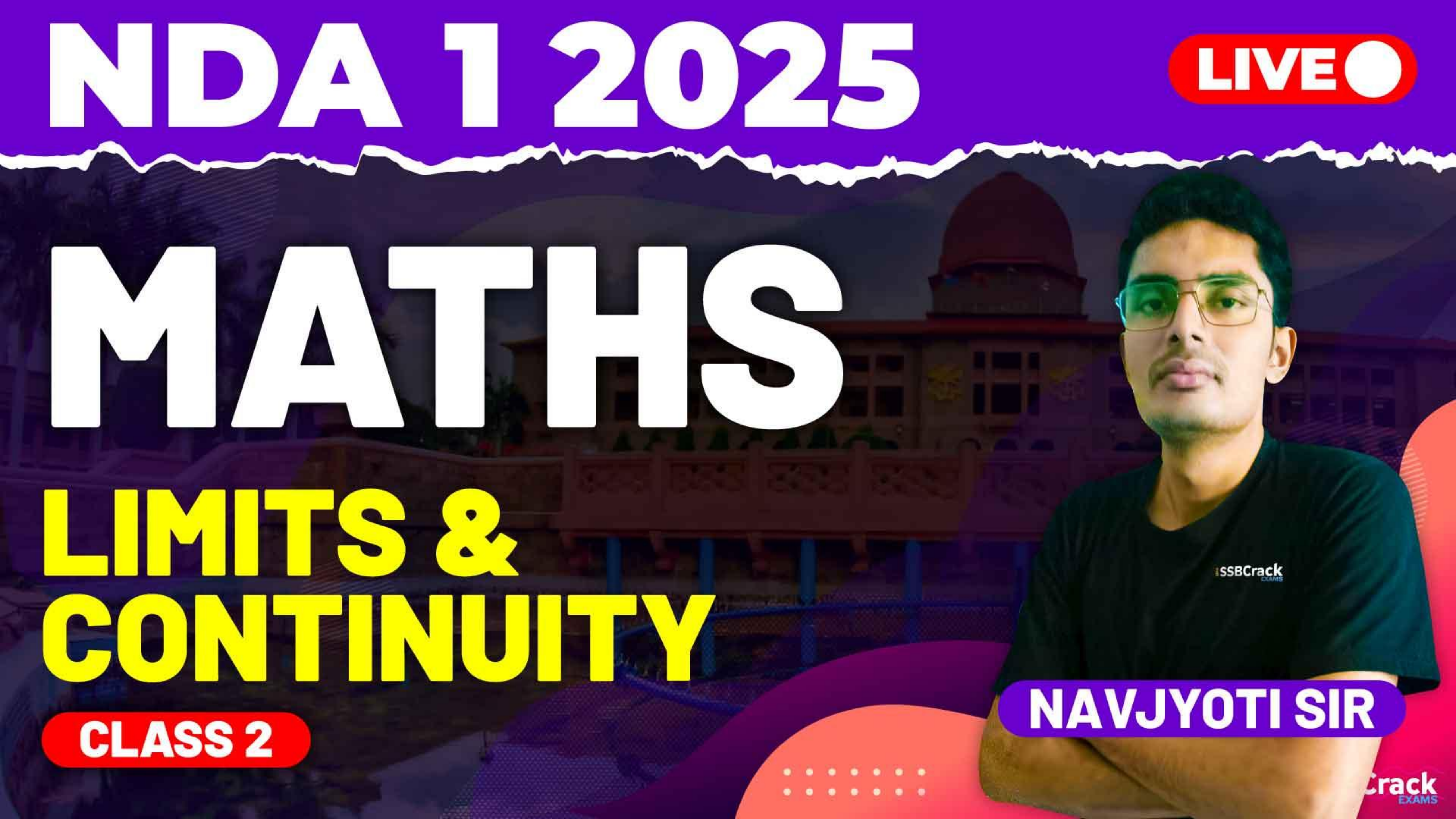

NDA 1 2025
LIVE
MATHS
LIMITS &
CONTINUITY
CLASS 2
NAVJYOTI SIR
SSBCrack EXAMS
Crack EXAMS

29 Nov 2024 Live Classes Schedule

8:00AM	29 NOVEMBER 2024 DAILY CURRENT AFFAIRS	RUBY MA'AM
9:00AM	29 NOVEMBER 2024 DAILY DEFENCE UPDATES	DIVYANSHU SIR

NDA 1 2025 LIVE CLASSES

1:00PM	PHYSICS - REFLECTION OF LIGHT - CLASS 2	NAVJYOTI SIR
4:30PM	ENGLISH - ADAPTATION OF BORROWED WORDS - CLASS 1	ANURADHA MA'AM
5:30PM	MATHS - LIMITS & CONTINUITY - CLASS 2	NAVJYOTI SIR

CDS 1 2025 LIVE CLASSES

1:00PM	PHYSICS - REFLECTION OF LIGHT - CLASS 2	NAVJYOTI SIR
4:30PM	ENGLISH - ADAPTATION OF BORROWED WORDS - CLASS 1	ANURADHA MA'AM
7:00PM	MATHS - TRIGONOMETRY - CLASS 3	NAVJYOTI SIR

SSBCrack EXAMS

LIMITS – OTHER RESULTS

$$\lim_{x\to 0}\frac{e^x-1}{x}=1$$

$$\left(\lim_{x\to 0}\frac{a^x-1}{x}=\log_e a\right)$$

$$\lim_{x\to 0}\frac{\log_e(1+x)}{x}=1$$

SSBCrack EXAMS

EXAMPLE

Evaluate the limit : $\lim_{x \to 0} \dfrac{a^x - b^x}{x}$

$$\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a$$

$$\lim_{x \to 0} \frac{a^x - 1 - (b^x - 1)}{x}$$

$$\lim_{x \to 0} \frac{a^x - 1}{x} - \lim_{x \to 0} \frac{b^x - 1}{x}$$

$$\log_e a - \log_e b = \log_e\left(\frac{a}{b}\right) \text{ or, } \ln\left(\frac{a}{b}\right)$$

$$\lim_{x \to 0} \frac{a^x - b^x}{x}$$

$$\lim_{x \to 0} \frac{b^x\left(\frac{a^x}{b^x} - 1\right)}{x}$$

$$1.\ \lim_{x \to 0} \frac{\left(\frac{a}{b}\right)^x - 1}{x} = \log_e\left(\frac{a}{b}\right)$$

of form,

$$\lim_{x \to 0} \frac{a^x - 1}{x} = 1$$

SSBCrack EXAMS

LIMITS OF FORM 1^{∞}

If $\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = 0$ such that $\lim_{x \to a} \frac{f(x)}{g(x)}$ exists, then

$$\lim_{x \to a}\{1 + f(x)\}^{\frac{1}{g(x)}} = e^{\lim_{x \to a} \frac{f(x)}{g(x)}}$$

$$\lim_{x \to a}\{1 + 0\}^{\frac{1}{0}} = \lim_{x \to a} 1^{\infty}$$

SSBCrack EXAMS

LIMITS OF FORM 1^{∞}

If $\lim\limits_{x \to a} f(x) = 1$ and $\lim\limits_{x \to a} g(x) = \infty$ such that $\lim\limits_{x \to a} \{f(x) - 1\} g(x)$ exists, then

$$\lim_{x \to a} f(x)^{g(x)} = e^{\lim\limits_{x \to a} \{f(x) - 1\} g(x)}$$

SSBCrack EXAMS

EXAMPLE

Evaluate : $\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{x}$

$$\lim_{x\to\infty}\left(1+f(x)\right)^{\frac{1}{g(x)}}\quad \Bigg\}\quad f(x)=\frac{2}{x}\ ;\ g(x)=\frac{1}{x}$$

$$= e^{\lim_{x\to\infty}\frac{f(x)}{g(x)}} = e^{\lim_{x\to\infty}\frac{\left(\frac{2}{x}\right)}{\left(\frac{1}{x}\right)}} = e^{\lim_{x\to\infty} 2} = e^{2}$$

EXAMPLE

Evaluate : $\lim_{x \to 1} (\log_3 3x)^{\log_x 3}$

$$\lim_{x \to a} \left(1 + f(x)\right)^{\frac{1}{g(x)}} = e^{\lim_{x \to a} \frac{f(x)}{g(x)}}$$

$$(\log_3 3x)^{\log_x 3} = \left(\log_3 3 + \log_3 x\right)^{\frac{1}{\log_3 x}}$$

$$= \left(1 + \log_3 x\right)^{\frac{1}{\log_3 x}} \Rightarrow f(x) = g(x) = \log_3 x$$

$$e^{\lim_{x \to 1} \frac{\log_3 x}{\log_3 x}} = e^1 = e$$

PARTICULAR CASES

$$\lim_{x \to 0}(1 + x)^{\frac{1}{x}} = e$$

$$\lim_{x \to 0}(1 + \lambda x)^{\frac{1}{x}} = e^{\lambda}$$

PARTICULAR CASES

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x} = e$$

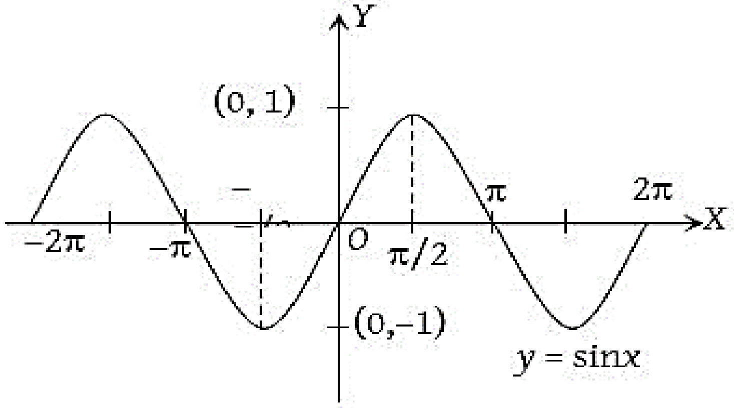
$$\lim_{x\to\infty}\left(1+\frac{\lambda}{x}\right)^{x} = e^{\lambda}$$

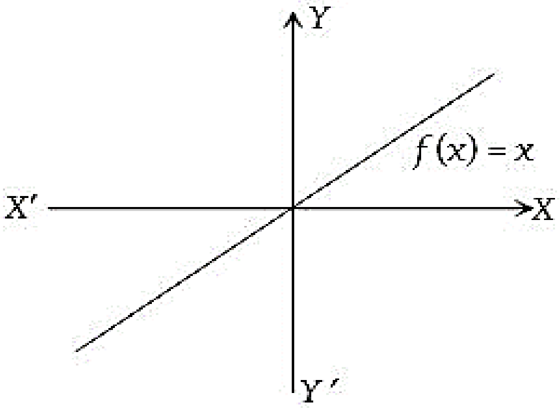
CONTINUITY

→ If the graph of function is drawn, it does not breaks, or no gaps.

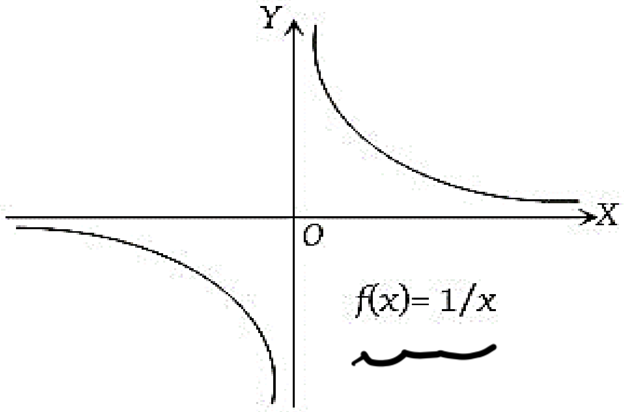
GRAPHS

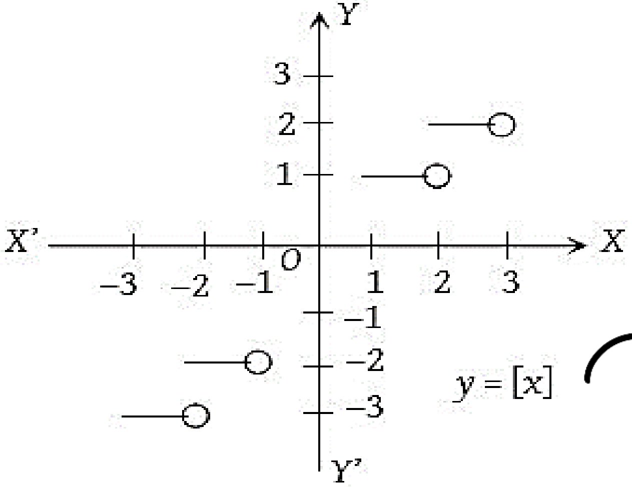
Continuous function



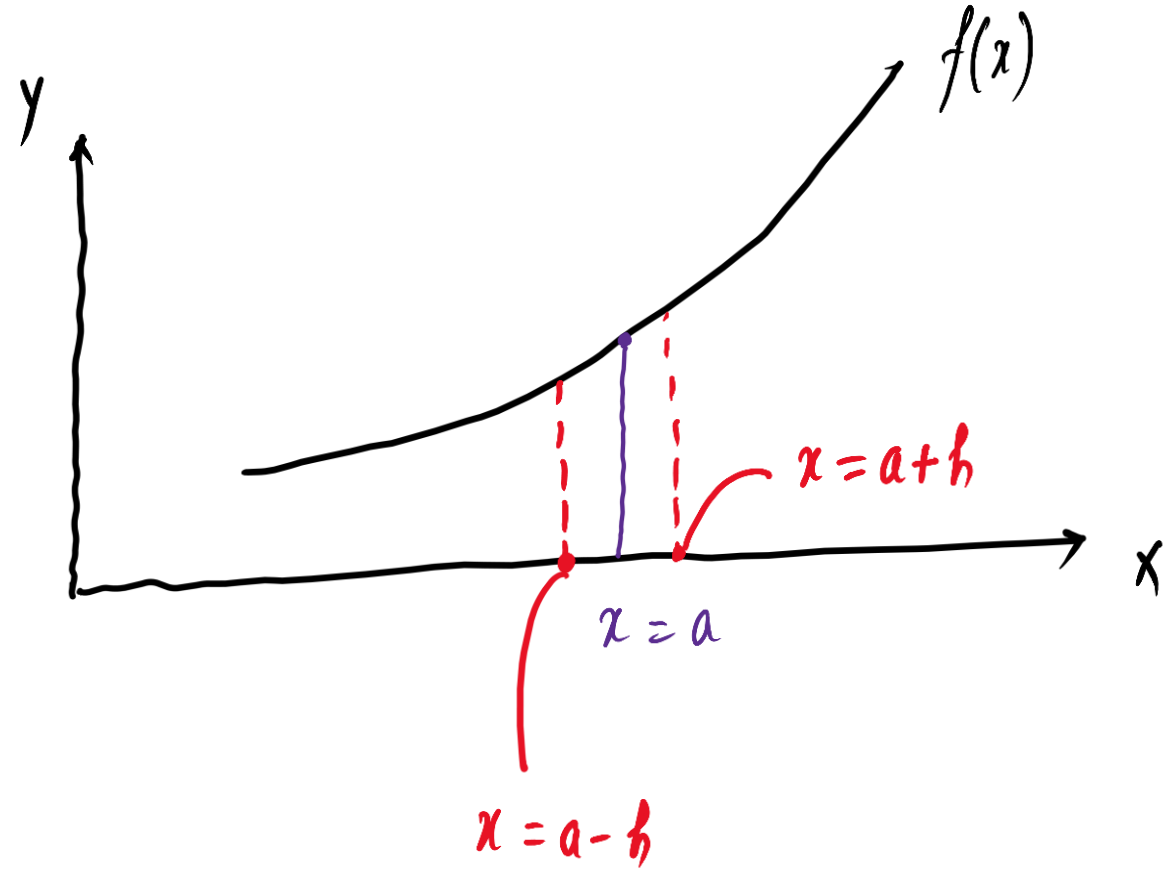


Discontinuous function





Greatest integer function.



$$\lim_{x \to a^-} f(x) = f(a) = \lim_{x \to a^+} f(x)$$

f(x) is continous at x = a, if and only if

$$\lim_{x \to a} f(x) = f(a)$$

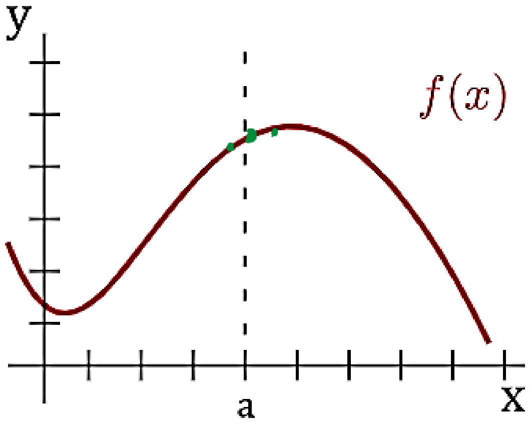
LHL (at x = a) of f(x) = f(a) = RHL (at x = a) of f(x)

SSBCrack EXAMS

CONTINUITY AT A POINT

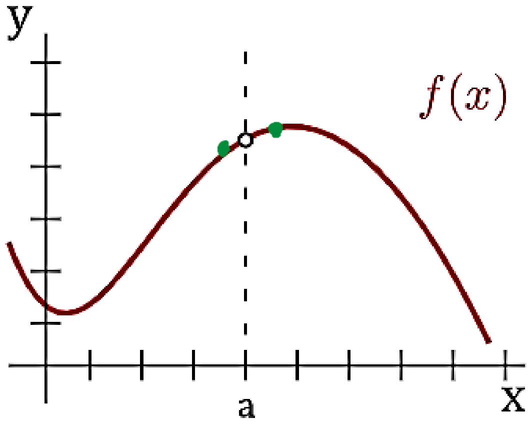
A function f is continuous at c if the following three conditions are met.

1. $f(c)$ is defined. (a point exists)
2. Lim $f(x)$ exists. (no gap or jump in the graph)
3. Lim $f(x) = f(c)$. (no hole in the graph)



continuous at $x = a$

$\left(\lim_{x \to a} f(x) = f(a)\right)$



11

f(a) not defined

(i) fails to hold

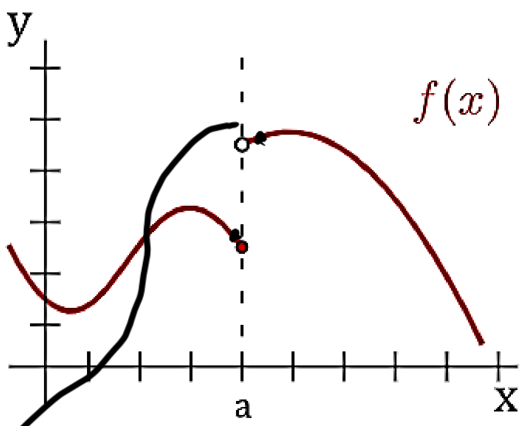
LHL and RHL exists.

SSBCrack EXAMS

CONTINUITY AT A POINT

A function f is continuous at c if the following three conditions are met.

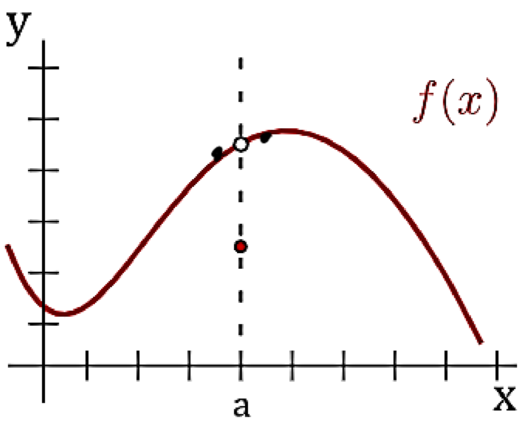
1. $f(c)$ is defined. (a point exists)
2. Lim $f(x)$ exists. (no gap or jump in the graph)
3. Lim $f(x) = f(c)$. (no hole in the graph)



$\lim_{x \to a} f(x)$ does not exist

(ii) fails to hold

III



$\lim_{x \to a} f(x) \neq f(a)$

(iii) fails to hold

IV

f(x) does not exists.

x = a is a point of discontinuity.

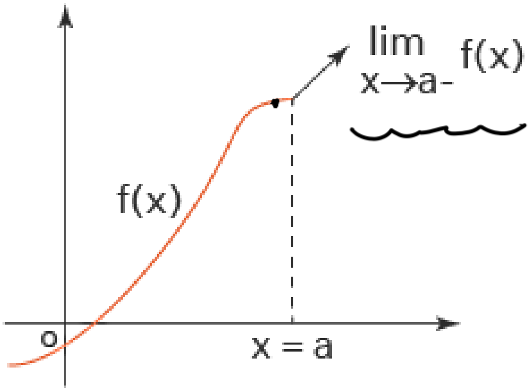
II, III and IV are graphs of discontinous function

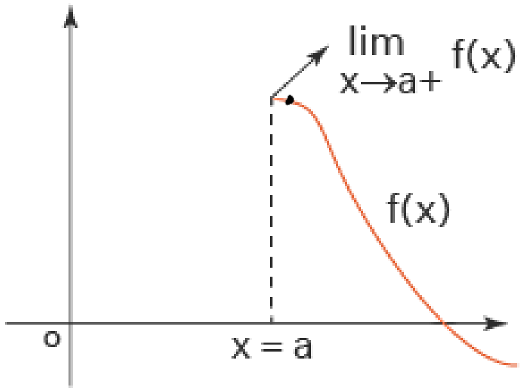
CONTINUITY AT A POINT

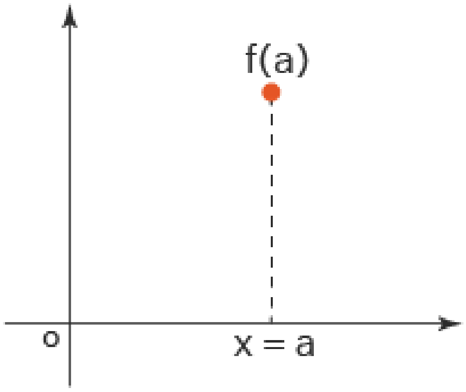
Continuity on an Open Interval:

A function is continuous on an open interval (a,b) if it is continuous at each point in the interval. A function that is continuous an the entire real line $(-\infty, \infty)$ is everywhere continuous.

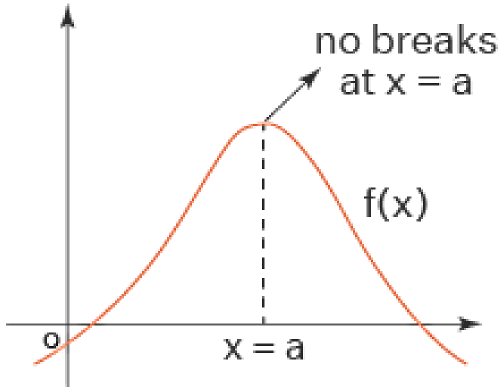
GRAPHICAL APPROACH TO DEFINITION







These three together will make the function f(x) continous at x = a



$\therefore \boxed{\lim_{x \to a} f(x) = f(a)} \Rightarrow$ f(x) is continuous at x = a

EXAMPLE

If $f(x) = \begin{cases} 2x+1, & x > 1 \\ k, & x = 1 \\ 5x-2, & x < 1 \end{cases}$, is continuous at $x = 1$, then the value of k is

(a) 1 (b) 2 (c) 3 (d) 4

$$\lim_{x \to 1^-} f(x) = \underline{\underline{f(1)}} = \lim_{x \to 1^+} f(x)$$

(LHL) (RHL)

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1} (5x-2)$$

$$= 5 - 2 = 3$$

$$f(1) = \underline{k = 3}$$

EXAMPLE

If $f(x)=\begin{cases}2x+1, & x>1\\ k, & x=1\\ 5x-2, & x<1\end{cases}$, is continuous at $x=1$, then the value of k is

(a) 1 (b) 2 (c) 3 (d) 4

Ans: (c)

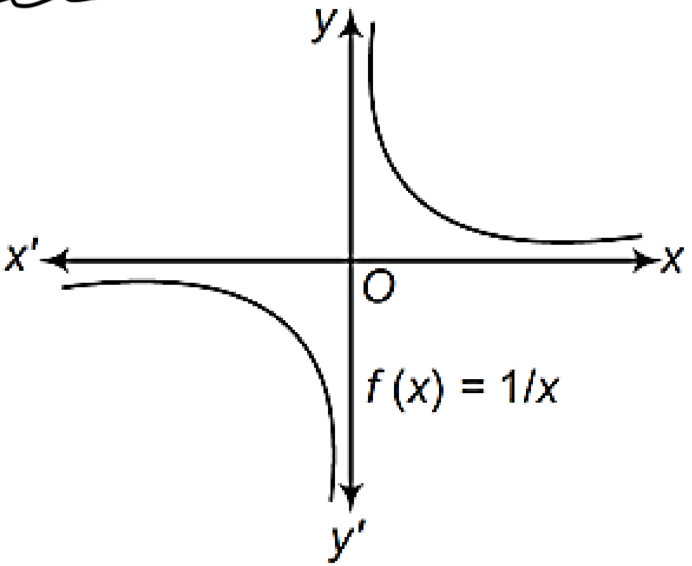
IMPORTANT PROPERTIES

If $y = f(x)$ and $y = g(x)$ are continuous functions at $x = a$, then functions $f(x) \frac{+}{\underset{\div}{\times}} g(x)$ are also continuous at $x = a$, only in case of $f(x) \div g(x)$, $g(a) \neq 0$ ✓

If $y = f(x)$ and $y = g(x)$ are discontinuous functions at $x = a$, then $f(x) \frac{+}{\underset{\div}{\times}} g(x)$ may be continuous function at $x = a$

DISCONTINOUS FUNCTION

A function f which is not continuous at a point $x = a$ in its domain is said to be discontinuous. The point a is called a point of discontinuity of the function.



CONTINUITY OF COMPOSITE FUNCTION

If the function $u = f(x)$ is continuous at the point $x = a$ and the function $y = g(u)$ is continuous at the point $u = f(a)$, then the composite function $y = gof\ (x) = g\,(f(x))$ is continuous at the point $x = a$.

IMPORTANT RESULTS

1. All polynomials, logarithmic functions, exponential functions, trigonometric functions, modulus function are continuous in their domains. The greatest integer function is ~~dis~~continuous at integers.

2. $\lim\limits_{x \to 0} e^{1/x}$ does not exist

Q) If $f(x) = \begin{cases} \dfrac{1-\cos x}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is

(a) 0 (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) $-\frac{1}{2}$

$x \neq 0 \rightarrow x < 0$

$x \neq 0 \rightarrow x > 0$

$$\lim_{x \to 0} \frac{1-\cos x}{x} = f(0)$$

$$k = \lim_{x \to 0} \frac{1-\cos x}{x} \quad \left(\frac{0}{0} \text{ form}\right)$$

$$= \lim_{x \to 0} \frac{\sin x}{1} = 0$$

$$k = 0$$

$(\cos x)' = -\sin x$

$(x^n)' = n x^{n-1} \Rightarrow (x^1)' = 1 \cdot x^0 = 1$

Q) If $f(x) = \begin{cases} \dfrac{1-\cos x}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$,

then the value of k is

(a) 0

(b) $\dfrac{1}{2}$

(c) $\dfrac{1}{4}$

(d) $-\dfrac{1}{2}$

Ans: (a)

SSBCrack EXAMS

Q) If the function

$$f(x) = \begin{cases} a + bx, & x < 1 \\ 5, & x = 1 \\ b - ax, & x > 1 \end{cases}$$

is continuous, then what is the value of $(a+b)$?

(a) 5 ✓ (b) 10

(c) 15 (d) 20

$5 = \lim_{x \to 1^+} f(x)$

$5 = b - a(1)$

$b = 5 + a$

not reqd.

$5 = \lim_{x \to 1^-} f(x)$

$5 = a + b(1) \Rightarrow \boxed{a + b = 5}$ — ①

$a = 0$

$b = 5$

Q) If the function

$$f(x) = \begin{cases} a + bx, & x < 1 \\ 5, & x = 1 \\ b - ax, & x > 1 \end{cases}$$

is continuous, then what is the value of $(a + b)$?

(a) 5 (b) 10

(c) 15 (d) 20

Ans: (a)

Q) Let $f(x) = \begin{cases} 1 + \dfrac{x}{2k}, & 0 < x < 2 \\ kx, & 2 \leq x < 4 \end{cases}$

If $\lim_{x \to 2} f(x)$ exists, then what is the value of k?

(a) – 2 (b) – 1

(c) 0 (d) 1 ✓

LHL = RHL

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$$

$$\lim_{x \to 2} 1 + \frac{x}{2k} = \lim_{x \to 2} kx$$

$$1 + \frac{1}{k} = 2k$$

$$k + 1 = 2k^2$$

$$2k^2 - k - 1 = 0$$

Check from options

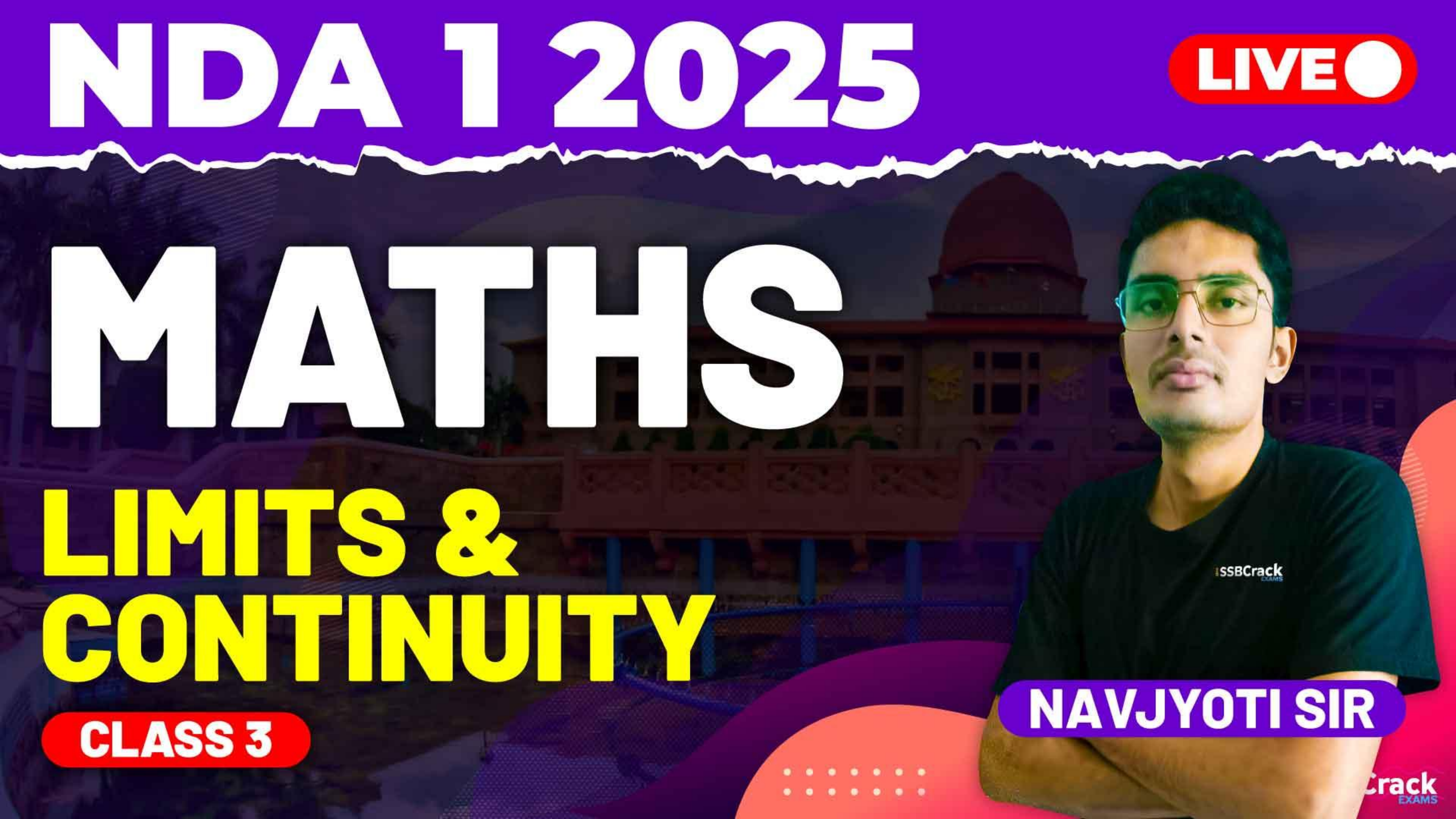
Q) Let $f(x) = \begin{cases} 1 + \dfrac{x}{2k}, & 0 < x < 2 \\ kx, & 2 \leq x < 4 \end{cases}$

If $\lim_{x \to 2} f(x)$ exists, then what is the value of k?

(a) – 2 (b) – 1

(c) 0 (d) 1

Ans: (d)


NDA 1 2025
LIVE
MATHS
LIMITS &
CONTINUITY
CLASS 3
NAVJYOTI SIR
SSBCrack EXAMS
Crack EXAMS