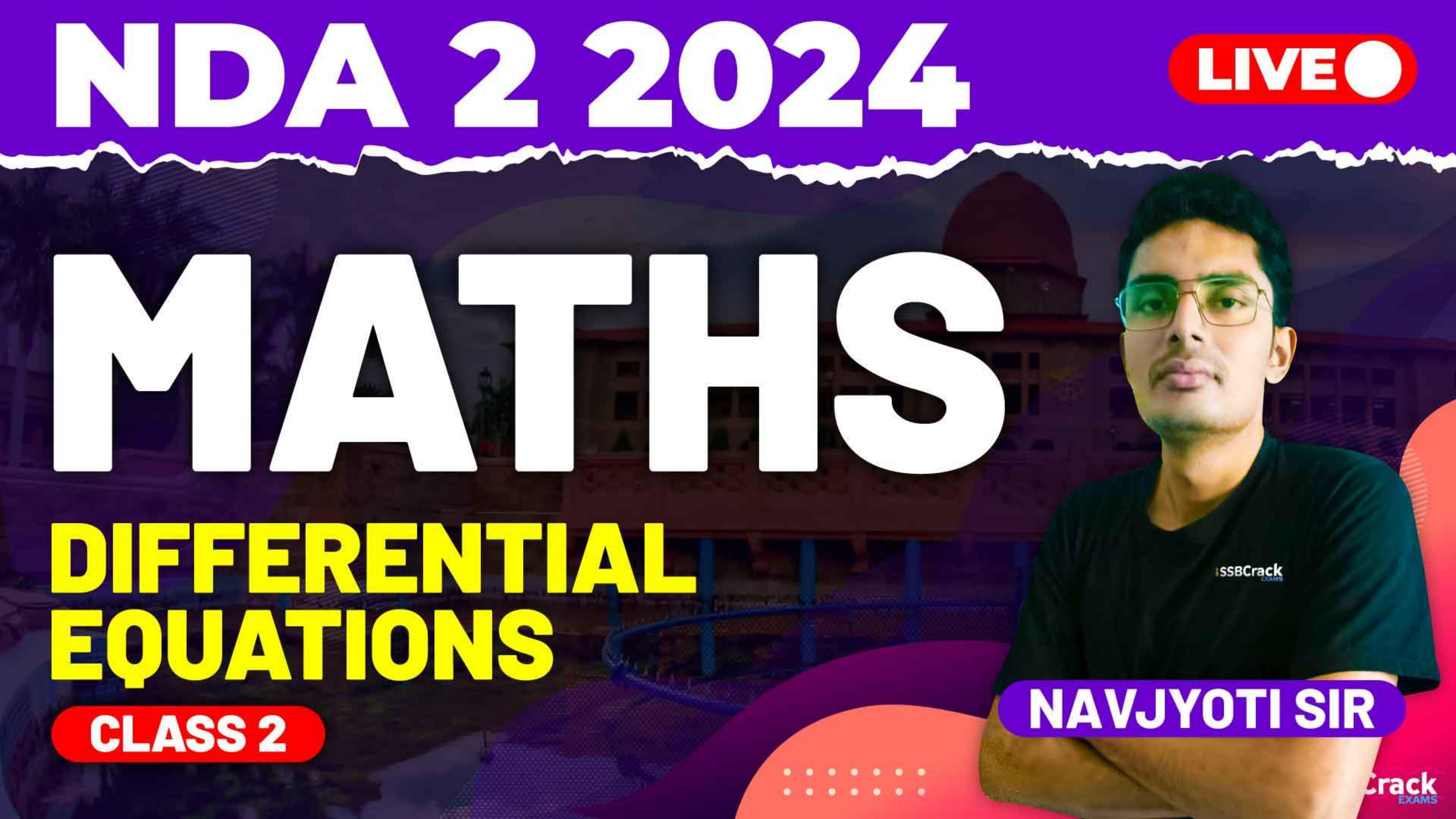
$$\int \frac{1}{y} dy = k \int 1 dt$$

$$\log|y| = kt + \log C$$

$$\log\left(\frac{y}{C}\right) = kt$$

$$y = Ce^{kt}$$

Such a growth is called exponential growth.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