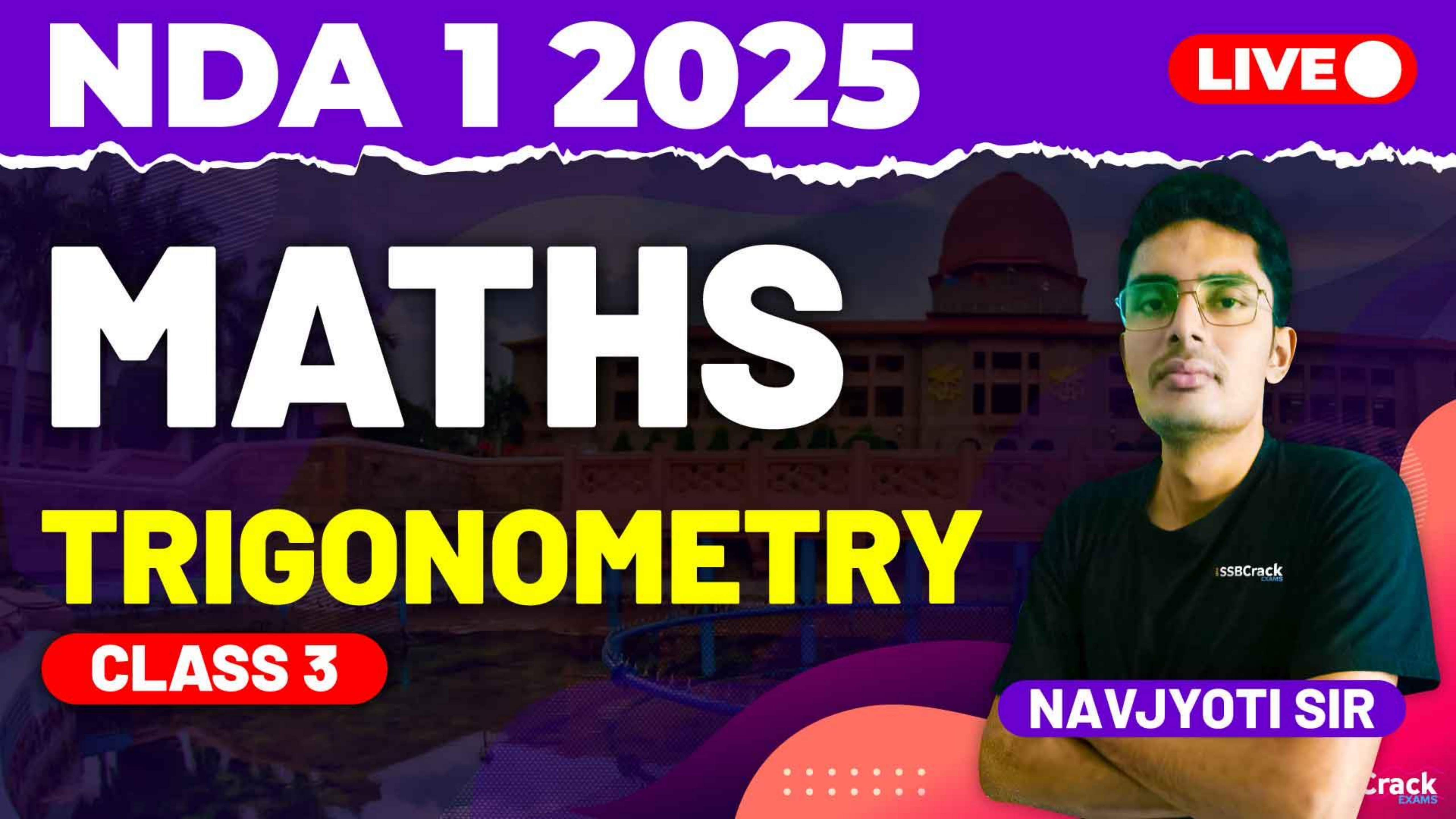

NDA 1 2025
LIVE
MATHS
TRIGONOMETRY
CLASS 3
NAVJYOTI SIR
SSBCrack EXAMS
Crack EXAMS

10 Oct 2024 Live Classes Schedule

8:00AM	10 OCTOBER 2024 DAILY CURRENT AFFAIRS	RUBY MA'AM
9:00AM	10 OCTOBER 2024 DAILY DEFENCE UPDATES	DIVYANSHU SIR

NDA 1 2025 LIVE CLASSES

11:30AM	GK - POLITY - CONSTITUTION	RUBY MA'AM
1:00PM	BIOLOGY - MCQ - CLASS 3	SHIVANGI MA'AM
4:00PM	MATHS - TRIGONOMETRY - CLASS 3	NAVJYOTI SIR

CDS 1 2025 LIVE CLASSES

11:30AM	GK - POLITY - CONSTITUTION	RUBY MA'AM
1:00PM	BIOLOGY - MCQ - CLASS 3	SHIVANGI MA'AM
7:00PM	MATHS - AVERAGE - CLASS 2	NAVJYOTI SIR

AFCAT 1 2025 LIVE CLASSES

4:00PM	STATIC GK - GI TAGS	DIVYANSHU SIR
7:00PM	MATHS - AVERAGE - CLASS 2	NAVJYOTI SIR

SSBCrack EXAMS

TRIGNOMETRIC EQUATIONS

Equations involving trigonometric functions of a variables are called trigonometric equations.

$2\sin^2 x - \cos x + 1 = 0$

$\sin x = 0$

$2\cos x = \sqrt{3}$

SSBCrack EXAMS

SOLUTIONS OF TRIGNOMETRIC EQUATIONS

$\sin x = 0 \Rightarrow$ x is any multiple of 180°. (GENERAL SOLUTION) | $x = n\pi$ ('for $n \in Z$)

$0 \leq x < 2\pi \longrightarrow$ x is 'PARTICULAR SOLUTION'

(least value of x)

SOLUTIONS OF TRIGNOMETRIC EQUATIONS

If $\sin\theta = \sin\alpha$ for some angle α, then

$\theta = n\pi + (-1)^n\alpha$ for $n \in \mathbf{Z}$, gives general solution of the given equation

α – least value (soln.)

$\sin\theta = \frac{\sqrt{3}}{2}$

$\theta = n\pi + (-1)^n\left(\frac{\pi}{3}\right)$

If $\cos\theta = \cos\alpha$ for some angle α, then

$\theta = 2n\pi \pm \alpha$, $n \in \mathbf{Z}$, gives general solution of the given equation

$\cos\theta = 1$

$\theta = 2n\pi \pm \frac{\pi}{2}$

If $\tan\theta = \tan\alpha$ or $\cot\theta = \cot\alpha$, then

$\theta = n\pi + \alpha$, $n \in \mathbf{Z}$, gives general solution for both equations

SOLUTIONS OF TRIGNOMETRIC EQUATIONS

The general value of θ satisfying any of the equations $\sin^2\theta = \sin^2\alpha$, $\cos^2\theta = \cos^2\alpha$ and $\tan^2\theta = \tan^2\alpha$ is given by $\theta = n\pi \pm \alpha$

The general value of θ satisfying equations $\sin\theta = \sin\alpha$ and $\cos\theta = \cos\alpha$ simultaneously is given by $\theta = 2n\pi + \alpha$, $n \in \mathbf{Z}$.

SSBCrack EXAMS

INVERSE TRIGNOMETRIC FUNCTIONS

$$\sin 30^\circ = \frac{1}{2}$$

$$\underbrace{\sin^{-1}\left(\frac{1}{2}\right)} = 30^\circ = \frac{\pi}{6}$$

$(\sin^{-1}x)$

$\rightarrow \sin^{-1}x \neq \frac{1}{\sin x} = (\sin x)^{-1}$

Trignometric function,

$\text{angle (input)} \rightarrow \boxed{} \rightarrow \text{value (output)}$

Inverse Trignometric function,

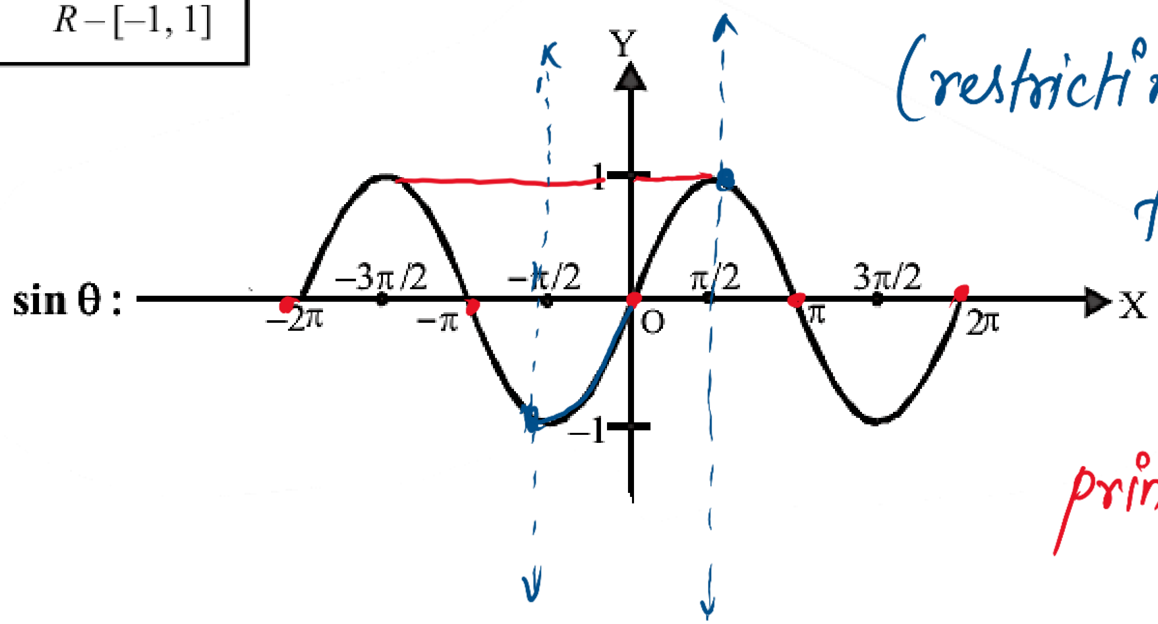
$\text{value (input)} \rightarrow \boxed{} \rightarrow \text{angle (output)}$

Function (y)	Domain of y	Range of y
$\sin\theta$	R	$[-1, 1]$
$\cos\theta$	R	$[-1, 1]$
$\tan\theta$	$R - \left\{(2n+1)\frac{\pi}{2}, n \in I\right\}$	$(-\infty, \infty)$
$\cot\theta$	$R - \{n\pi, n \in I\}$	$(-\infty, \infty)$
$\sec\theta$	$R - \left\{(2n+1)\frac{\pi}{2}, n \in I\right\}$	$R - [-1, 1]$
$\operatorname{cosec}\theta$	$R - \{n\pi, n \in I\}$	$R - [-1, 1]$

Trigonometric functions are not one-one functions.

One-one (restricting the domain to $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

principal value branch.



Functions	Domain	Range (Principal value branches)
$y = \sin^{-1}x$	$[-1,1]$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
$y = \cos^{-1}x$	$[-1,1]$	$[0,\pi]$
$y = \text{cosec}^{-1}x$	$\mathbf{R} - (-1,1)$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$y = \sec^{-1}x$	$\mathbf{R} - (-1,1)$	$[0,\pi] - \left\{\frac{\pi}{2}\right\}$
$y = \tan^{-1}x$	$\mathbf{R}$	$\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
$y = \cot^{-1}x$	$\mathbf{R}$	$(0,\pi)$

PROPERTIES OF INVERSE TRIGONOMETRIC FUNCTION

(i) $\sin^{-1}(\sin\theta) = \theta; \forall\ \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(ii) $\cos^{-1}(\cos\theta) = \theta; \forall\ \theta \in [0, \pi]$

(iii) $\tan^{-1}(\tan\theta) = \theta; \forall\ \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(iv) $\operatorname{cosec}^{-1}(\operatorname{cosec}\theta) = \theta; \forall\ \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right], \theta \neq 0$

(v) $\sec^{-1}(\sec\theta) = \theta; \forall\ \theta \in [0, \pi], \theta \neq \frac{\pi}{2}$

(vi) $\cot^{-1}(\cot\theta) = \theta; \forall\ \theta \in (0, \pi)$

$fn^{-1}(fn(x)) = x$, only if $x \in$ (principal value branch)

EXAMPLE

The value of

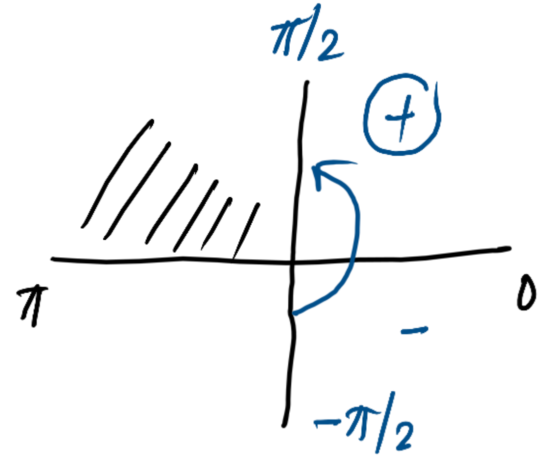
$$\sin\left[\cos^{-1}\left(-\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right] \text{ is}$$

(a) 4 (b) $\frac{1}{2}$

(c) 3 (d) None of these

$\cos^{-1}x \rightarrow [0, \pi]$

$\tan^{-1}x \rightarrow \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$



$\cos x = -\frac{1}{2}$

$x = \cos^{-1}\left(\frac{-1}{2}\right)$

$\sin\left[\left(\pi - \frac{\pi}{3}\right) + \frac{\pi}{6}\right]$

$\sin\left[\frac{2\pi}{3} + \frac{\pi}{6}\right] = \sin\left(\frac{4\pi+\pi}{6}\right) = \sin\left(\pi - \frac{\pi}{6}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$

EXAMPLE

The value of

$$\sin\left[\cos^{-1}\left(-\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right] \text{ is}$$

(a) 4

(b) $\frac{1}{2}$

(c) 3

(d) None of these

Ans: (b)

PROPERTIES OF INVERSE TRIGONOMETRIC FUNCTION

(i) $\sin^{-1}(-x) = -\sin^{-1}(x), \forall\ x \in [-1, 1]$

(ii) $\cos^{-1}(-x) = \pi - \cos^{-1}(x), \forall\ x \in [-1, 1]$

(iii) $\tan^{-1}(-x) = -\tan^{-1} x, \forall\ x \in R$

(iv) $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x,$
$\forall\ x \in (-\infty, -1] \cup [1, \infty)$

(v) $\sec^{-1}(-x) = \pi - \sec^{-1} x, \forall\ x \in (-\infty, -1] \cup [1, \infty)$

(vi) $\cot^{-1}(-x) = \pi - \cot^{-1} x, \forall\ x \in R$

$$\sin^{-1}(-x) = -\sin^{-1}x$$

$$\tan^{-1}(-x) = -\tan^{-1}x$$

$$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x$$

$$\cos^{-1}(-x) = \pi - \cos^{-1}x$$

$$\sec^{-1}(-x) = \pi - \sec^{-1}x$$

$$\cot^{-1}(-x) = \pi - \cot^{-1}x$$

INVERSE TRIGONOMETRIC FORMULAE

(i) $\tan^{-1} x + \tan^{-1} y$

$$= \left\{ \tan^{-1}\left(\frac{x+y}{1-xy}\right), \qquad \text{if } xy < 1 \right.$$

(ii) $\tan^{-1} x - \tan^{-1} y$

$$= \left\{ \tan^{-1}\left(\frac{x-y}{1+xy}\right), \qquad \text{if } xy > -1 \right.$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

EXAMPLE

$\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{4}$ is equal to

(a) $\tan^{-1}\frac{3}{5}$

(b) $\tan^{-1}\frac{5}{3}$ ✓

(c) $\tan^{-1}\frac{1}{5}$

(d) $\tan^{-1}\frac{7}{3}$

$$\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{3}\right)=\tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)}\right)$$

$$=\tan^{-1}\left(\frac{5}{5}\right)=\underline{\tan^{-1}(1)}$$

$$\tan^{-1}(1)+\tan^{-1}\left(\frac{1}{4}\right)$$

$$=\tan^{-1}\left(\frac{5/4}{3/4}\right)$$

$$=\tan^{-1}\left(\frac{5}{3}\right)$$

EXAMPLE

$\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{4}$ is equal to

(a) $\tan^{-1}\frac{3}{5}$

(b) $\tan^{-1}\frac{5}{3}$

(c) $\tan^{-1}\frac{1}{5}$

(d) $\tan^{-1}\frac{7}{3}$

Ans: (b)

INVERSE TRIGONOMETRIC FORMULAE

(i) $2\tan^{-1} x = \left\{\tan^{-1}\left(\dfrac{2x}{1-x^2}\right),\right.$ if $-1 < x \le 1$

(ii) $3\tan^{-1} x$

$= \left\{\tan^{-1}\left(\dfrac{3x - x^3}{1 - 3x^2}\right),\right.$ if $-\dfrac{1}{\sqrt{3}} \le x < \dfrac{1}{\sqrt{3}}$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

$$\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$$

EXAMPLE

$\tan\left[2\tan^{-1}\left(\frac{1}{5}\right)-\frac{\pi}{4}\right]$ is equal to

(a) $\frac{17}{7}$

(b) $-\frac{17}{7}$

(c) $\frac{7}{17}$

(d) $-\frac{7}{17}$

EXAMPLE

$\tan\left[2\tan^{-1}\left(\frac{1}{5}\right)-\frac{\pi}{4}\right]$ is equal to

(a) $\frac{17}{7}$ (b) $-\frac{17}{7}$

(c) $\frac{7}{17}$ (d) $-\frac{7}{17}$

Ans: (d)

INVERSE TRIGONOMETRIC FORMULAE

(i) $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}, \forall\, x \in [-1, 1]$

(ii) $\underline{\tan}^{-1} x + \underline{\cot}^{-1} x = \frac{\pi}{2}, \forall\, x \in R$

(iii) $\underline{\sec^{-1} x} + \underline{\text{cosec}}^{-1} x = \frac{\pi}{2}, \forall\, x \in (-\infty, -1] \cup [1, \infty)$

EXAMPLE

$\sin^{-1} x + \sin^{-1} \frac{1}{x} + \cos^{-1} x + \cos^{-1} \frac{1}{x}$ is equal to

(a) π ✓

(b) $\frac{\pi}{2}$

(c) $\frac{3\pi}{2}$

(d) None of these

$\left(\sin^{-1} x + \cos^{-1} x\right) + \left(\sin^{-1} \frac{1}{x} + \cos^{-1} \frac{1}{x}\right)$

$\frac{\pi}{2} + \frac{\pi}{2} = \pi$

EXAMPLE

$\sin^{-1} x + \sin^{-1} \frac{1}{x} + \cos^{-1} x + \cos^{-1} \frac{1}{x}$ is equal to

(a) π

(b) $\frac{\pi}{2}$

(c) $\frac{3\pi}{2}$

(d) None of these

Ans: (a)

INVERSE TRIGONOMETRIC FORMULAE

(i) $\sin^{-1} x + \sin^{-1} y$

$$= \begin{cases} \sin^{-1}\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\}, \text{ if } -1 \le x, y \le 1 \\ \text{and } x^2 + y^2 \le 1 \text{ or } xy < 0 \text{ and } x^2 + y^2 > 1 \end{cases}$$

(ii) $\sin^{-1} x - \sin^{-1} y$

$$= \begin{cases} \sin^{-1}\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\}, \text{ if } -1 \le x, y \le 1 \\ \text{and } x^2 + y^2 \le 1 \text{ or if } xy > 0 \text{ and } x^2 + y^2 > 1 \end{cases}$$

(i) $\cos^{-1} x + \cos^{-1} y$

$$= \begin{cases} \cos^{-1}\{xy - \sqrt{1-x^2}\sqrt{1-y^2}\}, \text{ if } -1 \le x, y \le 1 \\ \text{and } x + y \ge 0 \end{cases}$$

(ii) $\cos^{-1} x - \cos^{-1} y$

$$= \begin{cases} \cos^{-1}\{xy + \sqrt{1-x^2}\sqrt{1-y^2}\}, \\ \text{if } -1 \le x, y \le 1 \text{ and } x \le y \end{cases}$$

$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos(A \pm B) \quad \cos A \cos B \mp \sin A \sin B$

$\cos A = \sqrt{1 - \sin^2 A}$

$\sin A = \sqrt{1 - \cos^2 A}$

EXAMPLE

What is the value of $\cos\left\{\cos^{-1}\frac{4}{5}+\cos^{-1}\frac{12}{13}\right\}$?

(a) 63/65 (b) 33/65 ✓

(c) 22/65 (d) 11/65

$\frac{48-15}{65} = \frac{33}{65}$

$\cos\left\{\cos^{-1}\left(\left(\frac{4}{5}\right)\left(\frac{12}{13}\right) - \sqrt{1-\left(\frac{4}{5}\right)^2}\sqrt{1-\left(\frac{12}{13}\right)^2}\right)\right\}$

$\cos\left\{\cos^{-1}\left(\frac{48}{65} - \frac{3}{5}\times\frac{5}{13}\right)\right\} = \cos\left\{\cos^{-1}\left(\frac{48}{65}-\frac{3}{13}\right)\right\} = \cos\left\{\cos^{-1}\left(\frac{33}{65}\right)\right\}$

$$\cos\left(\cos^{-1}\left(\frac{33}{65}\right)\right)$$

$$= \frac{33}{65}$$

$\cos^{-1}x \longrightarrow [0, \pi]$

$0 < \frac{33}{65} < 1$ $\quad \frac{33}{65} \in [0, \pi]$

EXAMPLE

What is the value of $\cos\left\{\cos^{-1}\frac{4}{5}+\cos^{-1}\frac{12}{13}\right\}$?

(a) 63/65 (b) 33/65

(c) 22/65 (d) 11/65

Ans: (b)

INVERSE TRIGONOMETRIC FORMULAE

(i) $2\sin^{-1}x$

$$= \left\{ \sin^{-1}(2x\sqrt{1-x^2}), \quad \text{if } -\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}} \right.$$

$\sin 2x = 2\sin x \cos x$

(ii) $3\sin^{-1}x$

$$= \left\{ \sin^{-1}(3x-4x^3), \quad \text{if } -\frac{1}{2} \le x \le \frac{1}{2} \right.$$

$\sin 3x = 3\sin x - 4\sin^3 x$

(i) $2\cos^{-1}x = \{\cos^{-1}(2x^2-1), \quad \text{if } 0 \le x \le 1$

$\cos 2x = 2\cos^2 x - 1$

(ii) $3\cos^{-1}x = \left\{ \cos^{-1}(4x^3-3x); \quad \text{if } \frac{1}{2} \le x \le 1 \right.$

$\cos 3x = 4\cos^3 x - 3\cos x$

EXAMPLE

The value of $\sin(2\sin^{-1} 0.8)$ is

(a) 0.96 (b) 0.48

(c) 0.64 (d) None of these

$$\begin{array}{r} 0.5 \\ 125\overline{)720} \end{array}$$

$\underline{\sin 2x} = 2\underline{\sin x}\,\underline{\cos x}$

$2\sin^{-1}x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

$\sin(2\sin^{-1} 0.8)$

$= \sin\left(\sin^{-1}\left(2\times 0.8 \times (1-(0.8)^2\right)\right)$

$= \sin\left(\sin^{-1}\left(1.6 \times 0.36\right)\right)$

$= \frac{72}{125} = 0.5$

$\frac{\cancel{16}^{4}}{\cancel{10}_{5}} \times \frac{\cancel{36}^{18}}{\cancel{100}_{25}} = \frac{72}{125}$

$\sin^{-1}x \rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\frac{72}{125} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

EXAMPLE

The value of $\sin(2\sin^{-1} 0.8)$ is

(a) 0.96 (b) 0.48
(c) 0.64 (d) None of these

Ans: (d)

OTHER RESULTS

- If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$, then $xy + yz + zx = 1$
- If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, then $x + y + z = xyz$
- If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{\pi}{2}$, then

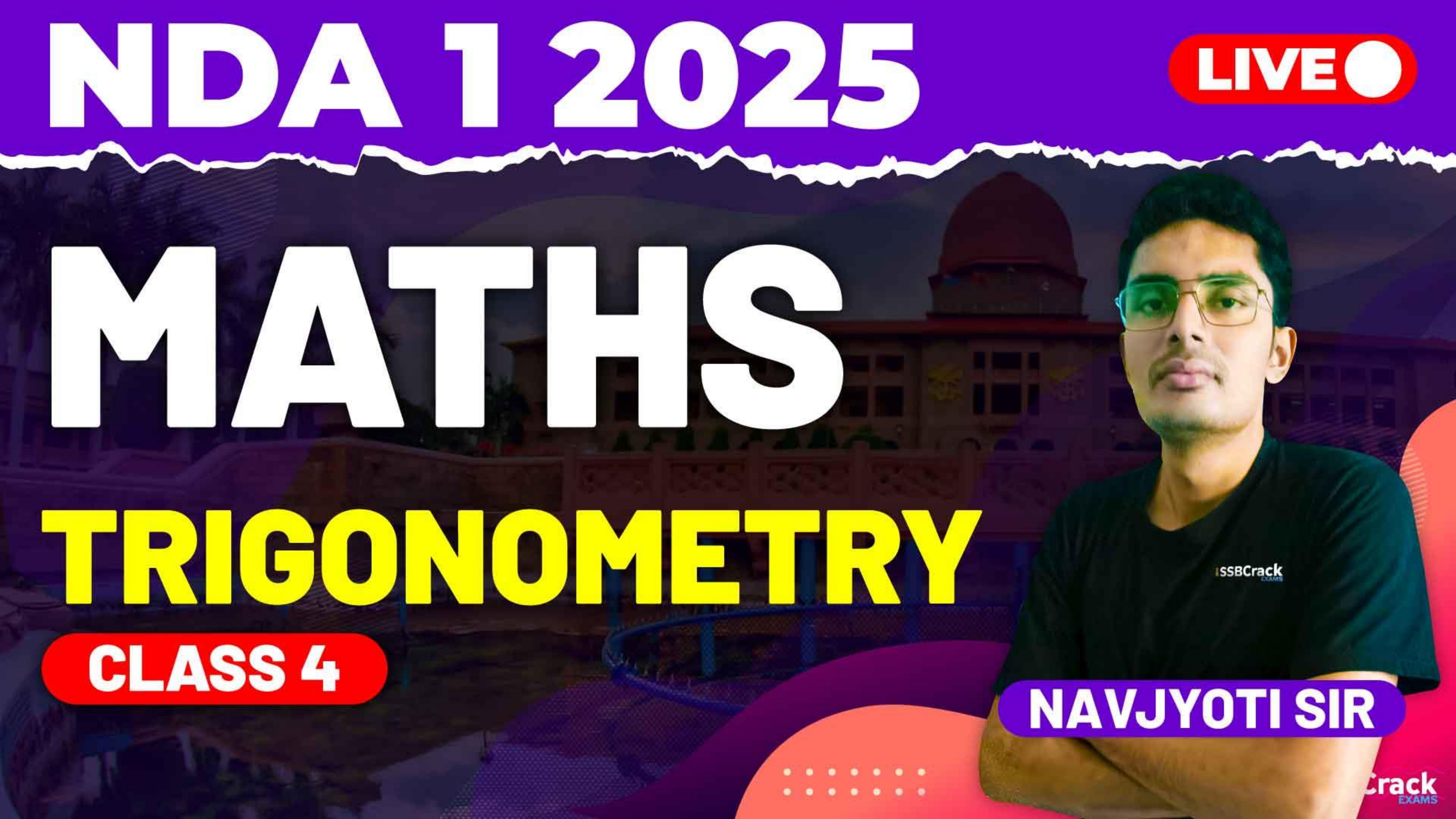
  $$x^2 + y^2 + z^2 + 2xyz = 1$$
- If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi$, then

  $$x\sqrt{1 - x^2} + y\sqrt{1 - y^2} + z\sqrt{1 - z^2} = 2xyz$$
- If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$, then

  $$xy + yz + zx = 3$$

OTHER RESULTS

- If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, then
  $$x^2 + y^2 + z^2 + 2xyz = 1$$
- If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$, then
  $$xy + yz + zx = 3$$
- If $\sin^{-1}x + \sin^{-1}y = \theta$, then $\cos^{-1}x + \cos^{-1}y = \pi - \theta$
- If $\cos^{-1}x + \cos^{-1}y = \theta$, then $\sin^{-1}x + \sin^{-1}y = \pi - \theta$
- If $\tan^{-1}x + \tan^{-1}y = \frac{\pi}{2}$, then $xy = 1$
- If $\cot^{-1}x + \cot^{-1}y = \frac{\pi}{2}$, then $xy = 1$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