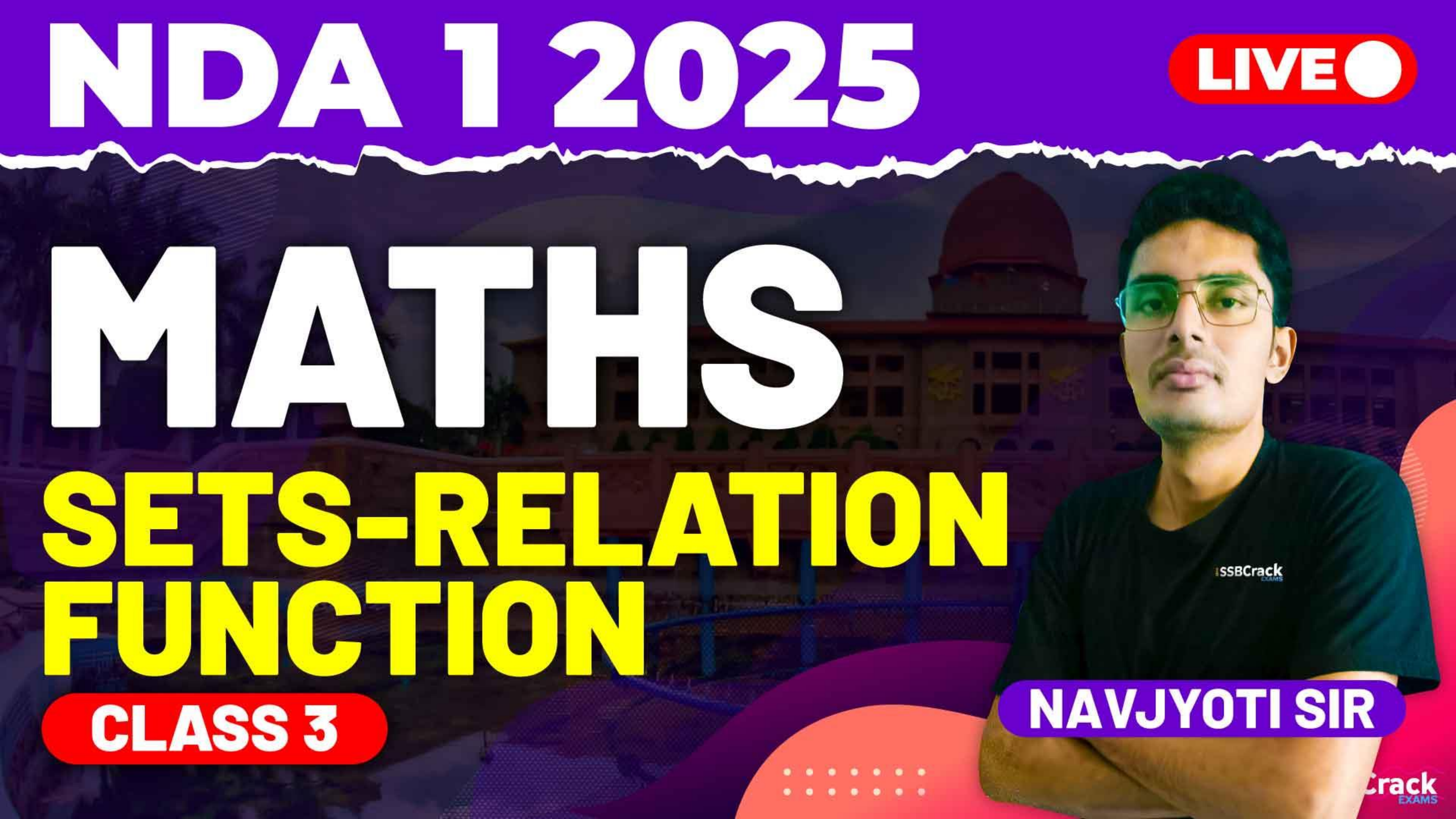
NDA 1 2025
LIVE
MATHS
SETS-RELATION
FUNCTION
CLASS 3
NAVJYOTI SIR
SSBCrack EXAMS
Crack EXAMS

05 Oct 2024 Live Classes Schedule

8:00AM	05 OCTOBER 2024 DAILY CURRENT AFFAIRS	RUBY MA'AM
9:00AM	05 OCTOBER 2024 DAILY DEFENCE UPDATES	DIVYANSHU SIR

NDA 1 2025 LIVE CLASSES

11:30AM	GK : INDIAN GEOGRAPHY CLASS 2	RUBY MA'AM
4:00PM	MATHS : SETS, RELATION AND FUNCTION - 3	NAVJYOTI SIR

CDS 1 2025 LIVE CLASSES

11:30AM	GK : INDIAN GEOGRAPHY CLASS 2	RUBY MA'AM
2:30PM	MATHS : TIME & WORK - CLASS 1	NAVJYOTI SIR

AFCAT 1 2025 LIVE CLASSES

10:00AM	REASONING : DIRECTION AND DISTANCES	RUBY MA'AM
2:30PM	MATHS : TIME & WORK - CLASS 1	NAVJYOTI SIR

Q) A relation R is defined on the set Z of integers as follows :

$mRn \Leftrightarrow m + n$ is odd.

Which of the following statements is/are true for R ?

1. R is reflexive ✗
2. R is symmetric ✓
3. R is transitive ✗

Select the correct answer using the code given below :

(a) 2 only ✓
(b) 2 and 3
(c) 1 and 2
(d) 1 and 3

1.) $a + a = 2a \rightarrow$ even

2.) odd + even $\rightarrow$ odd

even + odd $\rightarrow$ odd

$3 + 3 = 6 \rightarrow$ even

odd + even $\rightarrow$ odd

even + odd $\rightarrow$ odd

odd + odd $\rightarrow$ even

o o e

$1 + 3 = 4$

e + e e

$4 + 6 = 10$

$2 + 3 = 5$

e o o

Q) A relation R is defined on the set Z of integers as follows :

$$mRn \Leftrightarrow m + n \text{ is odd.}$$

Which of the following statements is/are true for R ?

1. R is reflexive
2. R is symmetric
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Select the correct answer using the code given below :

(a) 2 only
(b) 2 and 3
(c) 1 and 2
(d) 1 and 3

Ans: (a)

OPEN AND CLOSED INTERVALS

- If $a \leq x \leq b \Rightarrow x \in [a,b]$ ---------- Closed Intervals
- If $a < x < b \Rightarrow x \in (a,b)$ ------------- Open Intervals

$(-\infty, \infty)$

$2 < x \leq 6 \Longrightarrow x \in (2,6]$

$x \geq 2 \Longrightarrow x \in [2,\infty)$

WHAT WILL WE STUDY ?

- Functions
- Types of Functions
- Composition of Functions
- Practise Questions

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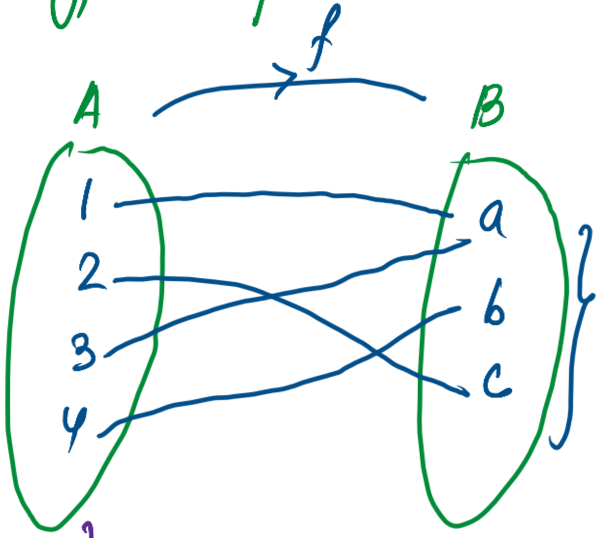
FUNCTION

→ special type of relation.



mapping

transformation

$f = \{(1, a), (2, c), (3, a), (4, b)\}$

$f(1) = a$ $\quad$ $f(3) = a$

$f(2) = c$ $\quad$ $f(4) = b$

images

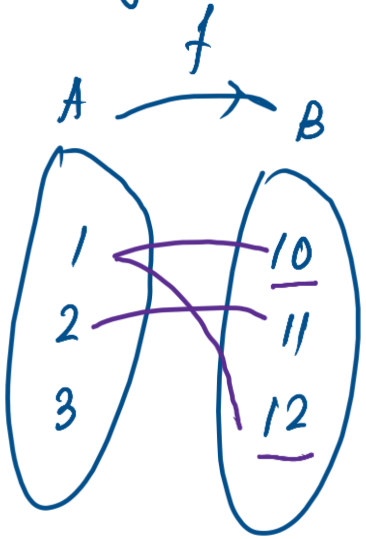
image of 1 under f

first elements can only come once

(pre-images) → pre-images cannot be repeated,

FUNCTION

→ images are unique.



f is not a function

(As 1 has two images)

$f(1) = \_\_$ (cannot write two numbers)

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FUNCTION

→ acts as machine.

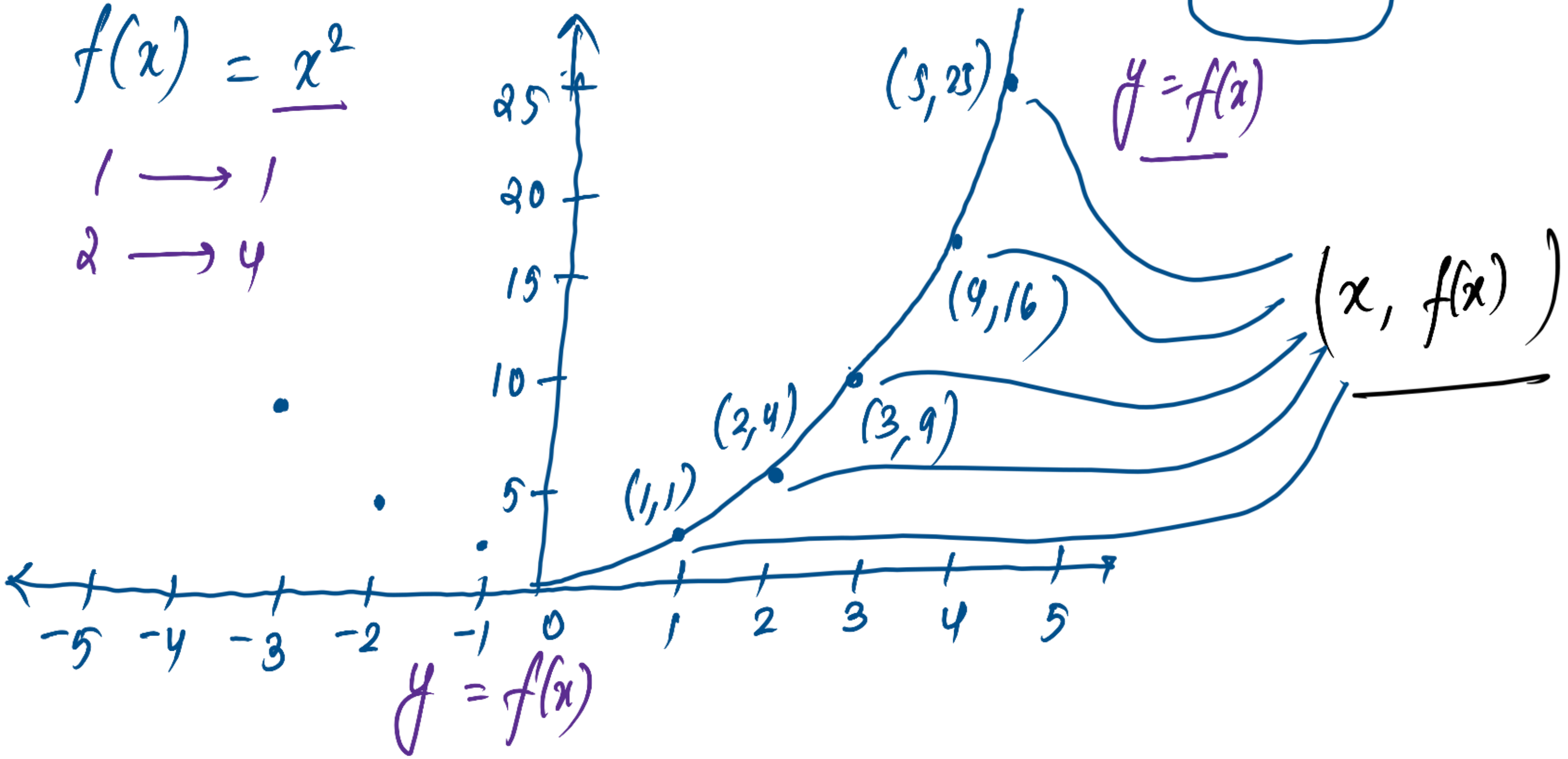
input, x → $\boxed{f(x)}$ → $f(x)$ values

Eg

$\underline{f(x) = x^2}$

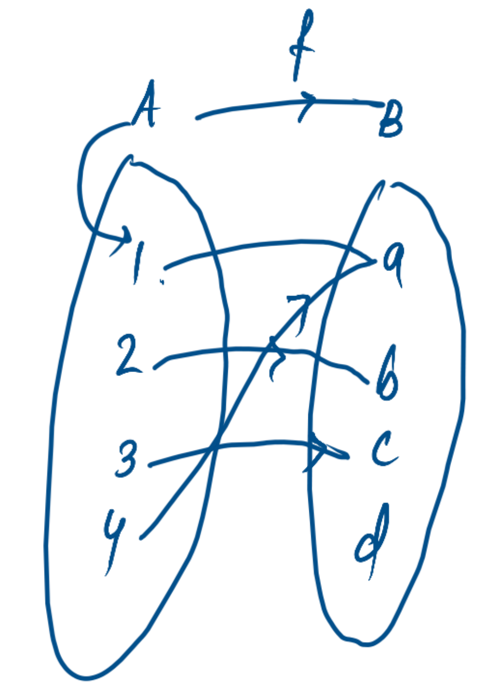
$1 \longrightarrow 1$

$2 \longrightarrow 4$



DOMAIN, CO-DOMAIN & RANGE

set of all pre-images



$f = \{(1, a), (2, b), (3, c), (4, a)\}$

Domain = $A = \{1, 2, 3, 4\}$

Range $\Rightarrow \{a, b, c\}$

codomain = $B \Rightarrow \{a, b, c, d\}$ (full set B)

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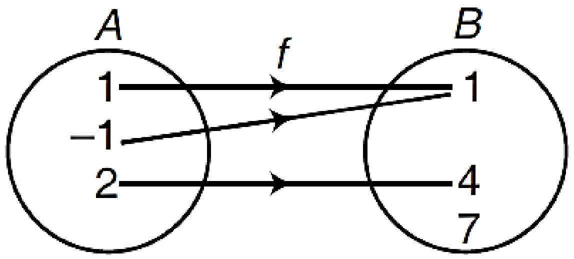
TYPES OF FUNCTION

Many-one function Let $f: A \to B$. If two or more than two elements have the same image in B, then f is said to be many-one function.

e.g., the function $f: A \to B$ given by $f(x) = x^2$ is a many-one function.



$f = \{(1, 1), (-1, 1), (2, 4)\}$

TYPES OF FUNCTION

(INJECTIVE)

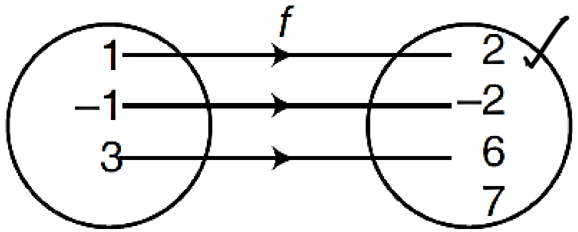
One-one function (injective) Let $f : A \to B$.
Then, f is said to be one-one function or an injective, if different elements of A have different images in B.

Thus, $f : A \to B$ is one-one

$$\Leftrightarrow \quad a \neq b \Rightarrow f(a) \neq f(b), \qquad \forall\, a, b \in A$$

$$\Leftrightarrow \quad f(a) = f(b) \Rightarrow a = b, \qquad \forall\, a, b \in A$$

e.g., the function $f : A \to B$ given by $f(x) = 2x$ is an one-one function

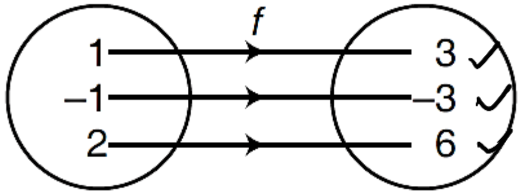


TYPES OF FUNCTION

Onto function (surjective) Let $f : A \to B$. If every element in B has atleast one preimage in A, then f is said to be an onto function.

(SURJECTIVE)

Thus, $f : A \to B$ is a surjective, iff for each $b \in B$, $\exists\ a \in A$ such that $f(a) = b$ clearly, f is onto $\Leftrightarrow$ range $(f) = B$.



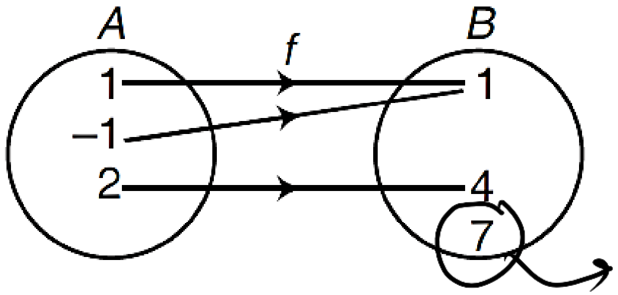
range of f = codomain of f = $\underline{B}$

e.g., the function $f : A \to B$ is given by $f(x) = 3x$ is an onto function.

TYPES OF FUNCTION

Into function Let $f: A \to B$. If there exists even a single element in B having no preimage in A, then f is said to be an into function.

e.g., the function $f: A \to B$ given by $f(x) = x^2$ is an into function.



atleast one element should not be an image of f.

Into Function

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TYPES OF FUNCTION

Bijective function A one-one and onto function is said to be bijective.

A bijective function is also known as a one-to-one correspondence.

In other words, a function $f : A \to B$ is a bijection, if

(a) it is one-one *i.e.*, $f(x) = f(y) \Rightarrow x = y, \forall\ x, y \in A$.

(b) it is onto *i.e.*, $\forall\ y \in B$, there exists $y \in A$ such that $f(x) = y$.

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TYPES OF FUNCTION

Even and odd functions A function $f : A \to B$ is said to be an even or odd function according as $f(-x) = f(x)$, $\forall\ x \in A$ and $f(-x) = -f(x)$, $\forall\ x \in A$, respectively.

If $f(-x) = -f(x) \longrightarrow$ odd function

$f(-x) = f(x) \longrightarrow$ even function

$f(x) = 3x + 2$

$f(-x) = 3(-x) + 2$

$= -3x + 2 = -(3x - 2)$

$\neq -f(x)$ } neither even nor odd

X

Identity function Let A be a non-empty set. Then, the function, defined by $I_A : A \to A$. $I_A(x) = x, \forall$ $x \in A$, is called an identity function on A.

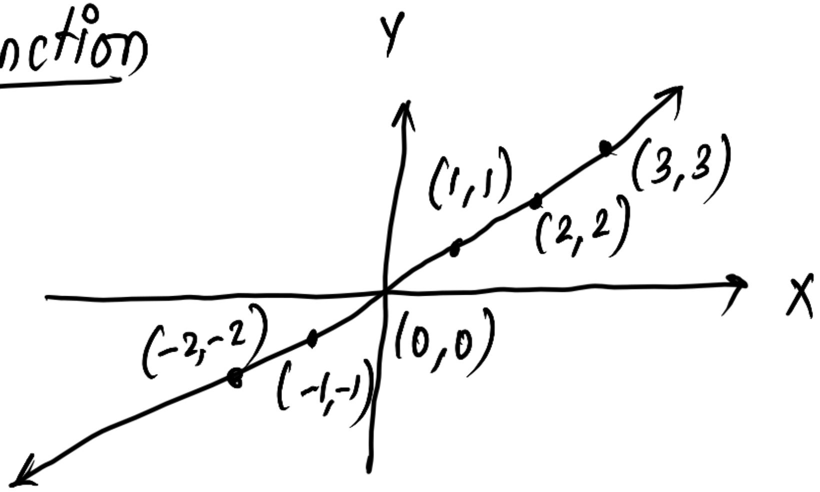
This is clearly a one-one onto function with domain A and range A.

$f(x) = x$

① Identity function

$f(x) = x$

$y = x$



Domain = R

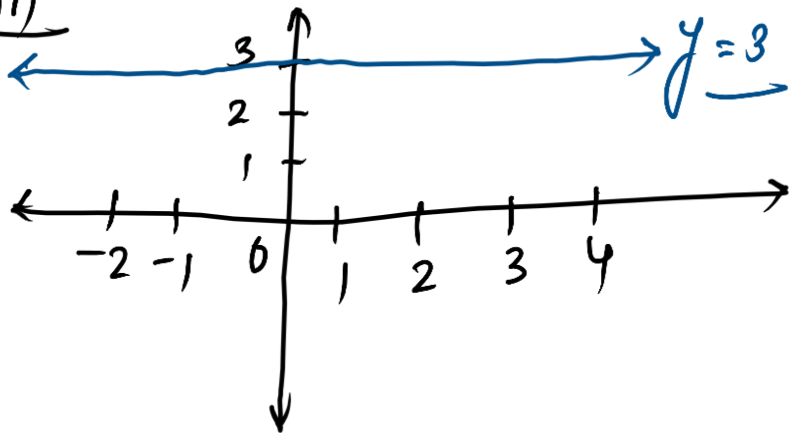
Range = R

② constant function

$f(x) = c$

~~y~~ - $f(x) = 3$

$y = 3$



Domain = R

Range = c

③ Modulus function :

$f(x) = |x| \rightarrow \begin{cases} +x, & x > 0 \\ 0, & x = 0 \\ -x, & x < 0 \end{cases}$

Domain = R

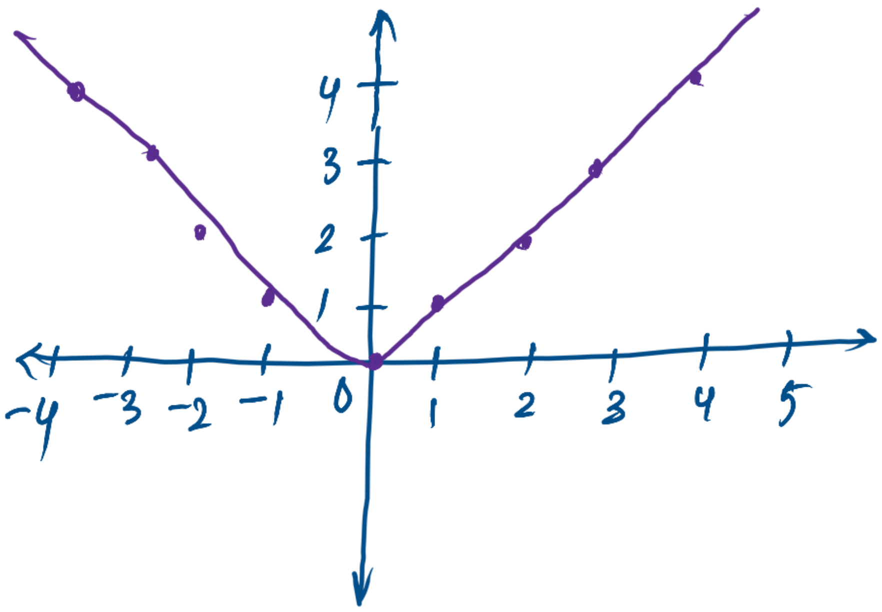
Range = $R^+ \cup \{0\}$

(set of all non-negative real numbers)

$f(-3) = |-3| = 3$

$f(4) = |4| = 4$

$f(-5.5) = 5.5$



4 Greatest Integer Function :

$f(x) = \lceil x \rceil$ or $[x]$

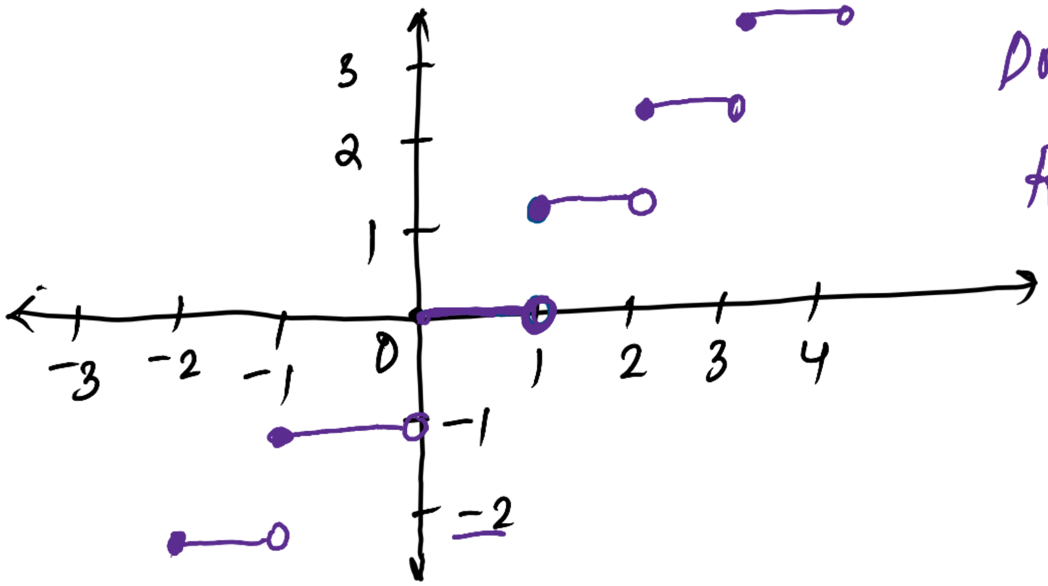
$f(2.4) = 2$

2   3

$f(1.6) = 1$

$f(-3.4) = -4$

$f(4) = 4$



Domain = R

Range = Z (integers only)

IMPORTANT RESULTS

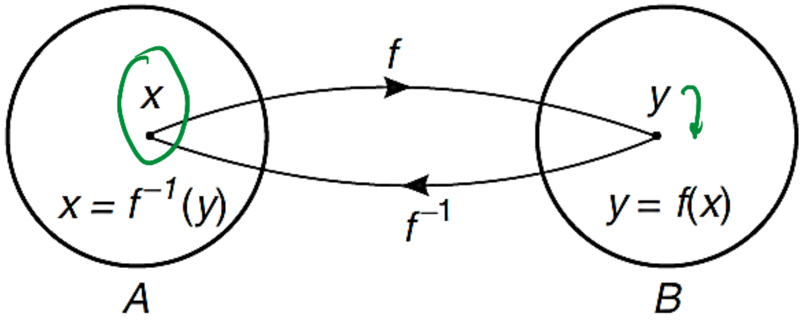
- The product of two even or odd function is an even function.
- The product of an even and an odd function is an odd function.
- Every function $f(x)$ can be expressed as the sum of an even and an odd function.

IMPORTANT RESULTS

- If A and B have n and m distinct elements respectively, then the number of mappings from A to B is equal to m^n.
- If A and B have n equal number of distinct elements, then the number of mappings from A to B is equal to n^n.
- The number of onto functions that can be defined from a finite set A containing n elements on finite set B containing 2 elements $= 2^n - 2$.

TYPES OF FUNCTION

Inverse function Let f be a one-one onto function from A to B.



Let y be an arbitrary element of B. Then, f being onto, there exists an element $x \in A$, such that $f(x) = y$.

Also, f being one-one, this x must be unique.

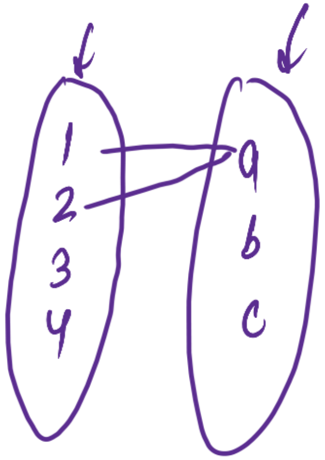
Thus, for each $y \in B$, there exists a unique element $x \in A$, such that $f(x) = y$.

So, we may define a function, denoted by f^{-1} as

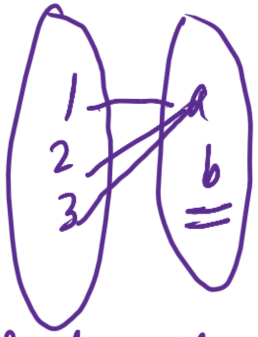
$$f^{-1} : B \rightarrow A, \text{ such that } f^{-1}(y) = x \Leftrightarrow f(x) = y$$

The above function f^{-1} is called the inverse of f.

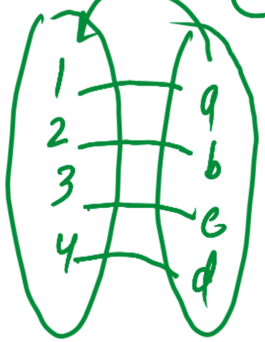
- A function f is invertible if and only if f is one-one onto. (its inverse is possible)



many-one X



into X



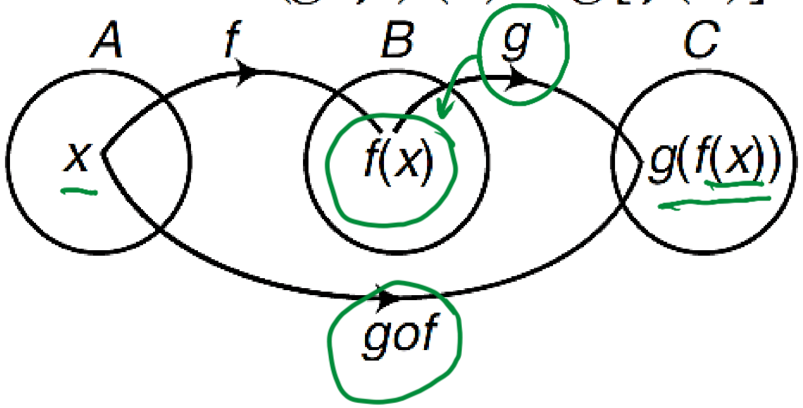
one-one & onto ✓

$x = f^{-1}(y)$

COMPOSITION OF FUNCTION

Let A, B and C be three non-empty sets. Let $f : A \to B$ and $g : B \to C$. Since, $f : A \to B$, for each $x \in A$, there exists a unique element $g[f(x)]$ of C. Thus, for each $x \in A$, there is associated a unique element $g[f(x)]$ of C. Thus, from f and g, we can define a new function from A to C. This function is called the product or composite of f and g, denoted by gof and defined by

$(gof) : A \to C$ such that $(gof)(x) = g[f(x)]$ for all $x \in A$.



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PROPERTIES OF COMPOSITION OF FUNCTION

1. The ~~product~~ composition of any function with the identity function is the function itself.
2. The product of any invertible function f with its inverse function f^{-1} is an identity function.
3. Let $f: A \to B$ and $g: B \to A$, such that gof is an identity function on A and fog is an identity function on B. Then, $g = f^{-1}$.
4. Let $f: A \to B$ and $g: B \to C$ be a one-one onto functions.

   Then, gof is also one-one onto and $(gof)^{-1} = f^{-1}og^{-1}$

$f(x) = 2x+3$

$g(x) = x$

$gof(x)$

$g[f(x)] = g(2x+3)$

$= 2x+3$

$= f(x)$

$fog = f[g(x)]$ $f^{-1}(g^{-1}(x))$

Q) If $f(x) + 2f\left(\frac{1}{x}\right) = 3x$, $x \neq 0$ and

$S = \{x \in R : f(x) = f(-x)\}$; then S:

(a) contains exactly two elements.

(b) contains more than two elements.

(c) is an empty set.

(d) contains exactly one element.

$$f(x) + f\left(\frac{1}{x}\right) = 3x$$

$$f\left(\frac{1}{x}\right) + f\left(\frac{1}{\frac{1}{x}}\right) = 3\left(\frac{1}{x}\right) = f\left(\frac{1}{x}\right) + f(x) = \frac{3}{x}$$

SSBCrack EXAMS

Q) If $f(x) + 2f\left(\frac{1}{x}\right) = 3x$, $x \neq 0$ and

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(a) contains exactly two elements.

(b) contains more than two elements.

(c) is an empty set.

(d) contains exactly one element.

Ans: (a)

Q) Let $f(x) = x^2 + 2x - 5$

and $g(x) = 5x + 30$

What are the roots of the equation $g[(f(x)] = 0$?

(a) 1, –1 (b) –1, –1 ✓

(c) 1, 1 (d) 0, 1

$g[f(x)] = 0$

$g(x^2 + 2x - 5) = 0$

$5(x^2 + 2x - 5) + 30 = 0$

$x^2 + 2x - 5 + 6 = 0$

$x^2 + 2x + 1 = 0$

$(x+1)^2 = 0$

$x = -1, -1$

SSBCrack EXAMS

Q) Let $f(x) = x^2 + 2x - 5$

and $g(x) = 5x + 30$

What are the roots of the equation $g[(f(x)] = 0$?

(a) 1, –1
(b) –1, – 1
(c) 1, 1
(d) 0, 1

Ans: (b)

Q) If $f(x) = \dfrac{\sqrt{x-1}}{x-4}$, defines a function on $\boldsymbol{R}$, then what is its domain ?

(a) $(-\infty, 4) \cup (4, \infty)$

(b) $[4, \infty)$

(c) $(1, 4) \cup (4, \infty)$

(d) $[1, 4) \cup (4, \infty)$

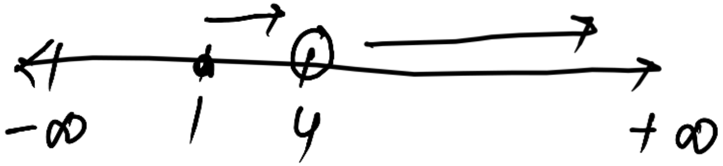
$$f(x) = \frac{\sqrt{x-1}}{x-4}$$

$x \neq 4$

$x - 1 \geq 0$

$x \geq 1$

$[1, 4) \cup (4, \infty)$



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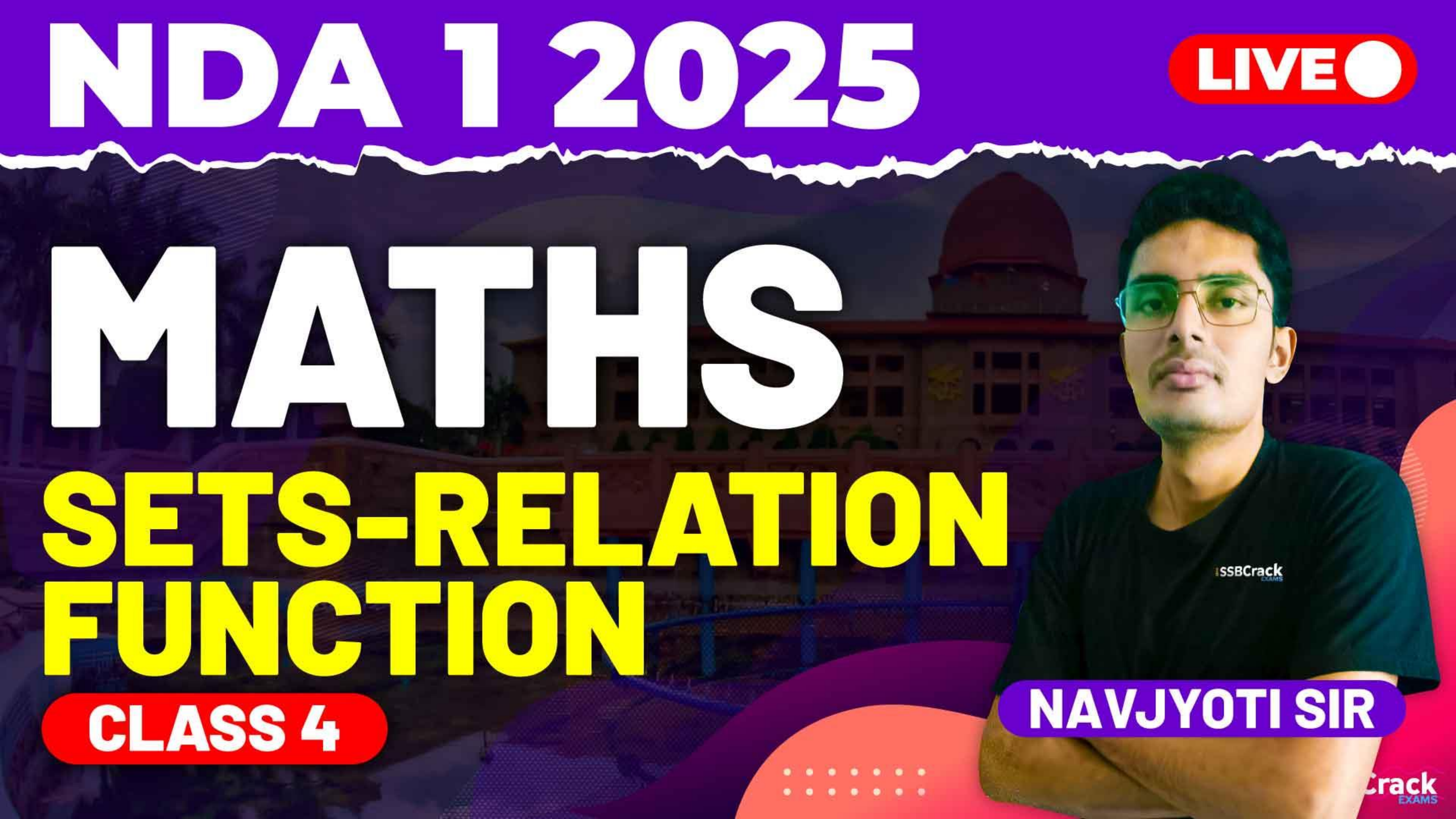
(c) $(1, 4) \cup (4, \infty)$

(d) $[1, 4) \cup (4, \infty)$

Ans: (d)

Summary

- Functions
- Types of Functions
- Composition of Functions
- Practise Questions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