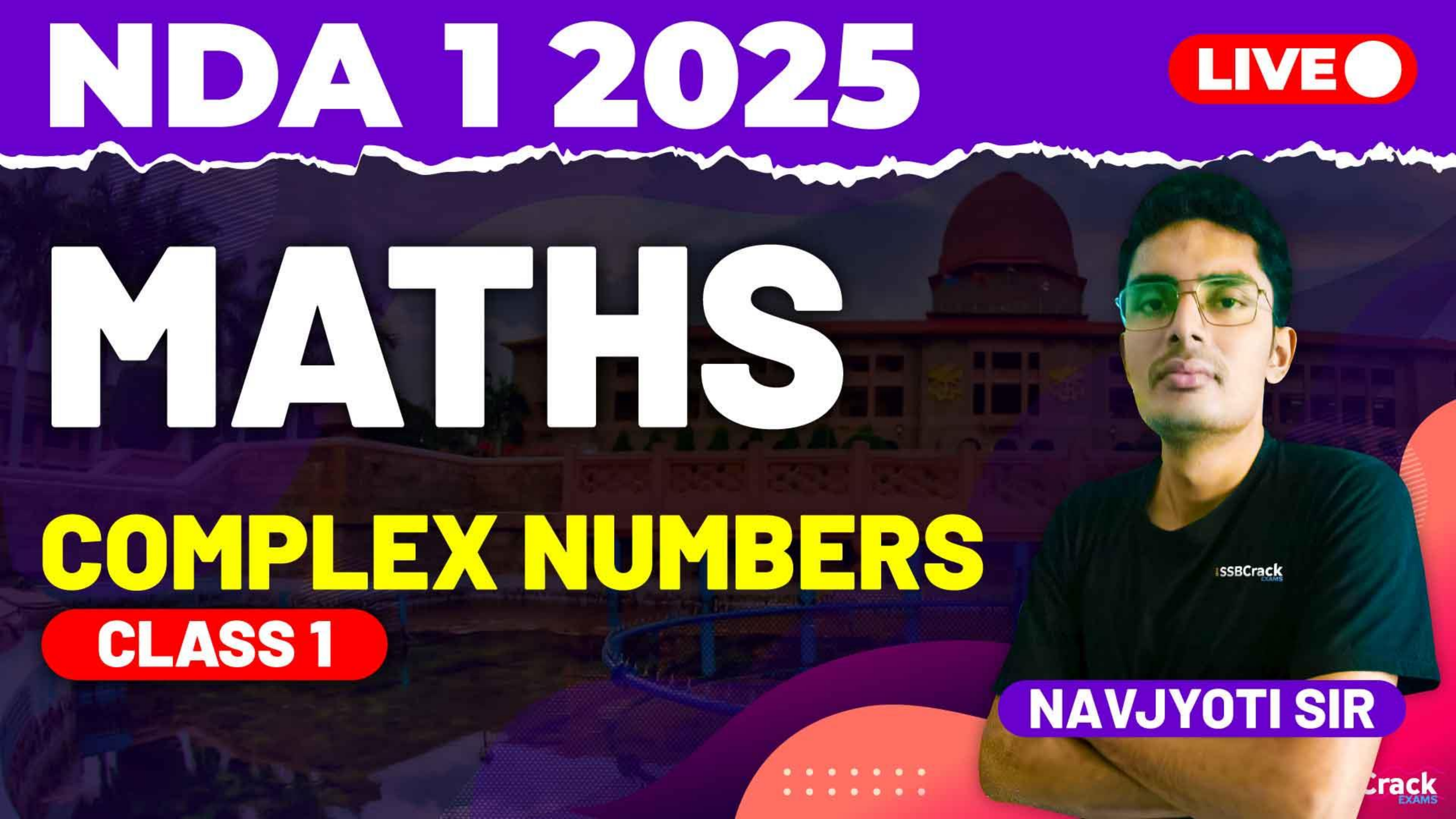

NDA 1 2025
LIVE
MATHS
COMPLEX NUMBERS
CLASS 1
NAVJYOTI SIR
SSBCrack EXAMS
Crack EXAMS

16 Oct 2024 Live Classes Schedule

9:00AM	16 OCTOBER 2024 DAILY DEFENCE UPDATES	DIVYANSHU SIR

SSB INTERVIEW LIVE CLASSES

9:30AM	OVERVIEW ON GPE & PRACTICE	ANURADHA MA'AM

NDA 1 2025 LIVE CLASSES

1:00PM	BIOLOGY - MCQ - CLASS 7	SHIVANGI MA'AM
4:00PM	MATHS - COMPLEX NUMBERS - CLASS 1	NAVJYOTI SIR
5:30PM	ENGLISH - ANTONYMS - CLASS 1	ANURADHA MA'AM

CDS 1 2025 LIVE CLASSES

1:00PM	BIOLOGY - MCQ - CLASS 7	SHIVANGI MA'AM
5:30PM	ENGLISH - ANTONYMS - CLASS 1	ANURADHA MA'AM
7:00PM	MATHS - PROBABILITY - CLASS 1	NAVJYOTI SIR

AFCAT 1 2025 LIVE CLASSES

4:00PM	STATIC GK - RAMSAR & LAKES IN INDIA	DIVYANSHU SIR
5:30PM	ENGLISH - ANTONYMS - CLASS 1	ANURADHA MA'AM
7:00PM	MATHS - PROBABILITY - CLASS 1	NAVJYOTI SIR

NUMBERS AND SETS

$N \longrightarrow$ Natural Numbers $\{1, 2, 3 \ldots\}$

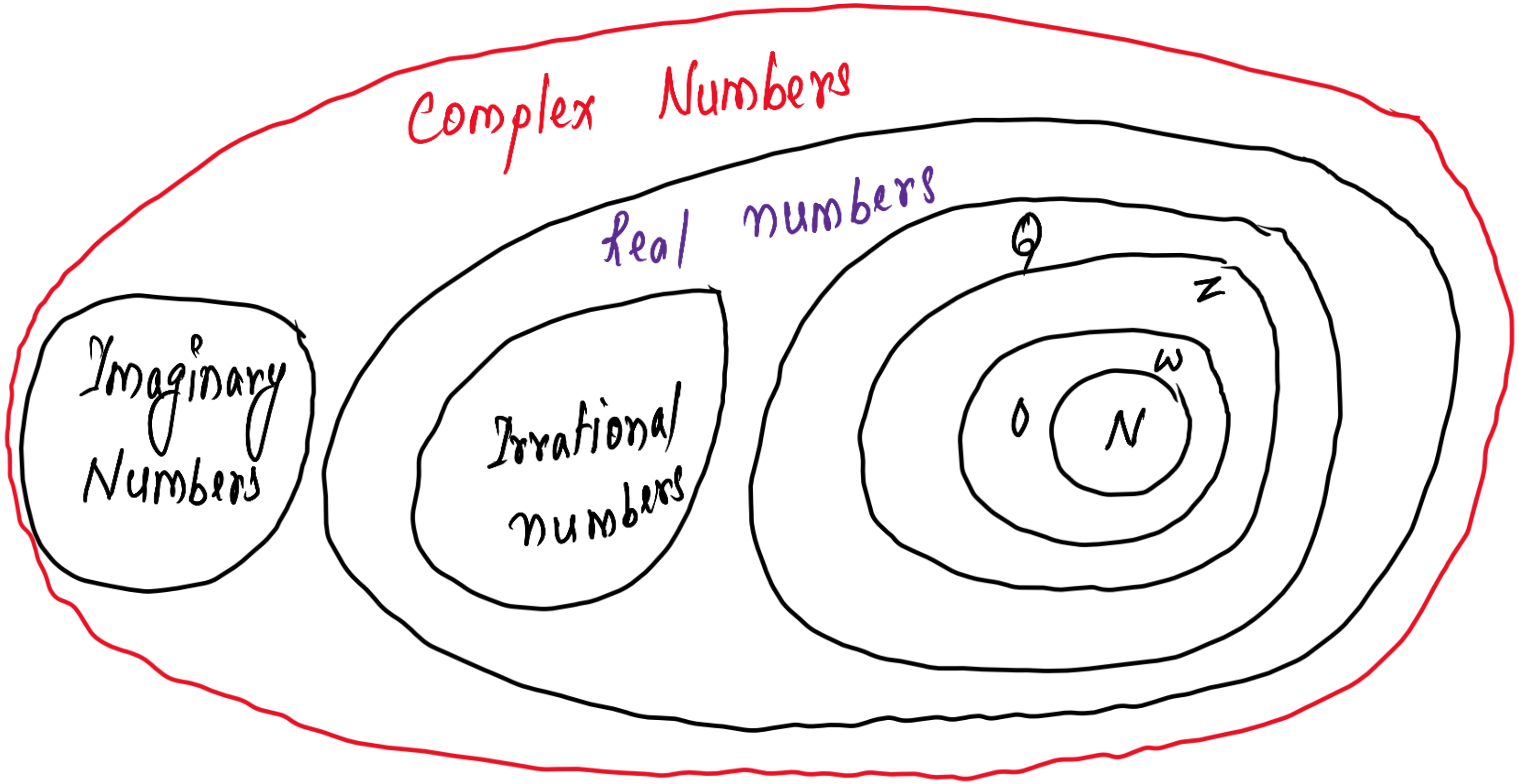
$W \longrightarrow$ Whole numbers $\{0, 1, 2, 3 \ldots\}$

$Z \longrightarrow$ Integers $\{\ldots -3, -2, -1, 0, 1, 2, 3 \ldots\}$

$Q \longrightarrow$ Rational numbers $\left(\frac{p}{q},\ p, q \in Z \ \& \ q \neq 0\right)$

$R \longrightarrow$ Real Numbers (containing rational and irrational numbers)

$\sqrt{-4} \longrightarrow$ Imaginary numbers



SSBCrack EXAMS

IMAGINARY NUMBERS AND i (IOTA)

$$\sqrt{-4} = \sqrt{4} \times \sqrt{-1}$$

$$= 2\sqrt{-1} = \underline{2i} \quad (\text{where } i = \sqrt{-1})$$

$$\sqrt{-23} = \sqrt{23} \times \sqrt{-1} = \sqrt{23}\, i$$

(of the form 'ki' where k is any real number)

SSBCrack EXAMS

COMPLEX NUMBERS

real numbers + imaginary numbers

(of the form 'a + ib' where a and b are real numbers)

$z = a + ib$

real part (a)

imaginary part (b)

→ Represented by z.

→ if $a = 0 \Rightarrow z = bi$ → z is called purely imaginary.

→ if $b = 0 \Rightarrow z = a$ → z is called purely real.

POWERS OF i

$i = \sqrt{-1}$

$i^1 = i \longrightarrow (r=1)$

$i^2 = (\sqrt{-1})^2 = -1 \longrightarrow (r=2)$

$i^3 = i^2 \cdot i = (-1)i = -i \longrightarrow (r=3)$

$i^4 = i^2 \cdot i^2 = (-1)(-1) = 1 \longrightarrow (r=0)$

repeats after every 4th power

$i^n = i^{4m+r} = i^r$

① Divide power on i by 4.

② Check the remainder,

$$\begin{cases} \text{if } r = 0 \Rightarrow 1 \\ \text{if } r = 1 \Rightarrow i \ (i^1) \\ r = 2 \Rightarrow -1 \ (i^2) \\ r = 3 \Rightarrow -i \ (i^3) \end{cases}$$

$\rightarrow i^{465} = i^1 = i$

$\rightarrow i^{-71} = \dfrac{1}{i^{71}} = \dfrac{1}{-i}$

$= -\dfrac{1}{i} \times \dfrac{i}{i} = \dfrac{-i}{(-1)}$

$= i$

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EQUALITY OF COMPLEX NUMBERS

$z_1 = a_1 + ib_1$ $\qquad$ $z_2 = a_2 + ib_2$

$z_1 = z_2$ only when

$$\left.\begin{array}{l} a_1 = a_2 \\ b_1 = b_2 \end{array}\right\}$$

Real parts and imaginary parts are equal to each other.

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OPERATIONS ON COMPLEX NUMBERS

Let $z_1 = a_1 + ib_1$ & $z_2 = a_2 + ib_2$ be two complex numbers,

① Addition

$$z_1 + z_2 = (a_1 + ib_1) + (a_2 + ib_2)$$
$$= (a_1 + a_2) + i(b_1 + b_2)$$

② Subtraction

$$z_1 - z_2 = (a_1 + ib_1) - (a_2 + ib_2)$$
$$= (a_1 - a_2) + i(b_1 - b_2)$$

Real with real part

imaginary with imaginary part

(3) Multiplication :

$$z_1 \cdot z_2 = (a_1 + ib_1)(a_2 + ib_2)$$

$$= a_1 a_2 + a_1(ib_2) + ib_1(a_2) + (ib_1)(ib_2)$$

$$= \underline{a_1 a_2} + i(a_1 b_2 + b_1 a_2) + \underline{i^2}\,\underline{(b_1 b_2)}$$

$$= a_1 a_2 + i(a_1 b_2 + b_1 a_2) - b_1 b_2$$

real

imaginary

$$= (a_1 a_2 - b_1 b_2) + i(a_1 b_2 + b_1 a_2) \qquad [A + iB]$$

(4) Division :

$z_1 = a_1 + ib_1 \qquad z_2 = a_2 + ib_2$

$$\frac{z_1}{z_2} = \frac{a_1 + ib_1}{a_2 + ib_2} \times \frac{a_2 - ib_2}{a_2 - ib_2}$$

$$= \frac{a_1 a_2 + i(b_1 a_2 - a_1 b_2) - i^2 b_1 b_2}{(a_2)^2 - (ib_2)^2}$$

$$= \frac{(a_1 a_2 + b_1 b_2) + i(b_1 a_2 - a_1 b_2)}{a_2^2 + b_2^2} \quad //$$

$$\left(\frac{a_1 a_2 + b_1 b_2}{a_2^2 + b_2^2}\right) + i\left(\frac{b_1 a_2 - a_1 b_2}{a_2^2 + b_2^2}\right)$$

What is $(1+i)^4 + (1-i)^4$ equal to, where $i = \sqrt{-1}$? **(PYQ)**

(a) 4

(b) 0

(c) −4

(d) −8 ✓

$$((1+i)^2)^2 + ((1-i)^2)^2$$

$$(1+i)^2 = 1 + 2i + i^2 = 1 + 2i - 1 = 2i$$

$$(1-i)^2 = 1 - 2i + i^2 = 1 - 2i - 1 = -2i$$

$$= (2i)^2 + (-2i)^2 = 4i^2 + 4i^2$$

$$= 8i^2 = 8(-1) = -8$$

CONJUGATE OF COMPLEX NUMBERS

Let $z = a + ib$, then conjugate of z,

$\bar{z} = a - ib$ (changing the sign of imaginary part)

$2 + 3i \longrightarrow 2 - 3i$

$4i - 6 \longrightarrow -4i - 6$

$-7i + \sqrt{3} \longrightarrow 7i + \sqrt{3}$

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PROPERTIES OF CONJUGATE OF COMPLEX NUMBERS

$\overline{(\overline{z})} = z$

$z + \overline{z} = 2\,Re(z)$, $z - \overline{z} = 2\,i\,Im(z)$

$$\begin{array}{r} a+ib \\ +\quad a-ib \\ \hline 2a = 2Re(z) \end{array} \qquad \begin{array}{r} a+ib \\ -\quad a-ib \\ \hline 2ib = i(2b) = 2i\,Im(z) \end{array}$$

$z = \overline{z}$, if z is purely real.

$z + \overline{z} = 0 \Leftrightarrow z$ is purely imaginary

$z = -\overline{z}$

$$\begin{array}{r} ib \\ +\quad -ib \\ \hline 0 \end{array}$$

$z = a$

$\overline{z} = a$

PROPERTIES OF CONJUGATE OF COMPLEX NUMBERS

$$\overline{z_1 + z_2} = \overline{z}_1 + \overline{z}_2$$

$$\overline{z_1 - z_2} = \overline{z}_1 - \overline{z}_2$$

$$\overline{z_1 z_2} = \overline{z}_1 \overline{z}_2$$

$$\overline{\left(\frac{z_1}{z_2}\right)} = \left(\frac{\overline{z}_1}{\overline{z}_2}\right), (z_2 \neq 0)$$

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PROPERTIES OF CONJUGATE OF COMPLEX NUMBERS

$z \cdot \bar{z} = \{Re\ (z)\}^2 + \{Im\ (z)\}^2$.

$$z = a + ib$$

$$\bar{z} = a - ib$$

$$z \cdot \bar{z} = (a+ib)(a-ib)$$

$$= a^2 - (i^2 b^2)$$

$$= a^2 + b^2 = [Re(z)]^2 + [Im(z)]^2$$

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EXAMPLE

For conjugate complex numbers $-3 + ix^2y$ and $x^2 + y + 4i$, the value of x and y is

(a) $x = 1, y = -4$ ✓
(b) $x = 1, y = 1$
(c) $x = 2, y = -4$
(d) $x = 3, y = 4$

$$z_1 = \bar{z}_2$$

$$-3 + ix^2y = (x^2 + y) - 4i$$

$$x^2 + y = -3 \qquad x^2y = -4$$

SSBCrack EXAMS

EXAMPLE

For conjugate complex numbers $-3 + ix^2y$ and $x^2 + y + 4i$, the value of x and y is

(a) $x = 1, y = -4$ (b) $x = 1, y = 1$

(c) $x = 2, y = -4$ (d) $x = 3, y = 4$

Ans: (a)

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MODULUS OF COMPLEX NUMBERS

let $z = a + ib$

$$|z| = \sqrt{a^2 + b^2} = \sqrt{[Re(z)]^2 + [Im(z)]^2}$$

Eg - $z = 3 - 4i$

$$|z| = \sqrt{3^2 + (-4)^2} = \underbrace{5}$$

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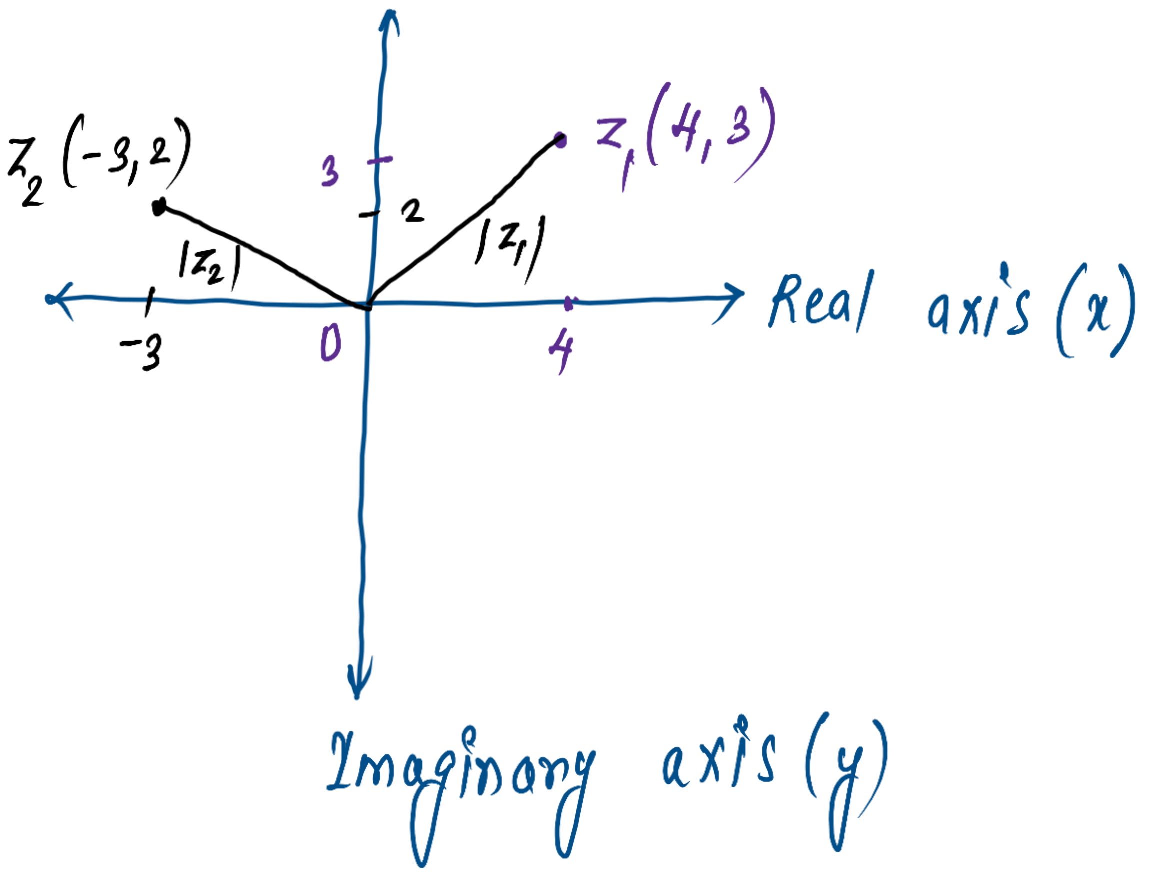
COMPLEX PLANE / ARGAND PLANE

$z = x + iy$



$z_1 = 4 + 3i$; $|z_1| = \sqrt{4^2 + 3^2}$

$z_2 = -3 + 2i$

$\sqrt{(4-0)^2 + (3-0)^2}$

(Modulus represents distance from origin)

Every point on the complex plane represents a complex number.

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PROPERTIES OF MODULUS OF COMPLEX NUMBERS

If $z, z_1,$ and z_2 are complex numbers, then

$$|z| \geq 0 \Rightarrow |z| = 0, \text{iff } z = 0 \text{ and } |z| > 0, \text{iff } |z| \neq 0$$

$|z| = 0 \Rightarrow z = 0 + 0i$

$\sqrt{0^2 + 0^2} = 0$

$$-|z| \leq \text{Re}(z) \leq |z| \text{ and } -|z| \leq \text{Im}(z) \leq |z|$$

$-\sqrt{a^2+b^2} \leq a, b \leq \sqrt{a^2+b^2}$

$$|z| = |\bar{z}| = |-z| = |-\bar{z}|$$

$z = a + ib$ $\quad -z = -a - ib$

$\bar{z} = a - ib$ $\quad -\bar{z} = -a + ib$

$$z\bar{z} = |z|^2$$

$|z| = |\bar{z}| = |-z| = |-\bar{z}| = \sqrt{a^2 + b^2}$

$$z\bar{z} = a^2 + b^2 = \left(\sqrt{a^2+b^2}\right)^2 = |z|^2$$

PROPERTIES OF MODULUS OF COMPLEX NUMBERS

$$|z_1 z_2| = |z_1||z_2|$$

$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}, z_2 \neq 0$$

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PROPERTIES OF MODULUS OF COMPLEX NUMBERS

$$|z_1 \pm z_2| \leq |z_1| + |z_2|$$

$$|z_1 \pm z_2| \geq ||z_1| - |z_2||$$

PROPERTIES OF MODULUS OF COMPLEX NUMBERS

$$|z^n| = |z|^n$$

$$||z_1| - |z_2|| \le |z_1 + z_2| \le |z_1| + |z_2|$$

PROPERTIES OF MODULUS OF COMPLEX NUMBERS

$$|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2\,\mathrm{Re}\,(z_1 \bar{z}_2)$$

$$|z_1 - z_2|^2 = |z_1|^2 + |z_2|^2 - 2\,\mathrm{Re}\,(z_1 \bar{z}_2)$$

$$|z_1 - z_2|^2 + |z_1 + z_2|^2 = 2\,(|z_1|^2 + |z_2|^2)$$

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PROPERTIES OF MODULUS OF COMPLEX NUMBERS

$$|az_1 - bz_2|^2 + |bz_1 + az_2|^2 = (a^2 + b^2)(|z_1|^2 + |z_2|^2)$$

SSBCrack EXAMS

MULTIPLICATIVE INVERSE (RECIPROCAL) OF A COMPLEX NUMBER

$$z = a + ib$$

$$z^{-1} = \frac{1}{z} = \frac{1}{a+ib} \times \frac{a-ib}{a-ib} = \frac{a-ib}{a^2+b^2} = \frac{\bar{z}}{\underbrace{|z|^2}}$$

$$\left(z\bar{z} = |z|^2 \right) \rightarrow \frac{\bar{z}}{|z|^2} = \frac{1}{z} = z^{-1}$$

Let z_1 and z_2 be two complex numbers such that $\left|\frac{z_1+z_2}{z_1-z_2}\right|=1$, then what is $Re\left(\frac{z_1}{z_2}\right)+1$ equal to ?

(a) -1

(b) 0

(c) 1 ✓

(d) 5

(PYQ)

$$\frac{|z_1+z_2|}{|z_1-z_2|} = 1$$

$$|z_1+z_2|^2 = |z_1-z_2|^2$$

$$\left(\begin{array}{l} z_1 = a_1 + ib_1 \\ z_2 = a_2 + ib_2 \end{array}\right)$$

$$(a_1+a_2)^2+(b_1+b_2)^2 = (a_1-a_2)^2+(b_1-b_2)^2$$

$$4a_1a_2 + 4b_1b_2 = 0$$

$$a_1a_2 + b_1b_2 = 0 \quad \text{— } (1)$$

$$\frac{z_1}{z_2} = \frac{a_1 + ib_1}{a_2 + ib_2} \times \frac{a_2 - ib_2}{a_2 - ib_2}$$

$$Re\left(\frac{z_1}{z_2}\right) = \frac{a_1a_2 + b_1b_2}{a_2^2 + b_2^2} = 0 \quad (\text{from } 1)$$

$$Re\left(\frac{z_1}{z_2}\right) + 1 = 0 + 1 = \underbrace{1}$$

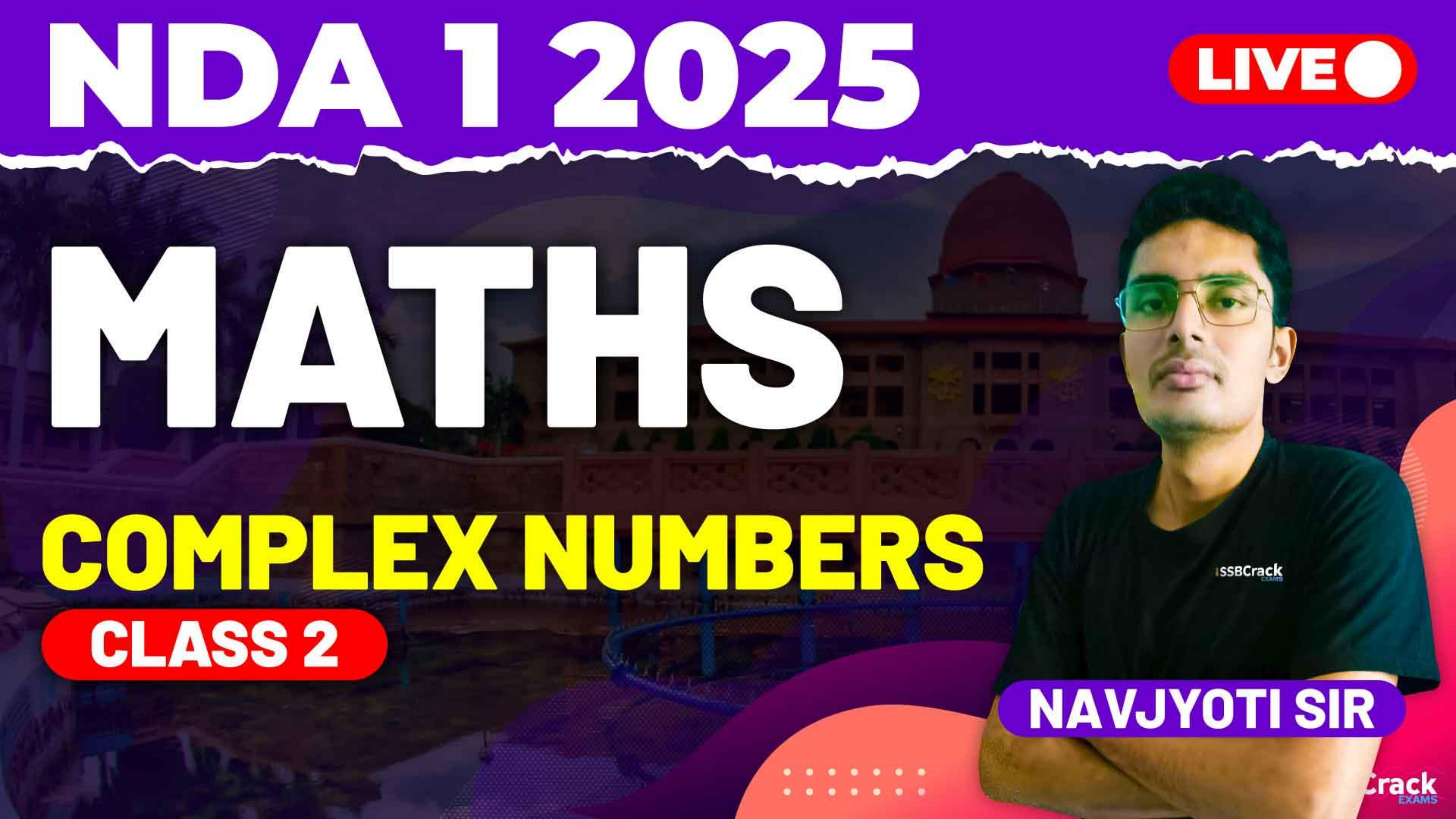
If z is any complex number and $iz^3 + z^2 - z + i = 0$, where $i = \sqrt{-1}$, then what is the value of $\left(|z|+1\right)^2$? **(PYQ)**

(a) 1

(b) 4

(c) 81

(d) 121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