NDA 1 2025

LIVE

MATHS

LIMITS & CONTINUITY

CLASS 1

NAVJYOTI SIR

28 Nov 2024 Live Classes Schedule

9:00AM	28 NOVEMBER 2024 DAILY DEFENCE UPDATES	DIVYANSHU SIR

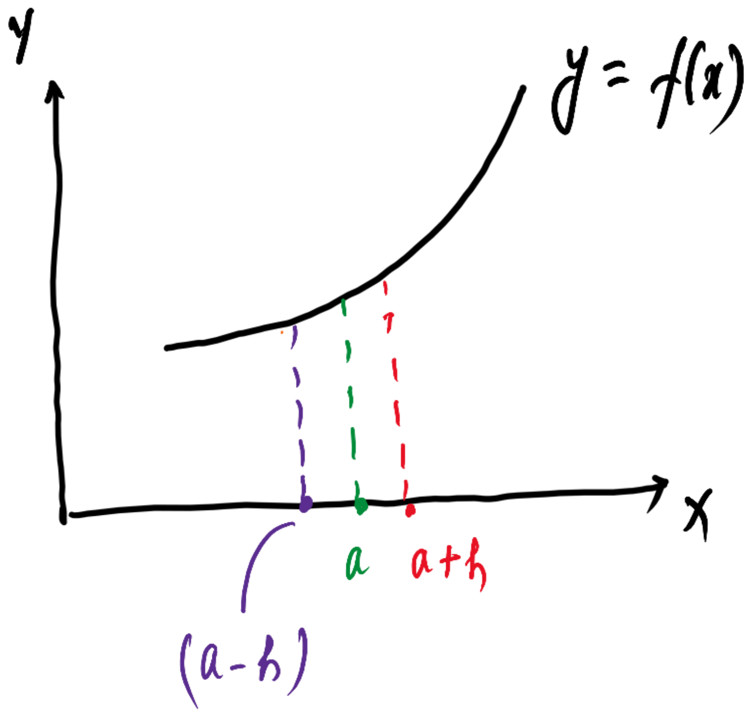
NDA 1 2025 LIVE CLASSES

1:00PM	PHYSICS - REFLECTION OF LIGHT - CLASS 1	NAVJYOTI SIR
4:30PM	ENGLISH - COMMONLY USED WORDS - CLASS 2	ANURADHA MA'AM
5:30PM	MATHS - LIMITS & CONTINUITY - CLASS 1	NAVJYOTI SIR

CDS 1 2025 LIVE CLASSES

1:00PM	PHYSICS - REFLECTION OF LIGHT - CLASS 1	NAVJYOTI SIR
4:30PM	ENGLISH - COMMONLY USED WORDS - CLASS 2	ANURADHA MA'AM
7:00PM	MATHS - TRIGONOMETRY - CLASS 2	NAVJYOTI SIR

LIMIT AT A POINT



(let $h > 0$ and is a very small)

When x approaches towards 'a', what does f(x) approach to ?

$\lim\limits_{x \to a^-} f(x) \longrightarrow$ Left hand limit at $x = a$

$\lim\limits_{x \to a^+} f(x) \longrightarrow$ Right hand limit at $x = a$.

$\lim_{x \to a^-} f(x) \longrightarrow$ Left hand limit at $x = a$

$\lim_{x \to a^+} f(x) \longrightarrow$ Right hand limit at $x = a$.

if both gives finite value and equal,

$\downarrow$

$\lim_{x \to a} f(x)$ exists.

PROPERTIES OF LIMIT

Let f and g be two functions such that both $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist. Then

(i) $\lim_{x\to a} [f(x) + g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$ ✓

(ii) $\lim_{x\to a} [f(x) - g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$

(iii) For every real number α

$\lim_{x\to a} (\alpha f)(x) = \alpha \lim_{x\to a} f(x)$ ✓

(iv) $\lim_{x\to a} [f(x)\, g(x)] = [\lim_{x\to a} f(x) \lim_{x\to a} g(x)]$

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}, \text{ provided } g(x) \neq 0$$

INDETERMINATE FORM

An **indeterminate form** is an expression involving two functions whose limit cannot be determined solely from the limits of the individual functions.

If a function $f(x)$ takes any of the following forms at $x = a, \frac{0}{0}, \frac{\infty}{\infty}, \infty - \infty, \underline{0 \times \infty}, \underline{0^0}, \underline{\infty^0}, \underline{1^\infty}$, then $f(x)$ is said to be indeterminate at $x = a$.

(majorly) — most important

$$\lim_{x \to 2} \frac{x^2 - 5x + 6}{x - 2}$$

When $x = 2$ is put in $\frac{x^2 - 5x + 6}{x-2}$, it becomes $\underline{\left(\frac{0}{0}\right)}$

$$\lim_{x \to 2} \frac{(x-3)\cancel{(x-2)}}{\cancel{(x-2)}} = x - 3 = -1$$

IMPORTANT RESULTS

$$\lim_{x \to a} \frac{\sin f(x)}{f(x)} = 1$$

$$\lim_{x \to a} \cos f(x) = 1$$

$$\lim_{x \to a} \frac{\tan f(x)}{f(x)} = 1$$

$f(a) = 0,$

similar

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$$\lim_{x \to 0} \frac{\sin x}{x} = 1 \qquad \lim_{x \to 0} \frac{\tan x}{x} = 1 \qquad \lim_{x \to 0} \cos x = 1$$

IMPORTANT RESULTS

$$\lim_{x \to a} (1 + f(x))^{1/f(x)} = e$$

$$\lim_{x \to a} \frac{e^{f(x)} - 1}{f(x)} = 1 \quad (\log e = 1)$$

$$\lim_{x \to a} \frac{b^{f(x)} - 1}{f(x)} = \log b \ \ (b > 0)$$

$(x \to 0$ and $f(x) = x) \longrightarrow$ simpler results,

IMPORTANT RESULTS

$$\lim_{x \to a} \frac{x^m - a^m}{x^n - a^n} = \frac{m}{n} a^{m-n} \quad (m, n > 0)$$

#1

$$\lim_{x \to a} \frac{x^m - a^m}{x - a} = m a^{m-1}$$

$$\lim_{x \to a} \frac{x^n - a^n}{x - a} = n a^{n-1}$$

$$\lim_{x \to a} \frac{x^m - a^m}{x^n - a^n} = \frac{\lim_{x \to a} \frac{x^m - a^m}{x - a}}{\lim_{x \to a} \frac{x^n - a^n}{x - a}} = \frac{m a^{m-1}}{n a^{n-1}} = \frac{m}{n} a^{m-n}$$

L'HOSPITAL RULE

Let $f(x)$ and $g(x)$ be two functions such that $f(a) = 0$ and $g(a) = 0$, then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

derivative w.r.t. x

derivative w.r.t. x,

→ For $\frac{0}{0}$ form and $\frac{\infty}{\infty}$ form,

→ For any other indeterminate form, if it can be converted to $\frac{0}{0}$ or $\frac{\infty}{\infty}$ form, L-Hospital rule can be applied.

Determinate-Indeterminate Forms Table	
Indeterminate Forms	Determinate Forms
$0/0$	$\infty + \infty = \infty$
$\pm\infty/\pm\infty$	$-\infty - \infty = -\infty$
$\infty - \infty$	$0^{\infty} = 0$
$0(\infty)$	$0^{-\infty} = \infty$
0^0	$(\infty)\cdot(\infty) = \infty$
1^{∞}	
∞^0	
Use L'Hôpital's Rule	Do *Not* Use L'Hôpital's Rule

IMPORTANT EXPANSIONS

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots\ldots\ldots\ldots$$

$$\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 + \frac{17}{315}x^7 \ldots\ldots\ldots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots\ldots\ldots\ldots$$

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IMPORTANT EXPANSIONS

$$\sin hx = x + \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots\ldots\ldots\ldots$$

$$\cos hx = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots\ldots\ldots\ldots$$

$$\tan hx = x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17}{315}x^7 \ldots\ldots\ldots$$

IMPORTANT EXPANSIONS

$$\sin^{-1}x = \mathrm{x} + \frac{x^3}{3!} + \frac{9x^5}{5!} + \ldots\ldots\ldots$$

$$\cos^{-1}x = \frac{\pi}{2} - \left(\mathrm{x} + \frac{x^3}{3!} + \frac{9x^5}{5!} + \ldots\ldots\ldots\right)$$

$$\tan^{-1}x = \mathrm{x} - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots\ldots\ldots$$

Q) What is the value of $\lim_{x \to \infty} \frac{\sin x}{x}$?

(a) 1 (b) 0

(c) ∞ (d) -1

$x = \frac{1}{y}$

$x \to \infty \Rightarrow y \to 0$

$\lim_{x \to \infty} \frac{\sin x}{x} = \lim_{\underbrace{y \to 0}} \frac{\sin\left(\frac{1}{y}\right)}{\left(\frac{1}{y}\right)} = \lim_{y \to 0} \cancel{y}\, \cancel{\sin\left(\frac{1}{y}\right)}$

can give any value between -1 and 1.

$= 0(\quad) = 0$

SSBCrack EXAMS

Q) What is the value of $\lim\limits_{x \to \infty} \dfrac{\sin x}{x}$?

(a) 1

(b) 0

(c) ∞

(d) -1

Ans: (b)

SSBCrack EXAMS

Q) What is the value of $\lim_{x \to \infty} \left\{ x \sin\left(\frac{2}{x}\right) \right\}$?

(a) 2 (b) 1

(c) 1/2 (d) ∞

$y \to 0$

$2y \to 0$

$x = \frac{1}{y}$

$\lim_{y \to 0} \frac{1}{y} \sin(2y) \Rightarrow \lim_{y \to 0} 2\left(\frac{1}{2y}\right) \sin(2y)$

$= \lim_{y \to 0} 2 \frac{\sin(2y)}{2y} = \lim_{2y \to 0} 2 \frac{\sin(2y)}{2y}$

$$= \lim_{2y \to 0} 2 \frac{\sin(2y)}{2y}$$

$$\Rightarrow 2 \lim_{2y \to 0} \frac{\sin(2y)}{2y}$$

$$= 2 \times 1 = 2$$

SSBCrack EXAMS

Q) What is the value of $\lim_{x \to \infty} \left\{x \sin\left(\frac{2}{x}\right)\right\}$?

(a) 2 (b) 1

(c) 1/2 (d) ∞

Ans: (a)

Q) If $\lim_{x \to 1} \frac{x^4 - 1}{x - 1} = \lim_{x \to k} \frac{x^3 - k^3}{x^2 - k^2}$, then k is

(a) $\frac{4}{3}$ (b) $\frac{3}{8}$ (c) $\frac{3}{2}$ (d) $\frac{8}{3}$

of the form $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$ $(a > 0)$

$4(1)^{4-1} = \frac{3}{2}(k)^{3-2}$

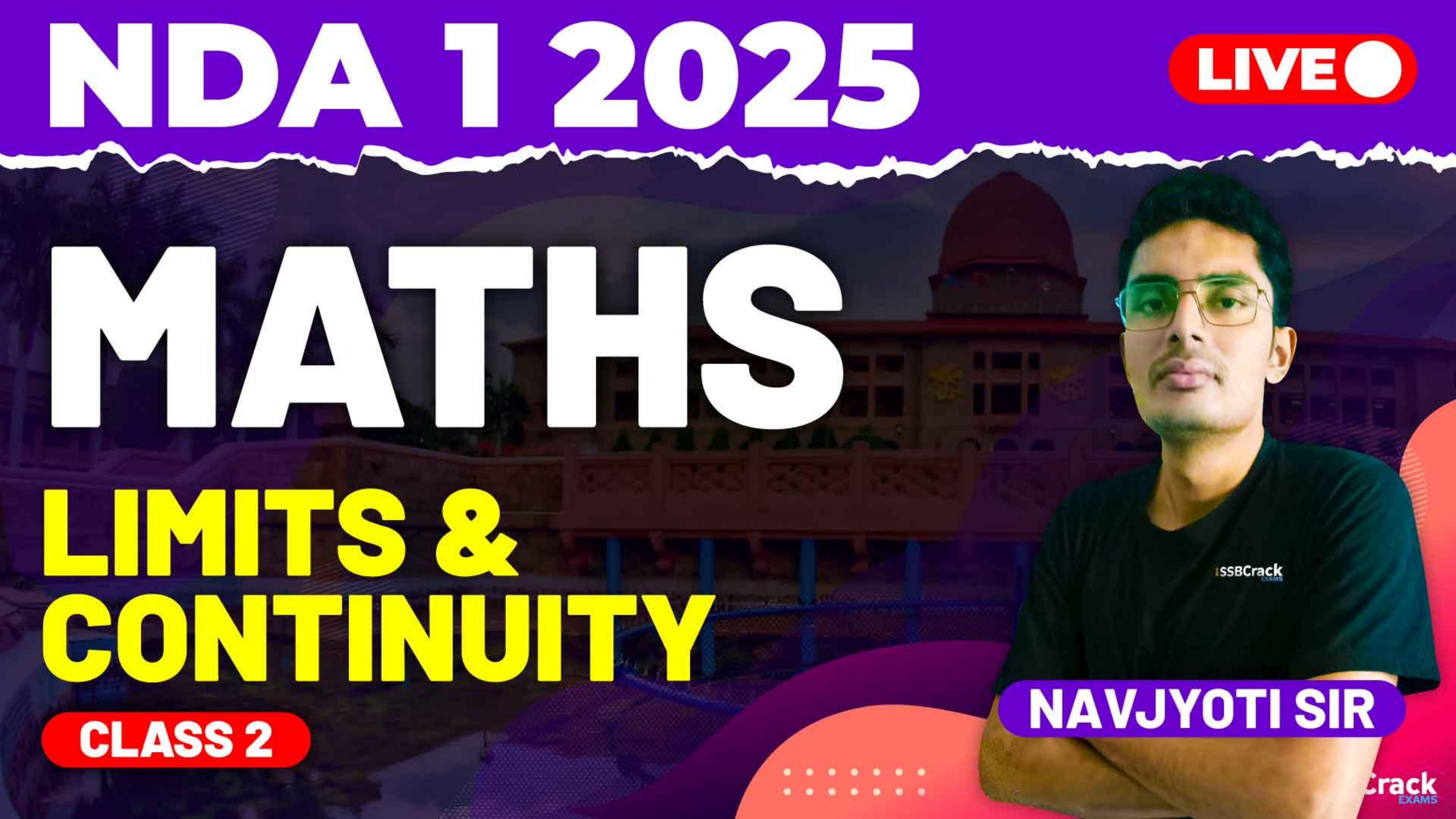
$\lim_{x \to a} \frac{x^m - a^m}{x^n - a^n} = \frac{m}{n} a^{m-n}$

$\frac{4 \times 2}{3} = k \Rightarrow k = \frac{8}{3}$

Q) If $\lim_{x \to 1} \frac{x^4 - 1}{x - 1} = \lim_{x \to k} \frac{x^3 - k^3}{x^2 - k^2}$, then k is

(a) $\frac{4}{3}$ (b) $\frac{3}{8}$ (c) $\frac{3}{2}$ (d) $\frac{8}{3}$

Ans: (d)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