

# NDA 1 2025

LIVE

# MATHS

## MATRICES & DETERMINANTS

CLASS 3



NAVJYOTI SIR

Crack  
EXAMS



## 21 Nov 2024 Live Classes Schedule

8:00AM	21 NOVEMBER 2024 DAILY CURRENT AFFAIRS	RUBY MA'AM
9:00AM	21 NOVEMBER 2024 DAILY DEFENCE UPDATES	DIVYANSHU SIR

### SSB INTERVIEW LIVE CLASSES

9:30AM	MOCK PERSONAL INTERVIEWS	ANURADHA MA'AM
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### NDA 1 2025 LIVE CLASSES

11:30AM	GK - ECONOMICS - CLASS 4	RUBY MA'AM
1:00PM	PHYSICS - UNITS & DIMENSIONS - CLASS 1	NAVJYOTI SIR
4:30PM	ENGLISH - USAGE OF PAIRED WORDS - CLASS 1	ANURADHA MA'AM
5:30PM	MATHS - MATRICES & DETERMINANTS - CLASS 3	NAVJYOTI SIR

### CDS 1 2025 LIVE CLASSES

11:30AM	GK - ECONOMICS - CLASS 3	RUBY MA'AM
1:00PM	PHYSICS - UNITS & DIMENSIONS - CLASS 1	NAVJYOTI SIR
4:30PM	ENGLISH - USAGE OF PAIRED WORDS - CLASS 1	ANURADHA MA'AM
7:00PM	MATHS - SPEED DISTANCE TIME - CLASS 3	NAVJYOTI SIR



# QUESTION

Let  $A$  and  $B$  be matrices of order  $3 \times 3$ .

PYQ - 24 - I

If  $|A| = \frac{1}{2\sqrt{2}}$  and  $|B| = \frac{1}{729}$ , then what

is the value of  $|2B(\text{adj}(3A))|$  ?

(a) 27

(b)  $\frac{-27}{2\sqrt{2}}$

(c)  $\frac{27}{2}$

(d) 1 ✓

$$\begin{aligned}
 & |2B| | \text{adj}(3A) | \\
 & 2^3 |B| \cdot 3^{3(3-1)} |A|^{3-1} \\
 & 8 \times \frac{1}{729} \times 3^6 \times \left(\frac{1}{2\sqrt{2}}\right)^2 \\
 & = 8 \times \frac{1}{729} \times 729 \times \frac{1}{8} = 1
 \end{aligned}$$

$|A|$  order

$$\begin{aligned}
 |kA| &= k^n |A| \\
 |\text{adj}A| &= |A|^{n-1} \\
 |\text{adj}(kA)| &= (|kA|)^{n-1} \\
 &= (k^n |A|)^{n-1} \\
 &= k^{n(n-1)} |A|^{n-1}
 \end{aligned}$$

# QUESTION

Consider the following statements in respect of two non-singular matrices  $A$  and  $B$  of the same order  $n$ :

PYQ - 24 - I

1.  $adj(AB) = (adjA)(adjB)$  ✗

2.  $adj(AB) = adj(BA)$  ✗

3.  $(AB)adj(AB) - |AB|I_n$  is a null matrix of order  $n$  ✓

How many of the above statements are correct?

- (a) None
- (b) Only one statement ✓
- (c) Only two statements
- (d) All three statements

①  $adj(AB) = adj(B)adj(A)$

②  $AB \neq BA$

$adj(AB) \neq adj(BA)$

③  $A(adjA) = (adjA)A$   
 $= |A|I_n$

$A adjA - |A|I_n = 0 \rightarrow$  zero matrix  
 A can be replaced by AB.

# INVERSE OF A MATRIX

$$A^{-1} = \frac{1}{|A|} \text{adj} A$$

→ If  $|A| = 0$ ,  $A^{-1}$  does not exist.

(A is invertible only when it is non-singular)

# PROPERTIES OF INVERSE

$$(A^{-1})^{-1} = A$$

$$(AB)^{-1} = \underline{B^{-1}} \cdot \underline{A^{-1}}$$

$$|A^{-1}| = |A|^{-1} = \frac{1}{|A|}$$

$$\left\{ \begin{array}{l} A \longrightarrow |A| \\ A^{-1} \longrightarrow |A|^{-1} = \frac{1}{|A|} \end{array} \right. \text{reciprocal}$$

# SYSTEM OF LINEAR EQUATIONS

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

(A)                      (X)                      (B)

→ soln. of given linear eqns,  
finding  $x$ .

$$AX = B$$

$$X = A^{-1}B$$

# CONSISTENT AND INCONSISTENT SYSTEM OF EQUATIONS

If soln. exists  $\longrightarrow$  consistent system of equations

" " does not exists  $\longrightarrow$  inconsistent " " "



# SOLUTIONS

If  $|A| \neq 0$ , then there exists unique solution. (single values each of  $x, y$  and  $z$ )

If  $|A| = 0$  and  $(adj A) B \neq 0$ , then there exists no solution. (inconsistent)

If  $|A| = 0$  and  $(adj A) B = 0$ , then the system of equation has infinitely many solutions.

Let  $x = k$ , then

find  $y$  and  $z$  in terms of  $k$ .

# HOMOGENEOUS SYSTEM OF LINEAR EQUATIONS

The same linear equations with,

$$\left. \begin{array}{l} d_1 \\ d_2 \\ d_3 \end{array} \right\} = 0 \Rightarrow \text{Homogeneous system,}$$

# HOMOGENEOUS SYSTEM OF LINEAR EQUATIONS AND SOLUTIONS

$|A| \neq 0$ , then  $x = y = z = 0$   $\longrightarrow$  Trivial solution

If  $|A| = 0$ , then the system has infinite many solutions  $\longrightarrow$  non-trivial solution.

# SPECIAL MATRICES

## Orthogonal Matrix

$$A \cdot A^T = I \Rightarrow \underbrace{A^{-1} = A^T}$$

$$\text{Eg: } \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

## Idempotent Matrix

$$A^2 = A$$

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} + \frac{1}{4} & \frac{1}{4} + \frac{1}{4} \\ \frac{1}{4} + \frac{1}{4} & \frac{1}{4} + \frac{1}{4} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

# SPECIAL MATRICES

## Involutory Matrix

$$A^2 = I$$

$$\Rightarrow A = A^{-1}$$

eg -  $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

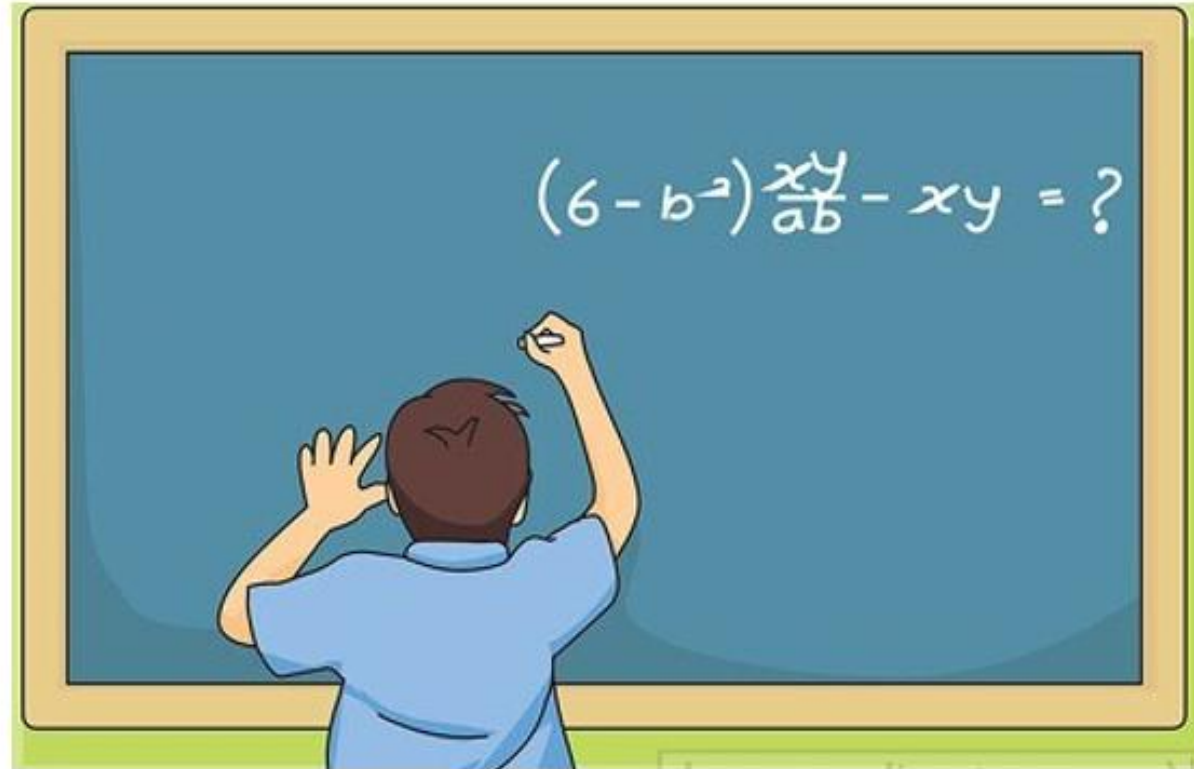
## Nilpotent Matrix

$$p \in \mathbb{N}$$

$$A^p = 0$$

(Zero matrix)

PRACTISE  
TIME !



Q) Consider the following statements:

1. If  $\det A = 0$ , then  $\det(\operatorname{adj} A) = 0$  ✓
  2. If  $A$  is non-singular, then  $\det(A^{-1}) = (\det A)^{-1}$  ✓
- (a) 1 only                                      (b) 2 only  
(c) Both 1 and 2 ✓                            (d) Neither 1 nor 2

①

$$\begin{aligned} |\operatorname{adj} A| &= |A|^{n-1} \\ &= 0^{n-1} = \underbrace{0} \end{aligned}$$

**Q)** Consider the following statements:

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2. If  $A$  is non-singular, then  $\det(A^{-1}) = (\det A)^{-1}$

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

**Ans: (c)**



Q) If  $l + m + n = 0$ , then the system of equations

$$-2x + y + z = l$$

$$x - 2y + z = m$$

$$x + y - 2z = n$$

has

(a) a trivial solution

(b) no solution

(c) a unique solution

(d) infinitely many solutions

$$A = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

$$\begin{aligned} |A| &= -2(4-1) - 1(-2-1) + 1(1+2) \\ &= -6 + 3 + 3 \\ &= 0 \end{aligned}$$

$$A = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

$$B = \begin{bmatrix} l \\ m \\ n \end{bmatrix}$$

$$\text{adj} A = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}$$

$|A| = 0$   
 $(\text{adj} A) B = 0$  }  
 infinitely many solutions

$$(\text{adj} A) B = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix} \begin{bmatrix} l \\ m \\ n \end{bmatrix} = \begin{bmatrix} 3(l+m+n) \\ 3(l+m+n) \\ 3(l+m+n) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \text{zero matrix}$$

Q) If  $l + m + n = 0$ , then the system of equations

$$-2x + y + z = l$$

$$x - 2y + z = m$$

$$x + y - 2z = n$$

has

- (a) a trivial solution                      (b) no solution  
(c) a unique solution                      (d) infinitely many solutions

**Ans: (d)**

**Q)** Consider the following statements in respect of symmetric matrices  $A$  and  $B$

1.  $AB$  is symmetric.
2.  $A^2 + B^2$  is symmetric.

Which of the above statement(s) is/are correct?

- |                  |                     |
|------------------|---------------------|
| (a) 1 only       | (b) 2 only          |
| (c) Both 1 and 2 | (d) Neither 1 nor 2 |

**Q)** Consider the following statements in respect of symmetric matrices  $A$  and  $B$

1.  $AB$  is symmetric.
2.  $A^2 + B^2$  is symmetric.

Which of the above statement(s) is/are correct?

- |                  |                     |
|------------------|---------------------|
| (a) 1 only       | (b) 2 only          |
| (c) Both 1 and 2 | (d) Neither 1 nor 2 |

**Ans: (b)**

Q) If  $A = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$ , where  $\omega$  is cube root of unity, then what is

$A^{100}$  equal to?

- (a)  $A$  (b)  $-A$   
(c) Null matrix (d) Identity matrix

Q) If  $A = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$ , where  $\omega$  is cube root of unity, then what is

$A^{100}$  equal to?

- |                 |                     |
|-----------------|---------------------|
| (a) $A$         | (b) $-A$            |
| (c) Null matrix | (d) Identity matrix |

**Ans: (a)**

**Q)** A matrix  $X$  has  $(a + b)$  rows and  $(a + 2)$  columns; and a matrix  $Y$  has  $(b + 1)$  rows and  $(a + 3)$  columns. If both  $XY$  and  $YX$  exist, then what are the values of  $a, b$  respectively?

(a) 3, 2

(b) 2, 3

(c) 2, 4

(d) 4, 3



**Q)** A matrix  $X$  has  $(a + b)$  rows and  $(a + 2)$  columns; and a matrix  $Y$  has  $(b + 1)$  rows and  $(a + 3)$  columns. If both  $XY$  and  $YX$  exist, then what are the values of  $a, b$  respectively?

(a) 3, 2

(b) 2, 3

(c) 2, 4

(d) 4, 3

**Ans: (b)**

Q) If  $f(x) = \begin{vmatrix} 1 + \sin^2 x & \cos^2 x & 4 \sin 2x \\ \sin^2 x & 1 + \cos^2 x & 4 \sin 2x \\ \sin^2 x & \cos^2 x & 1 + 4 \sin 2x \end{vmatrix}$

What is the maximum value of  $f(x)$  ?

- (a) 2                                      (b) 4  
(c) 6                                      (d) 8

**Q) If**  $f(x) = \begin{vmatrix} 1 + \sin^2 x & \cos^2 x & 4 \sin 2x \\ \sin^2 x & 1 + \cos^2 x & 4 \sin 2x \\ \sin^2 x & \cos^2 x & 1 + 4 \sin 2x \end{vmatrix}$

What is the maximum value of  $f(x)$  ?

- (a) 2
- (b) 4
- (c) 6
- (d) 8

**Ans: (c)**

Q) For a square matrix  $A$ , which of the following properties hold?

1.  $(A^{-1})^{-1} = A$

2.  $\det(A^{-1}) = \frac{1}{\det A}$

3.  $(\lambda A)^{-1} = \lambda A^{-1}$ , where  $\lambda$  is a scalar

Select the correct answer using the code given below.

- (a) 1 and 2      (b) 2 and 3      (c) 1 and 3      (d) 1, 2 and 3

Q) For a square matrix  $A$ , which of the following properties hold?

1.  $(A^{-1})^{-1} = A$

2.  $\det(A^{-1}) = \frac{1}{\det A}$

3.  $(\lambda A)^{-1} = \lambda A^{-1}$ , where  $\lambda$  is a scalar

Select the correct answer using the code given below.

- (a) 1 and 2      (b) 2 and 3      (c) 1 and 3      (d) 1, 2 and 3

**Ans: (d)**

Q) The system of equations

$$2x + y - 3z = 5$$

$$3x - 2y + 2z = 5 \text{ and } 5x - 3y - z = 16$$

- (a) is inconsistent
- (b) is consistent, with a unique solution
- (c) is consistent, with infinitely many solutions
- (d) has its solution lying along X-axis in three-dimensional space

Q) The system of equations

$$2x + y - 3z = 5$$

$$3x - 2y + 2z = 5 \text{ and } 5x - 3y - z = 16$$

- (a) is inconsistent
- (b) is consistent, with a unique solution
- (c) is consistent, with infinitely many solutions
- (d) has its solution lying along X-axis in three-dimensional space

**Ans: (d)**

Q) If  $\Delta = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$

then what is

$$\begin{vmatrix} 3d + 5g & 4a + 7g & 6g \\ 3e + 5h & 4b + 7h & 6h \\ 3f + 5i & 4c + 7i & 6i \end{vmatrix} \text{ equal to?}$$

(a)  $\Delta$

(b)  $7\Delta$

(c)  $72\Delta$

(d)  $-72\Delta$



Q) If  $\Delta = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$

then what is

$$\begin{vmatrix} 3d + 5g & 4a + 7g & 6g \\ 3e + 5h & 4b + 7h & 6h \\ 3f + 5i & 4c + 7i & 6i \end{vmatrix} \text{ equal to?}$$

(a)  $\Delta$

(b)  $7\Delta$

(c)  $72\Delta$

(d)  $-72\Delta$

**Ans: (d)**

Q) If  $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$ ,

where  $a \in \mathbb{N}$ , then what is  
 $A^{100} - A^{50} - 2A^{25}$  equal to?

- (a)  $-2I$                       (b)  $-I$   
(c)  $2I$                          (d)  $I$

where  $I$  is the identity matrix.

Q) If  $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$ ,

where  $a \in \mathbb{N}$ , then what is  $A^{100} - A^{50} - 2A^{25}$  equal to?

- (a)  $-2I$                       (b)  $-I$   
(c)  $2I$                         (d)  $I$

where  $I$  is the identity matrix.

**Ans: (a)**

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LIVE

# MATHS

## MATRICES & DETERMINANTS

CLASS 4



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