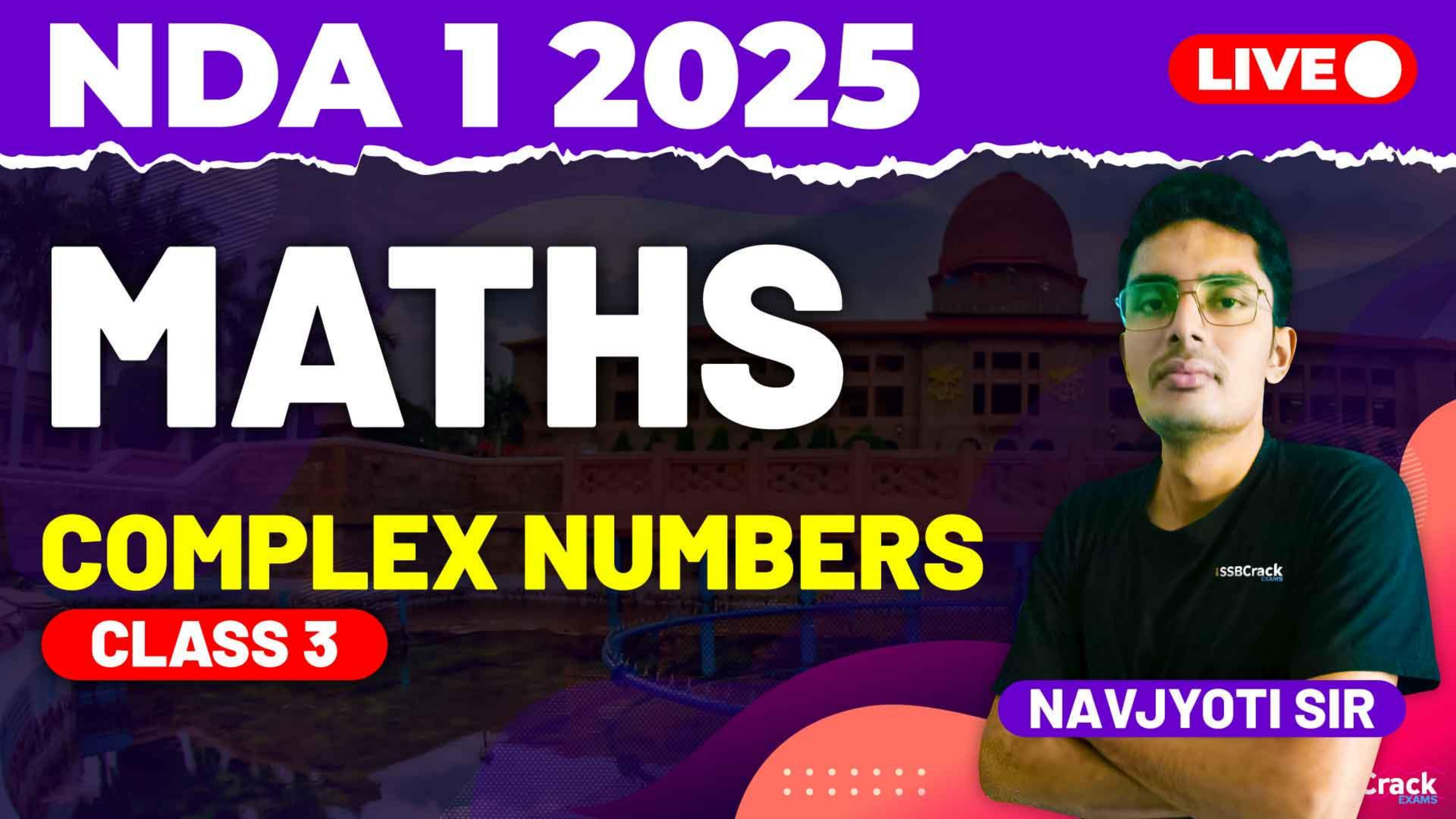

NDA 1 2025
LIVE
MATHS
COMPLEX NUMBERS
CLASS 3
NAVJYOTI SIR

18 Oct 2024 Live Classes Schedule

9:00AM	18 OCTOBER 2024 DAILY DEFENCE UPDATES	DIVYANSHU SIR

SSB INTERVIEW LIVE CLASSES

9:30AM	COMPLETE SCREENING TESTS	ANURADHA MA'AM

NDA 1 2025 LIVE CLASSES

1:00PM	CHEMISTRY - ACIDS-BASES-SALTS	SHIVANGI MA'AM
4:00PM	MATHS - COMPLEX NUMBERS - CLASS 3	NAVJYOTI SIR
5:30PM	ENGLISH - ANTONYMS - CLASS 3	ANURADHA MA'AM

CDS 1 2025 LIVE CLASSES

1:00PM	CHEMISTRY - ACIDS-BASES-SALTS	SHIVANGI MA'AM
5:30PM	ENGLISH - ANTONYMS - CLASS 3	ANURADHA MA'AM
7:00PM	MATHS - NUMBER SYSTEM - CLASS 1	NAVJYOTI SIR

AFCAT 1 2025 LIVE CLASSES

4:00PM	STATIC GK - HIGHEST-SMALLEST IN INDIA & WORLD	DIVYANSHU SIR
5:30PM	ENGLISH - ANTONYMS - CLASS 3	ANURADHA MA'AM
7:00PM	MATHS - NUMBER SYSTEM - CLASS 1	NAVJYOTI SIR

SSBCrack EXAMS

Q) Let z_1 and z_2 be two non-zero complex numbers such that

$$|z_1| = |z_2| = \left|\frac{1}{z_1} + \frac{1}{z_2}\right| = 2$$

What is the value of $|z_1 + z_2|$?

(a) 8 (b) 4

(c) 2 (d) 1

$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$$

$$|z_1 z_2| = |z_1||z_2|$$

$$\left|\frac{z_1 + z_2}{z_1 z_2}\right| = 2$$

$$\frac{|z_1 + z_2|}{|z_1||z_2|} = 2$$

$$|z_1 + z_2| = 2|z_1||z_2|$$

$$= 2 \times 2 \times 2 = 8$$

$$|z_1 + z_2| = 8$$

Q) Let z_1 and z_2 be two non-zero complex numbers such that

$$|z_1| = |z_2| = \left|\frac{1}{z_1} + \frac{1}{z_2}\right| = 2$$

What is the value of $|z_1 + z_2|$?

(a) 8 (b) 4

(c) 2 (d) 1

Ans: (a)

Q) What is one of the values of $\sqrt{i} + \sqrt{-i}$?

(a) $\sqrt{2}$ ✓ (b) 0

(c) $\pm\frac{1+i}{\sqrt{2}}$ ✗ (d) $\pm\frac{1-i}{\sqrt{2}}$ ✗

$i = \frac{(1+i)^2}{2}$ $\left((1+i)^2 = 1 + 2i + i^2 = 1 + 2i - 1\right)$
$= 2i$

$-i = \frac{(1-i)^2}{2}$

$\sqrt{i} = \pm\frac{(1+i)}{\sqrt{2}}$ $\sqrt{-i} = \pm\frac{(1-i)}{\sqrt{2}}$

$+\frac{1+i}{\sqrt{2}} + \frac{1-i}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

Q) What is one of the values of $\sqrt{i} + \sqrt{-i}$?

(a) $\sqrt{2}$

(b) 0

(c) $\pm\frac{1+i}{\sqrt{2}}$

(d) $\pm\frac{1-i}{\sqrt{2}}$

Ans: (a)

SSBCrack EXAMS

Q) If $1, \omega, \omega^2$ are the three cube roots of unity, then what is $\frac{(a\omega^6 + b\omega^4 + c\omega^2)}{(b + c\omega^{10} + a\omega^8)}$ equal to?

(a) $\frac{a}{b}$ (b) b

(c) ω ✓ (d) ω^2

$$\frac{a(1) + b\omega + c\omega^2}{b + c\omega + a\omega^2}$$

$$\frac{a\omega^3 + b\omega + c\omega^2}{a\omega^2 + b + c\omega}$$

$$\frac{\omega(a\omega^2 + b + c\omega)}{(a\omega^2 + b + c\omega)} = \omega$$

Q) If $1, \omega, \omega^2$ are the three cube roots of unity, then what is $\frac{(a\omega^6 + b\omega^4 + c\omega^2)}{(b + c\omega^{10} + a\omega^8)}$ equal to?

(a) $\frac{a}{b}$

(b) b

(c) ω

(d) ω^2

Ans: (c)

Q) If $\alpha = \frac{1+i\sqrt{3}}{2}$, then what is the value of $1 + \alpha^8 + \alpha^{16} + \alpha^{24} + \alpha^{32}$?

(a) 0 (b) 1

(c) ω (d) $-\omega^2$ ✓

$\omega = \frac{-1+\sqrt{3}i}{2}$; $\omega^2 = \frac{-1-\sqrt{3}i}{2} = -\left(\frac{1+\sqrt{3}i}{2}\right) = -\alpha$

$\alpha = -\omega^2$

$1 + (-\omega^2)^8 + (-\omega^2)^{16} + (-\omega^2)^{24} + (-\omega^2)^{32}$

$= 1+\omega+\omega^2 + 1 + \omega$

$= 0 + 1 + \omega = -\omega^2$

$(1+\omega+\omega^2 = 0)$

Q) If $\alpha = \frac{1+i\sqrt{3}}{2}$, then what is the value of $1 + \alpha^8 + \alpha^{16} + \alpha^{24} + \alpha^{32}$?

(a) 0

(b) 1

(c) ω

(d) $-\omega^2$

Ans: (d)

SSBCrack EXAMS

Q) What is $i^{1000} + i^{1001} + i^{1002} + i^{1003}$ equal to (where $i = \sqrt{-1}$)?

(a) 0 (b) i

(c) –i (d) 1

$i^{1000}(1 + i^1 + i^2 + i^3)$

$i^{1000}(1 + i - 1 - i)$

$i^{1000}(0) = 0$

sum of consecutive four powers in $i = 0$

$\rightarrow i^2 + i^3 + i^4 + i^5 = 0$

$\rightarrow i^{201} + i^{202} + i^{203} + i^{204} = 0$

Q) What is $i^{1000} + i^{1001} + i^{1002} + i^{1003}$ equal to (where $i = \sqrt{-1}$)?

(a) 0

(b) i

(c) –i

(d) 1

Ans: (a)

What is the value of the sum

$$\sum_{n=1}^{20} (i^{n-1} + i^{n} + i^{n+1})$$

where $i = \sqrt{-1}$?

(a) $-2i$

(b) 0 ✓

(c) 1

(d) $2i$

$$\sum_{n=1}^{20} (i^{n-1} + i^{n} + i^{n+1} + i^{n+2} - i^{n+2})$$

$$= \sum_{n=1}^{20} (i^{n-1} + i^{n} + i^{n+1} + i^{n+2}) - \sum_{n=1}^{20} i^{n+2}$$

$$i^3 + i^4 + i^5 + i^6 = 0$$

$$= \sum_{n=1}^{20} (0) - \sum_{i=1}^{20} i^{n+2} = 0 - 0 = 0$$

Q) $\left(i^{39} + \frac{1}{i^{69}}\right) = ?$

a) 0

b) 2i

c) −2i ✓

d) 1 − i

$$i^3 + \frac{1}{i^1}$$

$$= -i + \frac{1 \times i}{i \times i}$$

$$= -i + (-i) = -2i$$

Q) $\left(i^{39} + \frac{1}{i^{69}}\right) = ?$

a) 0

b) 2i

c) −2i

d) 1 − i

Ans: (c)

Q) What is the square root of the complex number $-5 + 12i$?

(a) $2 - 3i$ (b) $2 + 3i$ ✓

(c) $-2 + 3i$ (d) $\sqrt{-5} + \sqrt{12i}$

$\sqrt{-5+12i} = x + iy$

$x^2 - y^2 = -5$ $2xy = 12$ $(xy > 0)$

$x^2 + y^2 = \sqrt{(-5)^2 + (12)^2} = 13$

$2x^2 = 8 \Rightarrow x = \pm 2$

$2y^2 = 18 \Rightarrow y = \pm 3$

$2 + 3i$, $-2 - 3i$ → $\pm(2 + 3i)$

Q) What is the square root of the complex number $-5 + 12i$?

(a) $2 - 3i$ (b) $2 + 3i$

(c) $-2 + 3i$ (d) $\sqrt{-5} + \sqrt{12i}$

Ans: (b)

Q) If α and β are the roots of $x^2 + x + 1 = 0$, then what is $\sum_{j=0}^{3}(\alpha^j + \beta^j)$ equal to?

(a) 8 (b) 6

(c) 4 (d) 2 ✓

$x^2 + x + 1 = 0$

ω (α), ω^2 (β) (roots)

$1 + \omega + \omega^2 = 0$

$\omega + \omega^2 = -1$

$\omega^4 = \omega$

$$\sum_{j=0}^{3}(\alpha^j + \beta^j) = (1+1) + (\omega + \omega^2) + (\omega^2 + \omega^4) + (\omega^3 + \omega^6)$$

$$= 2 + (-1) + (-1) + (2)$$

$$= 2$$

Q) If α and β are the roots of $x^2 + x + 1 = 0$, then what is $\sum_{j=0}^{3}(\alpha^j + \beta^j)$ equal to?

(a) 8 (b) 6

(c) 4 (d) 2

Ans: (d)

Q) The modulus and principal argument of the complex number $\frac{1+2i}{1-(1-i)^2}$ are respectively

(a) 1, 0 ✓ (b) 1, 1 (c) 2, 0 (d) 2, 1

$$\frac{1+2i}{1-(1-1-2i)} = \frac{1+2i}{1+2i} = 1 \Rightarrow \underline{1+0i}$$

modulus = 1

argument $\Rightarrow \tan\theta = \frac{0}{1} = 0 \Rightarrow \underline{\theta = 0}$

Q) The modulus and principal argument of the complex number $\dfrac{1+2i}{1-(1-i)^2}$ are respectively

(a) 1, 0 (b) 1, 1 (c) 2, 0 (d) 2, 1

Ans: (a)

Q) If z is a complex number such that $z + z^{-1} = 1$, then what is the value of $z^{99} + z^{-99}$?

(a) 1 (b) –1

(c) 2 (d) –2 ✓

$z + \frac{1}{z} = 1$

$z^2 - z + 1 = 0$

Roots: $-\omega$, $-\omega^2$

$z^2 + z + 1 = 0$

Roots: ω, ω^2

$(-\omega)^{99} + (-\omega)^{-99}$

$(-1)\omega^{99} + \frac{(-1)}{\omega^{99}}$

$(-1)(1) + \frac{(-1)}{1} = -1 - 1 = -2$

Q) If z is a complex number such that $z + z^{-1} = 1$, then what is the value of $z^{99} + z^{-99}$?

(a) 1
(b) –1
(c) 2
(d) –2

Ans: (d)

Q) Consider the following statements

I. $(\omega^{10} + 1)^7 + \omega = 0$ ✗

II. $(\omega^{105} + 1)^{10} = p^{10}$ for some prime number p, where $\omega \neq 1$

Which of the above statement(s) is/are correct?

a) Only I

b) Only II ✓

c) Both I & II

d) Neither I nor II

I.) $(\omega + 1)^7 + \omega$

$(-\omega^2)^7 + \omega = -\omega^2 + \omega \neq 0$

II.) $(1+1)^{10} = 2^{10}$

2 is prime.

Q) Consider the following statements

I. $(\omega^{10} + 1)^7 + \omega = 0$

II. $(\omega^{105} + 1)^{10} = p^{10}$ for some prime number p, where $\omega \neq 1$

Which of the above statement(s) is/are correct?

a) Only I

b) Only II

c) Both I & II

d) Neither I nor II

Ans: (b)

Q) If $y = \cos\theta + i\sin\theta$, then the value of $y + \frac{1}{y}$ is

a) $2\cos\theta$

b) $2\sin\theta$

c) $2\operatorname{cosec}\theta$

d) $2\tan\theta$

$y = e^{i\theta}$

$\frac{1}{y} = y^{-1} = (e^{i\theta})^{-1}$

$= e^{-i\theta} = e^{i(-\theta)}$

$[e^{i\theta}] + [e^{i(-\theta)}]$

$[(\cos\theta + i\sin\theta)] + [(\cos(-\theta) + i\sin(-\theta))]$

$= 2\cos\theta + i\sin\theta - i\sin\theta$

$= 2\cos\theta$

$\cos(-\theta) = \cos\theta$

$\sin(-\theta) = -\sin\theta$

Q) If $y = \cos\theta + i\sin\theta$, then the value of $y + \frac{1}{y}$ is

a) $2\cos\theta$

b) $2\sin\theta$

c) $2\operatorname{cosec}\theta$

d) $2\tan\theta$

Ans: (a)

Q) If $A = \{x \in Z : x^3 - 1 = 0\}$ and $B = \{x \in Z : x^2 + x + 1 = 0\}$, where Z is set of complex numbers, then what is $A \cap B$ equal to ?

(a) Null set

(b) $\left\{\frac{-1+\sqrt{3}\,i}{2}, \frac{-1-\sqrt{3}\,i}{2}\right\}$ ✓

(c) $\left\{\frac{-1+\sqrt{3}\,i}{4}, \frac{-1-\sqrt{3}\,i}{4}\right\}$

(d) $\left\{\frac{1+\sqrt{3}i}{2}, \frac{1-\sqrt{3}\,i}{2}\right\}$

$A = \{1, \omega, \omega^2\}$

$B = \{\omega, \omega^2\}$

$A \cap B = \{\omega, \omega^2\}$

$= \left\{\frac{-1+\sqrt{3}i}{2}, \frac{-1-\sqrt{3}i}{2}\right\}$

Q) If $A = \{x \in Z : x^3 - 1 = 0\}$ and $B = \{x \in Z : x^2 + x + 1 = 0\}$, where Z is set of complex numbers, then what is $A \cap B$ equal to ?

(a) Null set

(b) $\left\{\frac{-1+\sqrt{3}\,i}{2}, \frac{-1-\sqrt{3}\,i}{2}\right\}$

(c) $\left\{\frac{-1+\sqrt{3}\,i}{4}, \frac{-1-\sqrt{3}\,i}{4}\right\}$

(d) $\left\{\frac{1+\sqrt{3}i}{2}, \frac{1-\sqrt{3}\,i}{2}\right\}$

Ans: (b)

Q) Which one of the following is correct in respect of the cube roots of unity?

(a) They are collinear

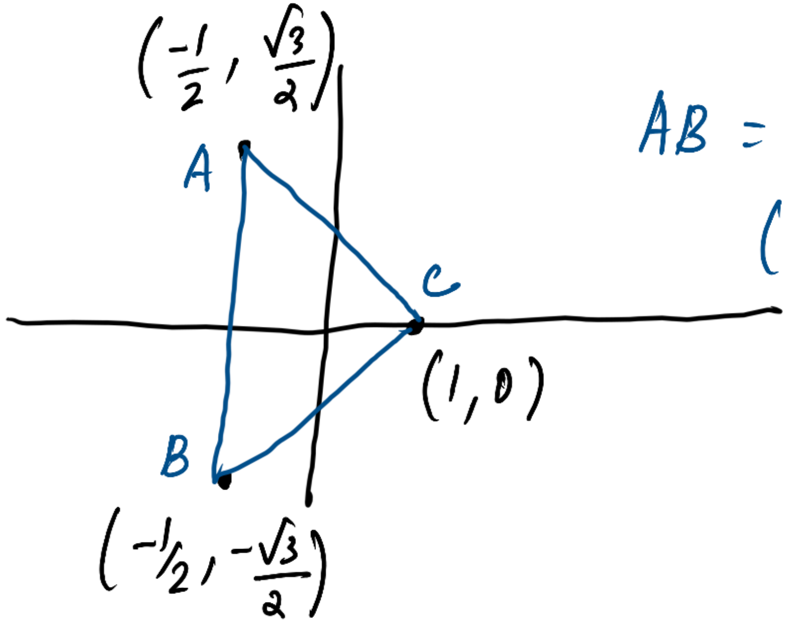
(b) They lie on a circle of radius $\sqrt{3}$

(c) They form an equilateral triangle ✓

(d) None of the above

$1, \omega, \omega^2$

$1 + 0i, \dfrac{-1 + \sqrt{3}i}{2}, \dfrac{-1 - \sqrt{3}i}{2}$



AB = BC = CA

(equilateral triangle)

Q) Which one of the following is correct in respect of the cube roots of unity?

(a) They are collinear

(b) They lie on a circle of radius $\sqrt{3}$

(c) They form an equilateral triangle

(d) None of the above

Ans: (c)

SSBCrack EXAMS

Q) If $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^{107} + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^{107}$, then what is the imaginary part of z equal to?

(a) 0 (b) $\frac{1}{2}$

(c) $\frac{\sqrt{3}}{2}$ (d) 1

$z_1 = \frac{\sqrt{3}}{2} + \frac{1}{2}i \longrightarrow |z_1| = 1$; $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$ | $\theta = \frac{\pi}{6}$

$z_2 = \frac{\sqrt{3}}{2} - \frac{1}{2}i \longrightarrow |z_2| = 1$; $\tan^{-1}\left|\frac{-1/2}{\frac{\sqrt{3}}{2}}\right| = \frac{\pi}{6}$ | $\theta = -\frac{\pi}{6}$

NDA 1 2025 - Maths Complex Numbers Class - 3

$$z = \left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^{107} + \left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right)^{107}$$

$$= \cos\frac{107\pi}{6} + i\sin\left(\frac{107\pi}{6}\right) + \cos\left(-\frac{107\pi}{6}\right) + i\sin\left(-\frac{107\pi}{6}\right)$$

$$= 2\cos\frac{107\pi}{6} + i\sin\left(\frac{107\pi}{6}\right) - i\sin\left(\frac{107\pi}{6}\right)$$

$$z = 2\cos\left(\frac{107\pi}{6}\right) + 0i$$

$$Im(z) = 0$$

$$z = (a+ib)^n + (a-ib)^n$$

$$Im(z) = 0$$

$$z = 2|z|\cos(n\theta)$$

θ - argument

Q) If $z = \left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^{107}+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^{107}$, then what is the imaginary part of z equal to?

(a) 0

(b) $\frac{1}{2}$

(c) $\frac{\sqrt{3}}{2}$

(d) 1

Ans: (a)

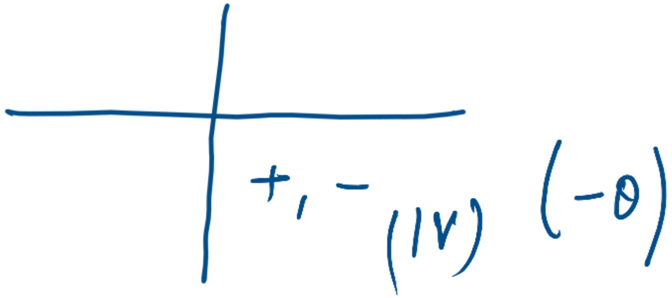
Q) Let $z = i^3(1 + i)$ be a complex number. What is its argument?

(a) π

(b) $\frac{\pi}{4}$

(c) $-\frac{\pi}{4}$ ✓

(d) $\frac{5\pi}{4}$



$z = (-i)(1+i) = -i + 1 = 1 - 1i$

$\tan^{-1}\left|\frac{-1}{1}\right| = \tan^{-1}(1) = \frac{\pi}{4}$ | $\theta = -\frac{\pi}{4}$

Q) Let $z = i^3(1 + i)$ be a complex number. What is its argument?

(a) π

(b) $\frac{\pi}{4}$

(c) $-\frac{\pi}{4}$

(d) $\frac{5\pi}{4}$

Ans: (c)

Q) If 1, ω, ω^2 are the cube roots of unity, then the value of $(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)$ is

(a) -1

(b) 0

(c) 1

(d) 2

$(-\omega^2)(-\omega)(-\omega^2)(-\omega)$

$= \omega^6 = 1$

Q) If 1, ω, ω^2 are the cube roots of unity, then the value of $(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)$ is

(a) -1 (b) 0

(c) 1 (d) 2

Ans: (c)

Q) If $z^2 + z + 1 = 0$, where z is complex number, then the value of

$$\left(z+\frac{1}{z}\right)^2+\left(z^2+\frac{1}{z^2}\right)^2+\left(z^3+\frac{1}{z^3}\right)^2+\ldots\ldots\ldots+\left(z^6+\frac{1}{z^6}\right)^2 \text{ is}$$

(a) 18 (b) 54

(c) 6 (d) 12 ✓

$z^2+z+1=0$ → roots ω, ω^2

$\omega \cdot \omega^2 = 1$

$\left(\omega = \frac{1}{\omega^2}\right)$

$$\left(\omega+\frac{1}{\omega}\right)^2+\left(\omega^2+\frac{1}{\omega^2}\right)^2+\left(\omega^3+\frac{1}{\omega^3}\right)^2+\left(\omega^4+\frac{1}{\omega^4}\right)+\left(\omega^5+\frac{1}{\omega^5}\right)+\left(\omega^6+\frac{1}{\omega^6}\right)$$

$$=(\omega+\omega^2)^2+(\omega^2+\omega)^2+(1+1)^2+(\omega+\omega^2)^2+(\omega^2+\omega)^2+(1+1)^2$$

$$=(-1)^2+(-1)^2+4+(-1)^2+(-1)^2+(2)^2=\boxed{12}$$

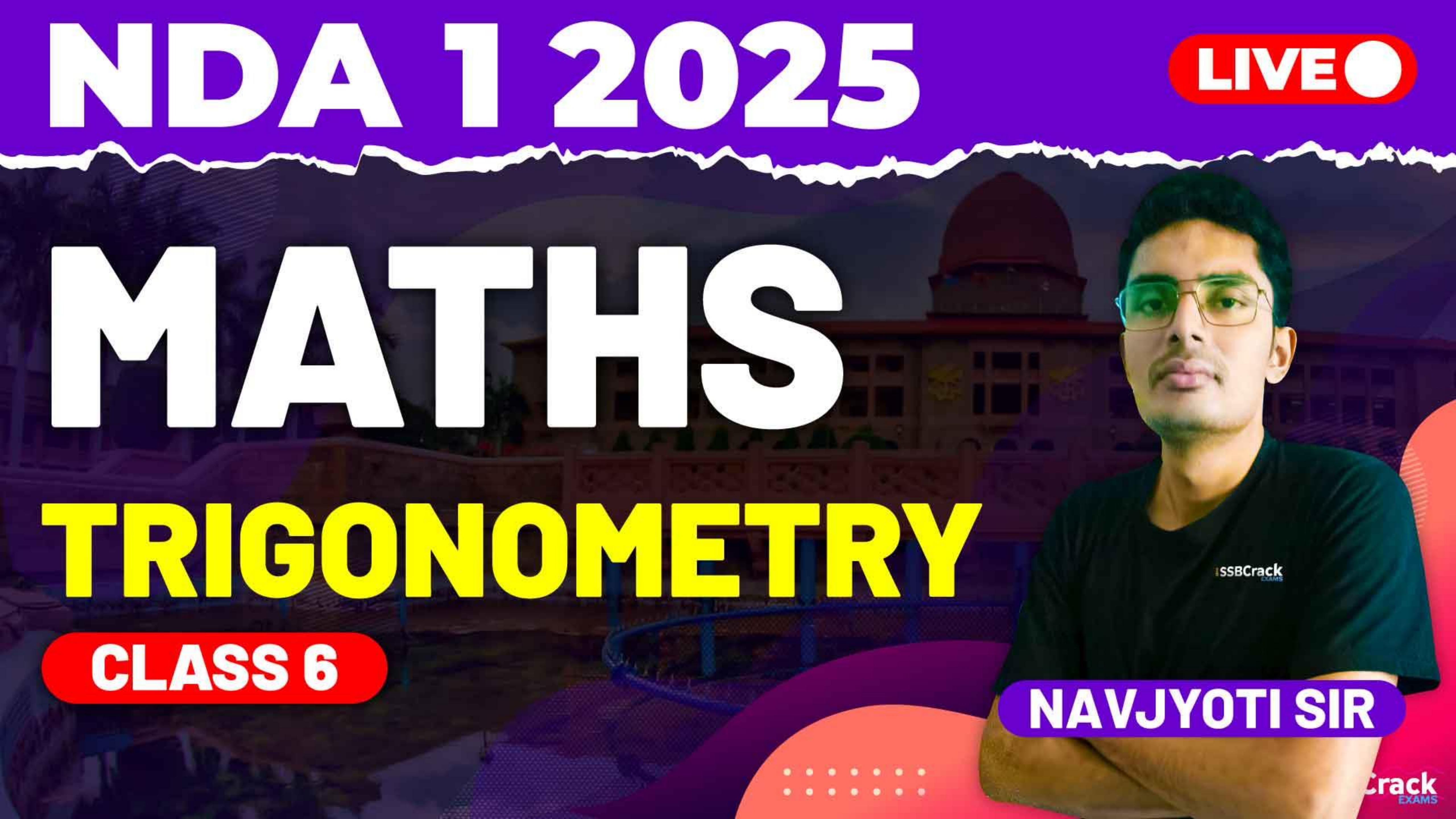
Q) If $z^2 + z + 1 = 0$, where z is complex number, then the value of

$$\left(z+\frac{1}{z}\right)^2 + \left(z^2+\frac{1}{z^2}\right)^2 + \left(z^3+\frac{1}{z^3}\right)^2 + \text{.........} + \left(z^6+\frac{1}{z^6}\right)^2$$ is

(a) 18 (b) 54

(c) 6 (d) 12

Ans: (d)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TRIGONOMETRY
CLASS 6
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SSBCrack EXAMS
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