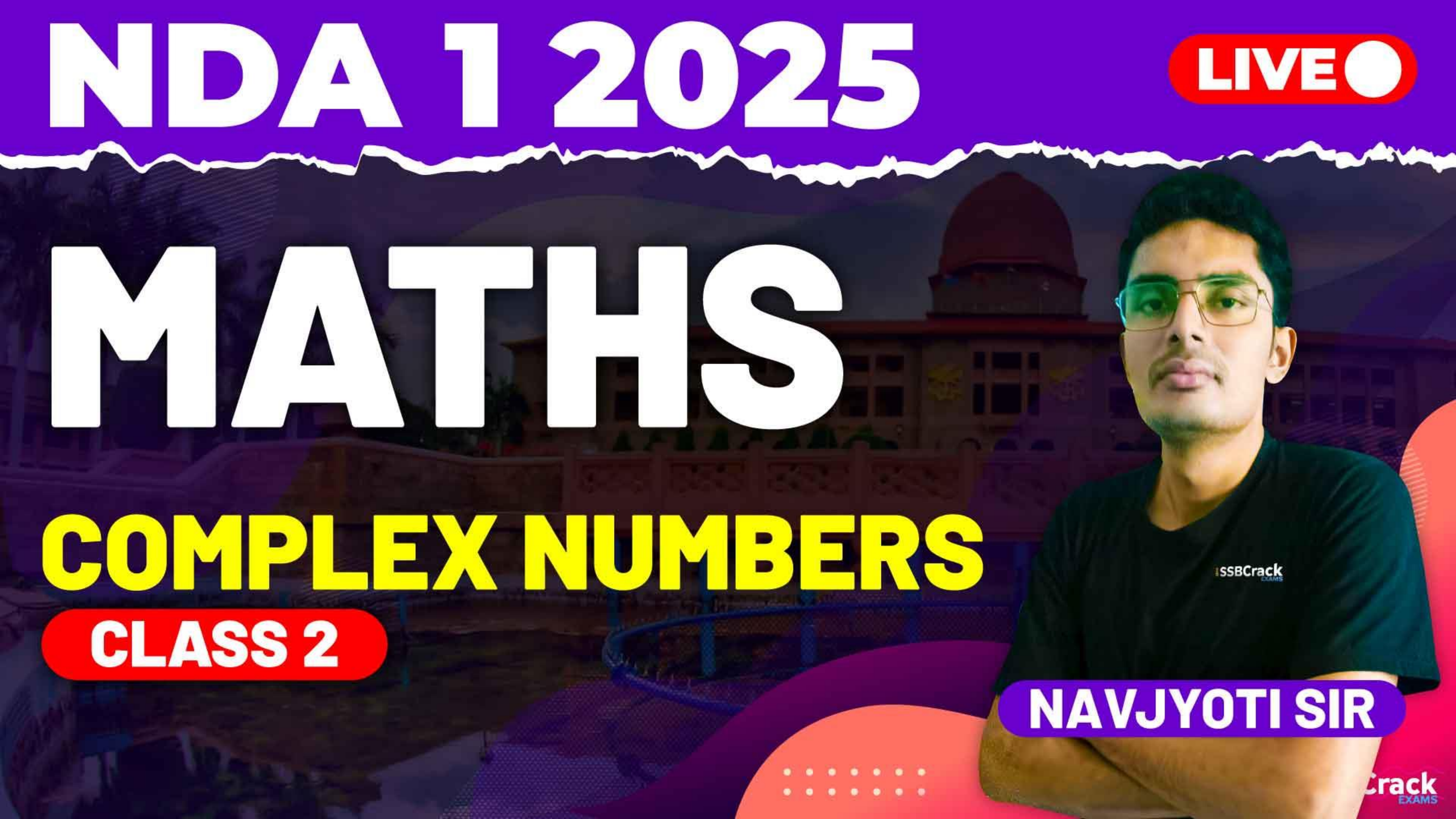
NDA 1 2025
LIVE
MATHS
COMPLEX NUMBERS
CLASS 2
SSBCrack EXAMS
NAVJYOTI SIR
Crack EXAMS

SSBCrack EXAMS

17 Oct 2024 Live Classes Schedule

9:00AM	17 OCTOBER 2024 DAILY DEFENCE UPDATES	DIVYANSHU SIR

SSB INTERVIEW LIVE CLASSES

9:30AM	COMPLETE PSYCH TESTS	ANURADHA MA'AM

NDA 1 2025 LIVE CLASSES

1:00PM	BIOLOGY - MCQ - CLASS 8	SHIVANGI MA'AM
4:00PM	MATHS - COMPLEX NUMBERS - CLASS 2	NAVJYOTI SIR
5:30PM	ENGLISH - ANTONYMS - CLASS 2	ANURADHA MA'AM

CDS 1 2025 LIVE CLASSES

1:00PM	BIOLOGY - MCQ - CLASS 8	SHIVANGI MA'AM
5:30PM	ENGLISH - ANTONYMS - CLASS 2	ANURADHA MA'AM
7:00PM	MATHS - PROBABILITY - CLASS 2	NAVJYOTI SIR

AFCAT 1 2025 LIVE CLASSES

4:00PM	STATIC GK - NATIONAL PARKS & WILDLIFE SANCTUARIES	DIVYANSHU SIR
5:30PM	ENGLISH - ANTONYMS - CLASS 2	ANURADHA MA'AM
7:00PM	MATHS - PROBABILITY - CLASS 2	NAVJYOTI SIR

If z is any complex number and $iz^3 + z^2 - z + i = 0$, where $i = \sqrt{-1}$, then what is the value of $(|z|+1)^2$?

(a) 1

(b) 4 ✓

(c) 81

(d) 121

$z = i = \underline{0 + 1i}$

$i \cdot i^3 + i^2 - i + i$

$i^4 + i^2 = 1 - 1 = 0$

$|z| = \sqrt{0^2 + (1)^2} = 1$

$(|z|+1)^2 = (1+1)^2 = 4$

SQUARE ROOT OF COMPLEX NUMBER

$z = a + ib$

$\sqrt{a+ib} = x + iy$

Squaring,

$a + ib = x^2 + (iy)^2 + 2xyi$

$a + ib = (x^2 - y^2) + i(2xy)$

Compare real and imaginary parts,

$a = x^2 - y^2$ — (1)

$b = 2xy$ — (2)

$(x^2+y^2)^2 = (x^2-y^2)^2 + 4x^2y^2$

$(x^2+y^2)^2 = a^2 + b^2 \rightarrow (2xy)^2$

$x^2+y^2 = +\sqrt{a^2+b^2}$ — (3)

Using eqns (1) and (3),

$$x^2 - y^2 = a \quad \text{——— (4)}$$

$$x^2 + y^2 = \sqrt{a^2+b^2} \quad \text{——— (5)}$$

Using (4) and (5),

(4) + (5),

$$x^2 = \frac{1}{2}\left(a + \sqrt{a^2+b^2}\right)$$

(2 values of x < +ve, -ve)

(5) − (4),

$$y^2 = \frac{1}{2}\left(a - \sqrt{a^2+b^2}\right)$$

(2 values of y < +ve, -ve)

from eqn (2),

$2xy = b$

If $b > 0 \Rightarrow xy > 0$] (Case - I)

$x \rightarrow (+)$; $y \rightarrow (+)$

$x \rightarrow (-)$; $y \rightarrow (-)$

one of them is solution.

giving a square roots.

If $b < 0 \Rightarrow xy < 0$ (Case - II)

$x \rightarrow (+)$; $y \rightarrow (-)$

$x \rightarrow (-)$; $y \rightarrow (+)$

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EXAMPLE

The square root of (3 – 4 *i*) is

(a) ± (2 – *i*) (b) ± 3 *i* (c) ± (3 – *i*) (d) ± (4 – *i*)

$\sqrt{3-4i} = x + iy$

$x^2 - y^2 = 3$

$2xy = -4$

$(xy = -ve)$

$a = 3$ (real part)

$b = -4$ (imaginary part)

$x^2 = \frac{1}{2}\left(3 + \sqrt{3^2 + (-4)^2}\right) = \frac{1}{2}(3+5)$

$x^2 = 4 \Rightarrow x = 2, -2$

$y^2 = \frac{1}{2}\left(3 - \sqrt{3^2 + (-4)^2}\right)$

$y^2 = -1 \Rightarrow y = \pm i$

x — $2, -2$

y — $i, -i$

$x + iy = 2 - i \; ; \; -2 + i$

$\pm(2 - i)$

SSBCrack EXAMS

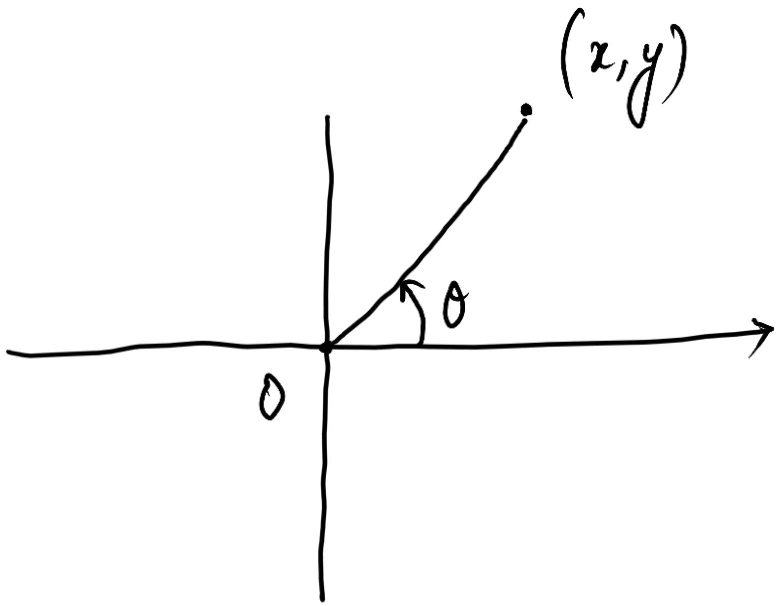
EXAMPLE

The square root of $(3 - 4i)$ is

(a) $\pm(2 - i)$ (b) $\pm 3i$ (c) $\pm(3 - i)$ (d) $\pm(4 - i)$

Ans: (a)

ARGUMENT



$z = x + iy$

$|z| = \sqrt{x^2 + y^2}$

$\theta = \tan^{-1}\left|\frac{y}{x}\right|$

$\theta \rightarrow$ argument / amplitude

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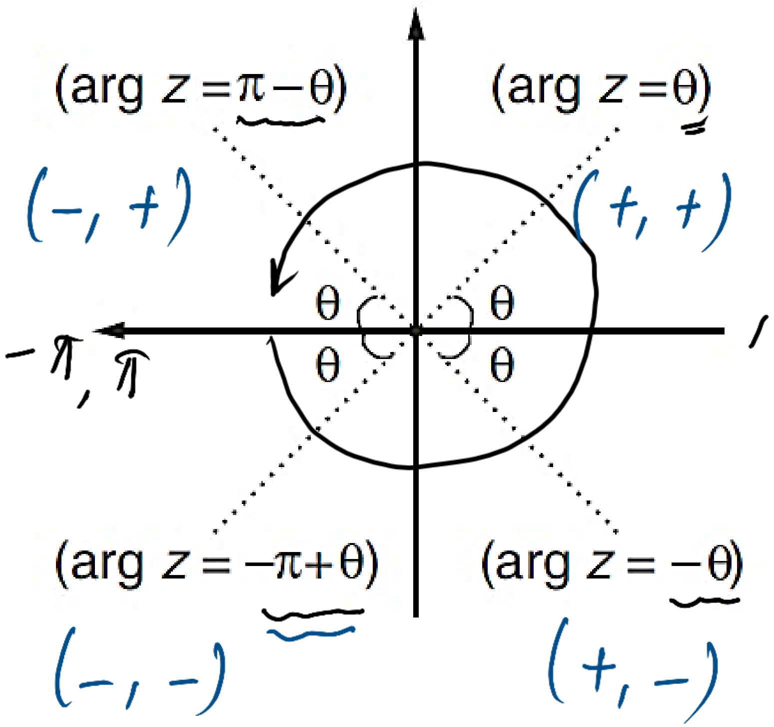
PRINCIPAL VALUE OF ARGUMENT

The unique value of θ such that $-\pi \le \theta \le \pi$ is called the principal argument.



+ve +ve

$$\arg(1+i) = \tan^{-1}\frac{1}{1} = \frac{\pi}{4}$$

$$\arg(-1-i) = -\pi + \frac{\pi}{4}$$

$$\arg(1-i) = -\frac{\pi}{4}$$

$$\arg(-1+i) = \pi - \frac{\pi}{4}$$

PROPERTIES OF ARGUMENT

$$\arg(z_1 \cdot z_2) = \arg(z_1) + \arg(z_2)$$

just like $\log(a \cdot b) = \log a + \log b$

$$\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$$

$\log\left(\frac{a}{b}\right) = \log a - \log b$

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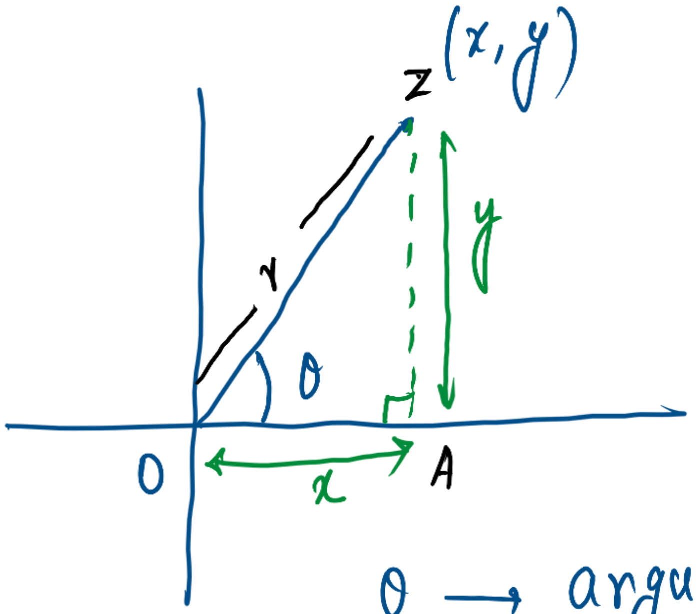
POLAR FORM



$\theta \rightarrow$ argument

$z = x + iy$

$|z| = r = \sqrt{x^2 + y^2}$

ΔOAZ,

$\cos\theta = \dfrac{x}{r}$; $\sin\theta = \dfrac{y}{r}$

$z = x + iy$

$z = r\cos\theta + i r\sin\theta$

polar form of $z \rightarrow$ $z = r(\cos\theta + i\sin\theta) = re^{i\theta}$

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QUESTION

Write the polar form of i + $\sqrt{3}$.

$\sqrt{3} + 1i$ | $x = \sqrt{3}$, $y = 1$

$r = |z| = \sqrt{(\sqrt{3})^2 + (1)^2} = \sqrt{3+1} = \underbrace{2}$

$\tan^{-1}\left|\frac{y}{x}\right| = \tan^{-1}\left|\frac{1}{\sqrt{3}}\right| = \frac{\pi}{6}$

$(x, y) \longrightarrow (+, +)$ [1st quadrant] — (θ) — $\theta = \frac{\pi}{6}$

$z = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

(OR)

$x = r\cos\theta$

$\cos\theta = \frac{x}{r} = \frac{\sqrt{3}}{2}$

$\sin\theta = \frac{y}{r} = \frac{1}{2}$

$\theta = \frac{\pi}{6}$ (1st quadrant)

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DE-MOIVRE's THEOREM

→ To find n^{th} power of a complex number, z.

$$z = x + iy = r(\cos\theta + i\sin\theta) = re^{i\theta}$$

$$z^n = \left(re^{i\theta}\right)^n = r^n e^{i(n\theta)} = \underbrace{r^n\left(\cos n\theta + i\sin n\theta\right)}$$

CUBE ROOTS OF UNITY

$$x^3 = 1$$

$$x^3 - 1^3 = 0$$

$$(x-1)(x^2+x+1) = 0$$

$$x - 1 = 0$$

$$x = 1$$

$$x = 1$$

$$x^2 + x + 1 = 0$$

$$x = \frac{-1 \pm \sqrt{1 - 4 \times 1 \times 1}}{2} = \frac{-1 \pm \sqrt{-3}}{2}$$

$$x = \frac{-1 + \sqrt{3}i}{2} \;;\; x = \frac{-1 - \sqrt{3}i}{2}$$

$x = 1$ —— real root

$\left(\frac{-1+\sqrt{3}i}{2}\right)$ $\left(\frac{-1-\sqrt{3}i}{2}\right)$ } imaginary roots

(ω) (ω^2)

(omega)

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PROPERTIES OF CUBE ROOTS OF UNITY

(1) product of cubic roots of unity is 1.

$$(1)(\omega)(\omega^2) = \omega^3 = 1$$

<u>powers of ω</u> : cyclic after every 3rd power.

$$\left.\begin{aligned} \omega^{3n} &= 1 \\ \omega^{3n+1} &= \omega^1 = \omega \\ \omega^{3n+2} &= \omega^2 \end{aligned}\right\}$$

Divide power by 3. Check the remainder (r).

$r = 0 \longrightarrow 1$

$r = 1 \longrightarrow \omega$

$r = 2 \longrightarrow \omega^2$

(2) $1 + \omega + \omega^2 = 0$ (sum of cubic roots of unity is 0)

If $a + b\omega + c\omega^2 = 0 \Rightarrow a = b = c$ where a, b & c are real numbers.

(3) $z^2 + z + 1 = 0$

roots $\rightarrow z = \omega, \omega^2$

(4) cubic roots of $-1 \Rightarrow -1, -\omega, -\omega^2$

(5) $(\omega^2)^2 = \omega$ | $\overline{\omega} = \omega^2$

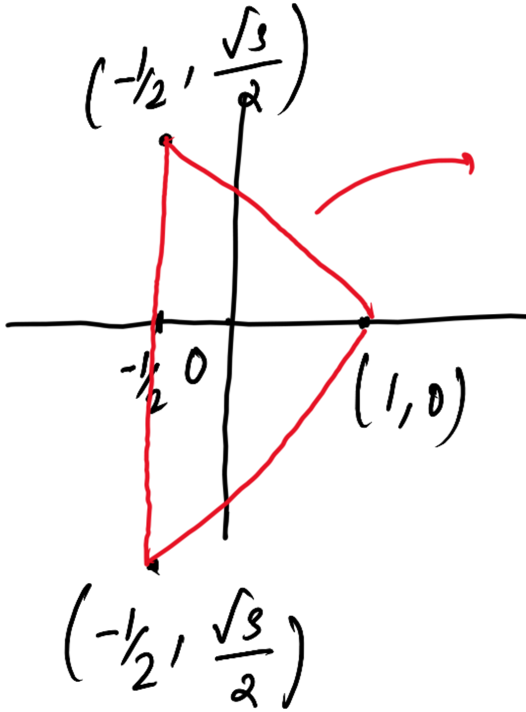
$(\omega)^2 = \omega^2$ | $\overline{\omega^2} = \omega$

6.

$1 \longrightarrow 1 + 0i$

$\omega \longrightarrow -\frac{1}{2} + \frac{\sqrt{3}}{2} i$

$\omega^2 \longrightarrow -\frac{1}{2} - \frac{\sqrt{3}}{2} i$



forms an equilateral triangle

7.

$z^3 - 1 = (z-1)(z-\omega)(z-\omega^2)$

If $\omega \neq 1$ is a cube root of unity, then what is $(1+\omega-\omega^2)^{100}+(1-\omega+\omega^2)^{100}$ equal to? **(PYQ)**

(a) $2^{100}\omega^2$

(b) $2^{100}\omega$

(c) 2^{100}

(d) -2^{100} ✓

$$1+\omega+\omega^2=0$$

$$(1+\omega-\omega^2)^{100}+(1-\omega+\omega^2)^{100}$$

$$(-\omega^2-\omega^2)^{100}+(-\omega-\omega)^{100}$$

$$=(-2\omega^2)^{100}+(-2\omega)^{100}$$

$$=2^{100}(\omega^{200}+\omega^{100})$$

$$=2^{100}(\omega^2+\omega^1) \quad \Big| \quad 2^{100}(-1)=-2^{100}$$

If x, y and z are the cube roots of unity, then what is the value of $xy + yz + zx$?

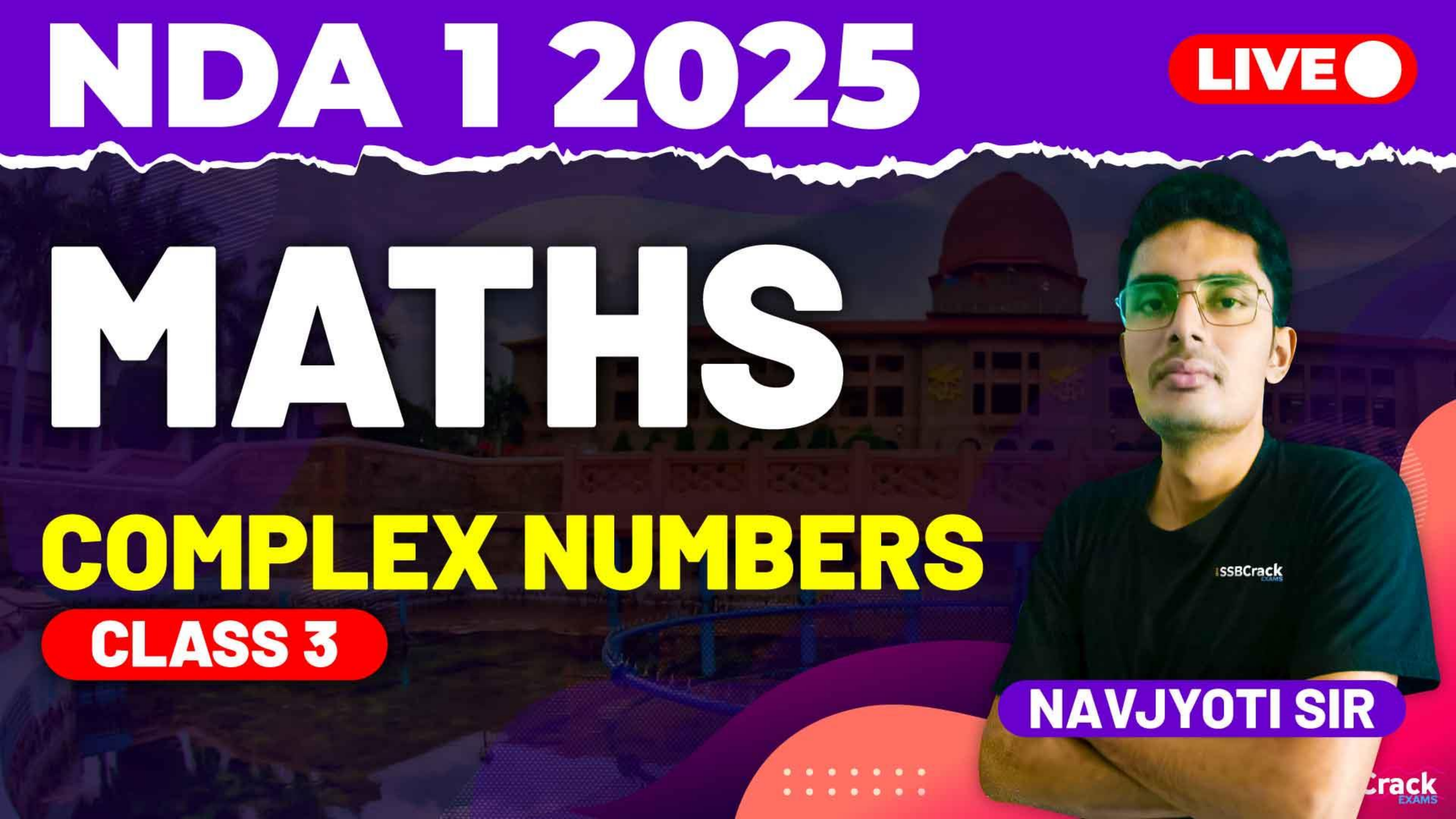
(a) 0

(b) 1

(c) 2

(d) 3

(PYQ)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