NDA 1 2025

LIVE

MATHS

BINOMIAL THEOREM

CLASS 1

NAVJYOTI SIR

Crack EXAMS

11 Nov 2024 Live Classes Schedule

8:00AM	11 NOVEMBER 2024 DAILY CURRENT AFFAIRS	RUBY MA'AM
9:00AM	11 NOVEMBER 2024 DAILY DEFENCE UPDATES	DIVYANSHU SIR

SSB INTERVIEW LIVE CLASSES

9:30AM	OVERVIEW OF SRT & SDT	ANURADHA MA'AM

NDA 1 2025 LIVE CLASSES

11:30AM	GK - MODERN HISTORY - CLASS 2	RUBY MA'AM
4:00PM	MATHS - BINOMIAL THEOREM - CLASS 1	NAVJYOTI SIR
5:30PM	ENGLISH - COMPREHENSION - CLASS 2	ANURADHA MA'AM

CDS 1 2025 LIVE CLASSES

11:30AM	GK - MODERN HISTORY - CLASS 2	RUBY MA'AM
5:30PM	ENGLISH - COMPREHENSION - CLASS 2	ANURADHA MA'AM
7:00PM	MATHS - GEOMETRY - CLASS 5	NAVJYOTI SIR

AFCAT 1 2025 LIVE CLASSES

5:30PM	ENGLISH - COMPREHENSION - CLASS 2	ANURADHA MA'AM

INTRODUCTION

$$(a+b)^2 = a^2 + 2ab + b^2 \longrightarrow a^2b^0 + 2a^1b^1 + b^2a^0$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 \longrightarrow a^3b^0 + 3a^2b^1 + 3a^1b^2 + b^3a^0$$

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BINOMIAL THEOREM

$$(a+b)^n = {}^nC_0\, a^{n-0} b^0 + {}^nC_1\, a^{n-1} b^1 + {}^nC_2\, a^{n-2} b^2 + \dots + {}^nC_{n-1}\, a^1 b^{n-1} + {}^nC_n\, a^{n-n} b^n$$

$$(a+b)^n = \sum_{r=0}^{n} {}^nC_r\, a^{n-r} b^r$$

${}^nC_0$, ${}^nC_1$, ${}^nC_2$ .... ${}^nC_n$ are called, 'Binomial coefficients'.

$$(a+b)^4 = {}^4C_0\, a^4 b^0 + {}^4C_1\, a^{4-1} b^1 + {}^4C_2\, a^{4-2} b^2 + {}^4C_3\, a^{4-3} b^3 + {}^4C_4\, a^{4-4} b^4$$

$$= 1 \cdot a^4 \cdot 1 + 4 \cdot a^3 b^1 + \frac{4 \times 3}{2} a^2 b^2 + 4 \cdot a^1 b^3 + 1 \cdot 1 \cdot b^4$$

$$= \underline{a^4} + 4a^3 b + 6a^2b^2 + 4ab^3 + \underline{b^4}$$

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OBSERVATIONS

- The total number of terms in the binomial expansion of $(a + b)^n$ is $n + 1$, i.e. one more than the exponent n.

$$(a+b)^n = {}^nC_0 + \underbrace{{}^nC_1 + \cdots {}^nC_n}_{n \text{ terms}}$$

Total $(n+1)$ terms,

OBSERVATIONS

- In the expansion, the first term is raised to the power of the binomial and in each subsequent terms the power of a reduces by one with simultaneous increase in the power of b by one, till power of b becomes equal to the power of binomial, i.e., the power of a is n in the first term, $(n - 1)$ in the second term and so on ending with zero in the last term.

  At the same time power of b is 0 in the first term, 1 in the second term and 2 in the third term and so on, ending with n in the last term.

OBSERVATIONS

- **In any term the sum of the indices (exponents) of '*a*' and '*b*' is equal to *n* (i.e., the power of the binomial).**

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OBSERVATIONS

- **The binomial coefficients in the binomial expansion equidistant from the beginning and the end are equal.**

coefficients

$(a+b)^2$ 1 2 1

$(a+b)^3$ 1 3 3 1

$(a+b)^4$ 1 4 6 4 1

PASCAL's TRIANGLE

- The coefficients in the expansion follow a certain pattern known as pascal's triangle.
- Each coefficient of any row is obtained by adding two coefficients in the preceding row, one on the immediate left and the other on the immediate right and each row is bounded by 1 on both sides.

$(a+b)^0$ $\quad$ 1

$(a+b)^1$ $\quad$ 1 + 1

$(a+b)^2$ $\quad$ 1 + 2 + 1

$(a+b)^3$ $\quad$ 1 3 3 1

$(a+b)^4$ $\quad$ 1 4 6 4 1

PASCAL's TRIANGLE

- The coefficients in the expansion follow a certain pattern known as pascal's triangle.

Index of Binomial (n)	Coefficient of various terms										
0						1					
1					1		1				
2				1		2		1			
3			1		3		3		1		
4		1		4		6		4		1	
5	1		5		10		10		5		1

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GREATEST BINOMIAL COEFFICIENTS

In a binomial expansion, binomial coefficients of the middle terms are greatest binomial coefficients.

(i) **If n is even :** ${}^{n}C_{n/2}$ takes maximum value.

(ii) **If n is odd :** Both ${}^{n}C_{\frac{n-1}{2}}$ and ${}^{n}C_{\frac{n+1}{2}}$ take maximum value.

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GENERAL TERM

$$(a+b)^n = {}^nC_0\, a^n b^0 + {}^nC_1\, a^{n-1} b^1 + {}^nC_2\, a^{n-2} b^2 + \dots {}^nC_n\, a^{n-n} b^0$$

1 2 3

$(r+1)^{th}$ term,

$$\boxed{T_{r+1} = {}^nC_r\, a^{n-r} b^r}$$

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QUESTION

Find the r^{th} term in the expansion of $\left(x+\frac{1}{x}\right)^{2r}$

r^{th} term,

$$T_r = {}^{2r}C_{r-1}\,(x)^{2r-(r-1)}\left(\frac{1}{x}\right)^{r-1}$$

$$= \frac{(2r)!}{(2r-r+1)!\,(r-1)!}\,x^{r+1} \times \frac{1}{x^{r-1}} = \frac{(2r)!}{(r+1)!\,(r-1)!}\,x^{r+1-r+1}$$

$$= \frac{(2r)!}{(r-1)!\,(r+1)!}\,x^2$$

MIDDLE TERM

$n = \text{odd}$

$(n+1) \rightarrow$ even

$\underbrace{\left(\frac{n+1}{2}\right)^{th} \text{ term and } \left(\frac{n+1}{2}+1\right)^{th} \text{ term}}_{2 \text{ middle terms}}$

$n = \text{even}$

$(n+1) \rightarrow$ odd

$\frac{n+1+1}{2} = \left(\frac{n+2}{2}\right)^{th}$ term

$= \underbrace{\left(\frac{n}{2}+1\right)^{th} \text{ term}}_{1 \text{ middle term}}$

QUESTION

Find the middle term (terms) in the expansion of $\left(\frac{p}{x}+\frac{x}{p}\right)^9$.

number of terms $= 9+1 = 10$

$\left(\frac{10}{2}\right)^{th}$ & $\left(\frac{10}{2}+1\right)^{th}$

$T_5 = {}^9C_4 \left(\frac{p}{x}\right)^{9-4}\left(\frac{x}{p}\right)^4 = \frac{9\times \cancel{8}\times \cancel{7}\times \cancel{6}}{\cancel{4}\times\cancel{3}\times\cancel{2}}\ \frac{p^5}{x^5}\times\frac{x^4}{p^4} = 126\left(\frac{p}{x}\right)$

$T_6 = {}^9C_5 \left(\frac{p}{x}\right)^{9-5}\left(\frac{x}{p}\right)^5 = 126\left(\frac{x}{p}\right)$

2 middle terms

$\left({}^nC_r = {}^nC_{n-r}\right)$

INDEPENDENT TERM

$$T_{r+1} = {}^{n}C_{r}\, a^{n-r}\, b^{r} \quad \text{(general term)}$$

Independent term $\longrightarrow$ no variable involved (only pure number)

QUESTION

Find the term independent of x in the expansion of $\left(\frac{\sqrt{x}}{\sqrt{3}}+\frac{\sqrt{3}}{2x^2}\right)^{10}$.

General term,

$$T_{r+1} = {}^{10}C_r \left(\frac{\sqrt{x}}{\sqrt{3}}\right)^{10-r}\left(\frac{\sqrt{3}}{2x^2}\right)^r$$

$$= {}^{10}C_r \frac{x^{\frac{10-r}{2}}}{3^{\frac{10-r}{2}}} \cdot \frac{3^{\frac{r}{2}}}{2^r \cdot x^{2r}}$$

$$= \frac{{}^{10}C_r}{2^r} 3^{\left(\frac{r}{2}-\frac{10-r}{2}\right)} \cdot x^{\frac{10-r}{2}-2r}$$

Independent term

power of $x = 0$

$$\frac{10-r}{2} - 2r = 0$$

$$10 - 5r = 0$$

$$r = 2$$

$$\frac{{}^{10}C_r}{2^r} \cdot 3^{\frac{2r-10}{2}} = \frac{{}^{10}C_r}{2^r} \cdot 3^{r-5}$$

Putting $r = 2$,

$${}^{10}C_2 \times \frac{1}{2^2} \times 3^{-3}$$

$$= \frac{\overset{5}{\cancel{10}} \times \cancel{9}}{\cancel{2}} \times \frac{1}{4} \times \frac{1}{\underset{3}{\cancel{27}}} = \frac{5}{12}$$

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SOME PARTICULAR CASES

$$(a+b)^n = a^n + {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 + {}^nC_3 a^{n-3} b^3 + \dots b^n \quad (1)$$

$$(a-b)^n = (a+(-b))^n = a^n + {}^nC_1 a^{n-1} (-b)^1 + {}^nC_2 a^{n-2} (-b)^2 + {}^nC_3 a^{n-3} (-b)^3 + \dots (-b)^n$$

$$= a^n - {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 - {}^nC_3 a^{n-3} b^3 + \dots \quad (2)$$

1 (1) + (2) $(a+b)^n + (a-b)^n = 2a^n + 2\,{}^nC_2 a^{n-2} b^2 + 2\,{}^nC_4 a^{n-4} b^4 + \dots$

$= 2$ (sum of terms at odd place) | even powers of b remain

2) 1 – 2,

$$(a+b)^n - (a-b)^n = 2\,{}^nC_1\, a^{n-1} b^1 + 2\,{}^nC_3\, a^{n-3} b^3 + \dots$$

$$= 2(\text{sum of terms at even places})$$

powers of b are odd.

3) Put $a = 1$, $b = x$,

$$(a+b)^n = a^n + {}^nC_1\, a^{n-1} b + {}^nC_2\, a^{n-2} b^2 + {}^nC_3\, a^{n-3} b^3 + \dots b^n$$

$$(1+x)^n = 1^n + {}^nC_1\, 1^{n-1}(x) + {}^nC_2\, 1^{n-2}(x)^2 + {}^nC_3\, (1)^{n-3} x^3 + \dots x^n$$

$$= 1 + {}^nC_1\, x + {}^nC_2\, x^2 + {}^nC_3\, x^3 + \dots x^n = \sum_{r=0}^{n} {}^nC_r\, x^r$$

(4) Put $a = 1$, $b = -x$,

$$(1-x)^n = (1+(-x))^n = 1 + {}^nC_1\, 1^{n-1}(-x)^1 + {}^nC_2 \cdot 1^{n-2}(-x)^{n-2}$$

$$+ {}^nC_3\, 1^{n-3}(-x)^{n-3} + \ldots$$

$$= 1 - {}^nC_1\, x + {}^nC_2\, x^2 - {}^nC_3\, x^3 + {}^nC_4\, x^4 + \ldots$$

$$= \sum_{r=0}^{n} (-1)^r\, {}^nC_r\, x^r$$

QUESTION

Evaluate: $\left(x^2 - \sqrt{1-x^2}\right)^4 + \left(x^2 + \sqrt{1-x^2}\right)^4$

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p^{th} TERM FROM THE END

The p^{th} term from the end in the expansion of $(a + b)^n$ is $(n - p + 2)^{th}$ term from the beginning.

$$T_{n-p+2} = {}^nC_{n-p+1} (a)^{p-1} (b)^{n-p+1}$$

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QUESTION

Find the 4th term from the end in the expansion of $\left(\frac{x^3}{2}-\frac{2}{x^2}\right)^9$

$n-p+2 = 9-4+2 = 7$

7th term from start,

$$T_7 = {}^9C_6\left(\frac{x^3}{2}\right)^{9-6}\left(-\frac{2}{x^2}\right)^6$$

$$= {}^9C_3\,\frac{x^9}{8}\times\frac{64}{x^{12}}$$

SSBCrack EXAMS

BINOMIAL COEFFICIENTS

$^nC_0$, $^nC_1$, $^nC_2$ . . . ^nC_n are binomial coefficients,

(1) $(1+x)^n = 1 + {^nC_1}x + {^nC_2}x^2 + {^nC_3}x^3 + \dots x^n$

$x = 1$,

$(1+1)^n = 1 + {^nC_1}1 + {^nC_2}1^2 + {^nC_3}1^3 + \dots 1^n$

$= 1 + {^nC_1} + {^nC_2} + {^nC_3} + {^nC_4} + \dots {^nC_n}$

$2^n = {^nC_0} + {^nC_1} + {^nC_2} + {^nC_3} + {^nC_4} + \dots {^nC_n}$

sum of binomial coefficients $= \underline{2^n}$

(2) $(1+x)^n = {}^nC_0 x^0 + {}^nC_1 x^1 + {}^nC_2 x^2 + \dots {}^nC_n x^n$

$x = -1,$

$(1-1)^n = {}^nC_0 - {}^nC_1 + {}^nC_2 (-1)^2 + {}^nC_3 (-1)^3 + \dots {}^nC_n (-1)^n$

$0^n = {}^nC_{\underline{0}} - {}^nC_{\underline{1}} + {}^nC_{\underline{2}} - {}^nC_3 + \dots$

$0 = ({}^nC_0 + {}^nC_2 + \dots) - ({}^nC_1 + {}^nC_3 + {}^nC_5 + \dots)$

Sum of even coefficients = Sum of odd coefficients $= \frac{2^n}{2} = 2^{n-1}$

${}^nC_{\underline{0}} + {}^nC_{\underline{2}} + {}^nC_{\underline{4}} + \dots = {}^nC_{\underline{1}} + {}^nC_{\underline{3}} + {}^nC_{\underline{5}} + \dots$

SSBCrack EXAMS

QUESTION

PYQ – 2024 - II

In the expansion of $(1+x)^p(1+x)^q$, if the coefficient of x^3 is 35, then what is the value of $(p+q)$?

(a) 5

(b) 6

(c) 7 ✓

(d) 8

$(1+x)^{p+q}$

$T_{r+1} = {}^{p+q}C_r (1)^{p+q-r} (x)^r$

${}^{p+q}C_r (x)^r$ — for $x^3 \longrightarrow r = 3$,

coefficient — $x^3 = {}^{p+q}C_3$

Put options

(a) ${}^5C_3 = 10$ ✗

(b) ${}^6C_3 = 20$ ✗

(c) ${}^7C_3 = 35$ ✓

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QUESTION

What is the remainder when $7^n - 6n$ is divided by 36 for $n = 100$?

PYQ – 2024 - II

(a) 0

(b) 1 ✓

(c) 2

(d) 6

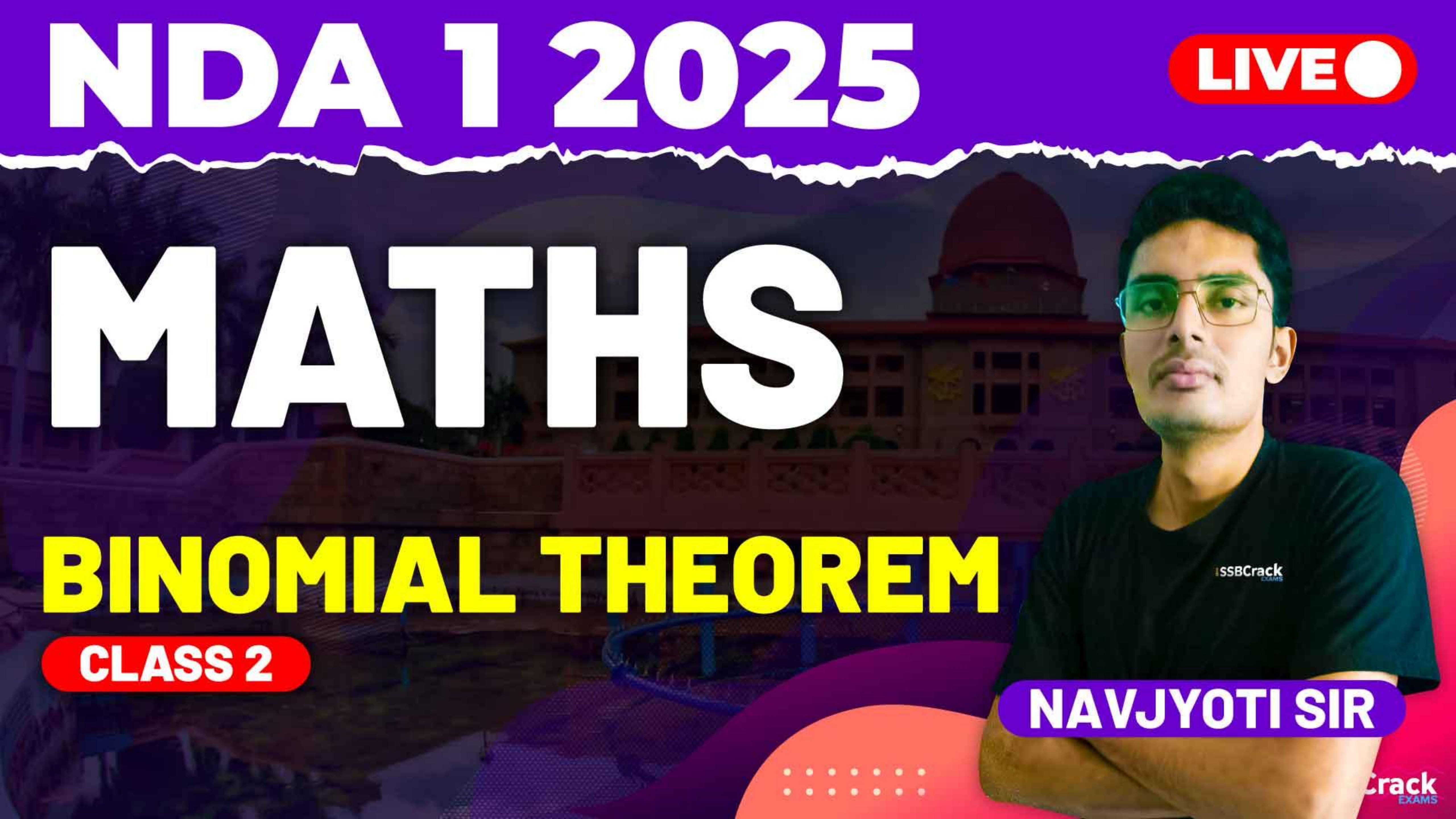
$$7^n = (1+6)^n$$

$$= 1 + {}^nC_1 6^1 + {}^nC_2 6^2 + {}^nC_3 6^3 + {}^nC_4 6^4 + \dots$$

$$7^n = 1 + 6n + 36\left({}^nC_2 + {}^nC_3 6 + {}^nC_4 6 + \dots\right)$$

$$7^n - 6n = 1 + 36m$$

remainder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