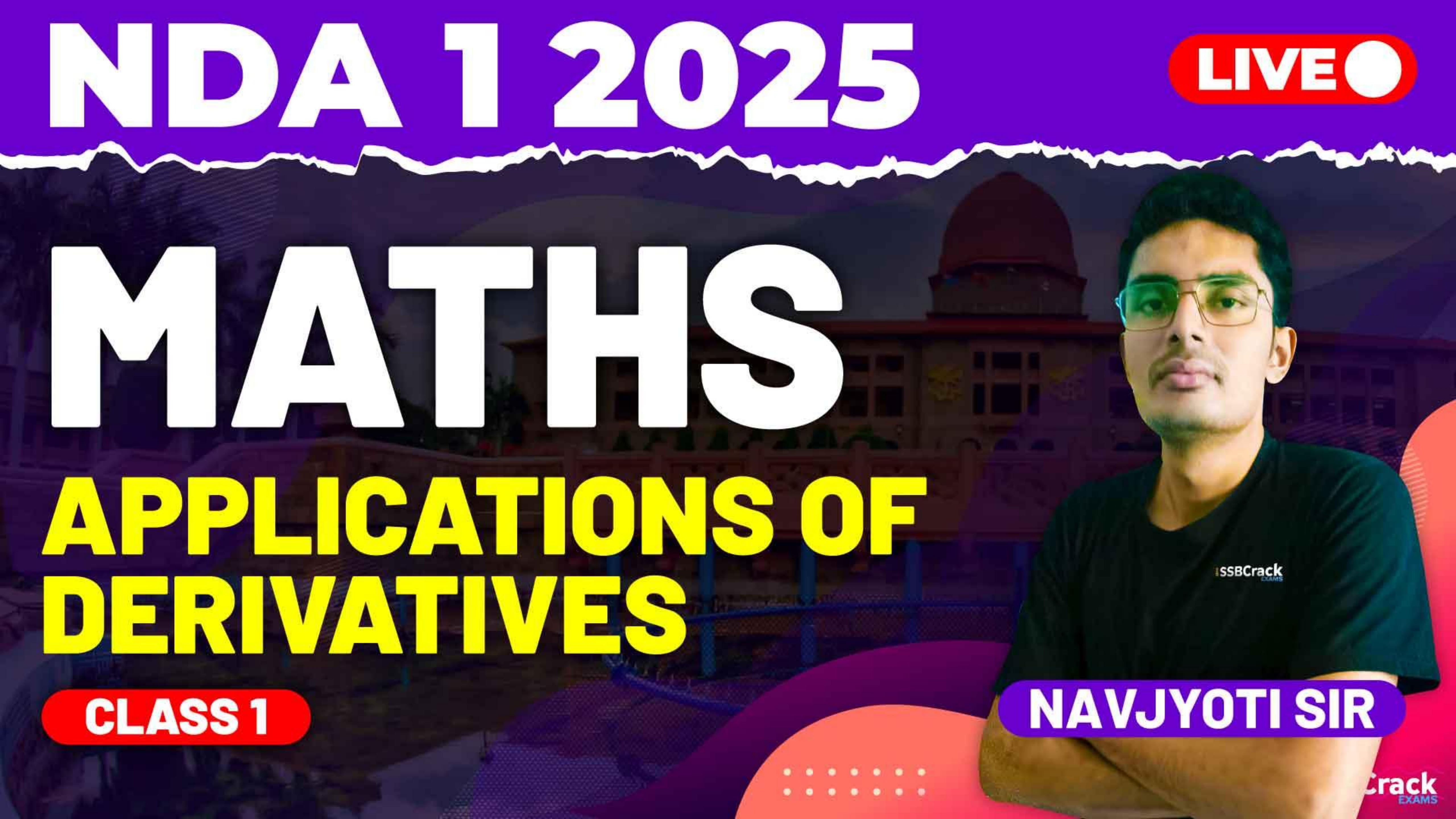

NDA 1 2025
LIVE
MATHS
APPLICATIONS OF DERIVATIVES
CLASS 1
SSBCrack EXAMS
NAVJYOTI SIR
Crack EXAMS

10 Dec 2024 Live Classes Schedule

8:00AM	10 DEC 2024 DAILY CURRENT AFFAIRS	RUBY MA'AM
9:00AM	10 DEC 2024 DAILY DEFENCE UPDATES	DIVYANSHU SIR

NDA 1 2025 LIVE CLASSES

1:00PM	PHYSICS - SOUND & WAVES MCQ	NAVJYOTI SIR
5:30PM	MATHS - APPLICATIONS OF DERIVATIVES - CLASS 1	NAVJYOTI SIR

CDS 1 2025 LIVE CLASSES

1:00PM	PHYSICS - SOUND & WAVES MCQ	NAVJYOTI SIR
7:00PM	MATHS - ALGEBRA - CLASS 5	NAVJYOTI SIR

RATE OF CHANGE OF QUANTITIES

For the function $y = f(x)$, $\frac{d}{dx}(f(x))$ represents the rate of change of y with respect to x.

Thus if 's' represents the distance and 't' the time, then $\frac{ds}{dt}$ represents the rate of change of distance with respect to time.

QUESTION

For the curve y = 5x – 2x^3 , if x increases at the rate of 2 units/sec, then how fast is the slope of curve changing when x = 3?

$\frac{dx}{dt} = +2$ unit/s

(increase)

Slope of curve, $\frac{dy}{dx} = 5 - 6x^2 = m$

$$\frac{dm}{dt} = \frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(5-6x^2\right) = -12x \cdot \frac{dx}{dt}$$

$$= -12x(2) = -24x$$

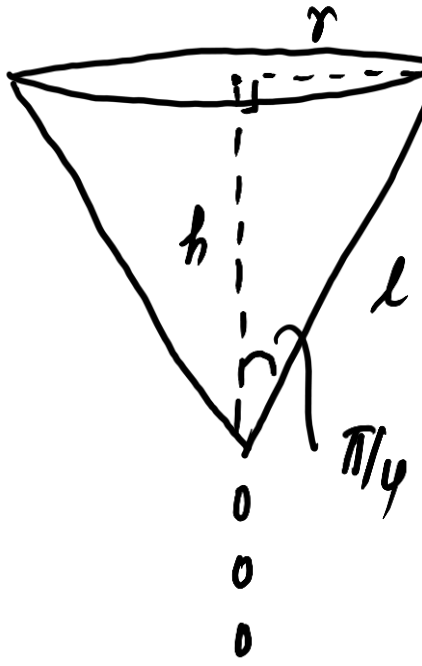
$$\left.\frac{dm}{dt}\right|_{at\, x=3} = -24 \times 3 = -72 \text{ /sec}$$

slope is reducing with time

QUESTION

Water is dripping out from a conical funnel of semi-vertical angle $\frac{\pi}{4}$ at the uniform rate of $2\ cm^2$/sec in the surface area, through a tiny hole at the vertex of the bottom. When the slant height of cone is 4 cm, find the rate of decrease of the slant height of water.

$$\frac{dl}{dt} = ?$$



$$\frac{ds}{dt} = 2\,cm^2/s$$

$r = h$

$l = \sqrt{2}r$

surface area = curved surface area of cone

$$S = \pi r l = \pi\left(\frac{l}{\sqrt{2}}\right) l$$

$$S = \frac{\pi l^2}{\sqrt{2}} \quad \Bigg| \quad \frac{ds}{dt} = \frac{\pi}{\sqrt{2}}(2l)\cdot\frac{dl}{dt}$$

$$\frac{ds}{dt} = \frac{\pi}{\sqrt{2}}(2l) \cdot \frac{dl}{dt}$$

$$2 = \pi\sqrt{2}(l)\frac{dl}{dt}$$

$$\frac{dl}{dt} = \frac{2}{\sqrt{2}\pi} \times \frac{1}{l}$$

$$\frac{dl}{dt} = \frac{\sqrt{2}}{\pi} \times \frac{1}{4} = \frac{2\sqrt{2}}{\pi} \text{ cm/s}$$

MOTION IN STRAIGHT LINE

If x and v denotes the displacement and velocity of a particle at any instant t, then velocity and acceleration is given by

$$v = \frac{dx}{dt}$$

and $$a = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d^2x}{dt^2}$$

Where, a is acceleration of particle. If the sign of acceleration is opposite to that of velocity, then the acceleration is called retardation which means decrease in magnitude of the velocity.

EXAMPLE

A particle moves in a straight line in such a way that its velocity at any point is given by $v^2 = 2 - 3x$, where x is measured from a fixed point. The acceleration is

(a) -4 (b) $-\frac{3}{2}$ ✓

(c) 3 (d) None of these

$$v = \frac{dx}{dt}$$

displacement / distance

$$a = \frac{dv}{dt}$$

velocity

$$a = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

$$v^2 = 2 - 3x$$

$$2v\frac{dv}{dt} = -3\frac{dx}{dt}$$

$$a = \frac{-3v}{2v} = -\frac{3}{2}$$

EXAMPLE

A particle moves in a straight line in such a way that its velocity at any point is given by $v^2 = 2 - 3x$, where x is measured from a fixed point. The acceleration is

(a) -4 (b) $-\frac{3}{2}$

(c) 3 (d) None of these

Ans: (b)

EXAMPLE

A point moves in a straight line during the time $t = 0$ to $t = 3$ according to the laws $s = 15t - 2t^2$. The average velocity of the point is

(a) 4 (b) 9 ✓

(c) 3 (d) 2

$$s = 15t - 2t^2$$

$$\frac{ds}{dt} = 15 - 4t$$

$$v = \frac{ds}{dt} = \underbrace{15 - 4t}$$

$$v_0 = 15 - 4(0) = 15$$

$$v_3 = 15 - 4(3) = \underbrace{3}$$

$$\text{Average velocity} = \frac{v_0 + v_3}{2} = \frac{15+3}{2} = 9$$

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EXAMPLE

A point moves in a straight line during the time $t = 0$ to $t = 3$ according to the laws $s = 15t - 2t^2$. The average velocity of the point is

(a) 4 (b) 9

(c) 3 (d) 2

Ans: (b)

TANGENTS AND NORMALS

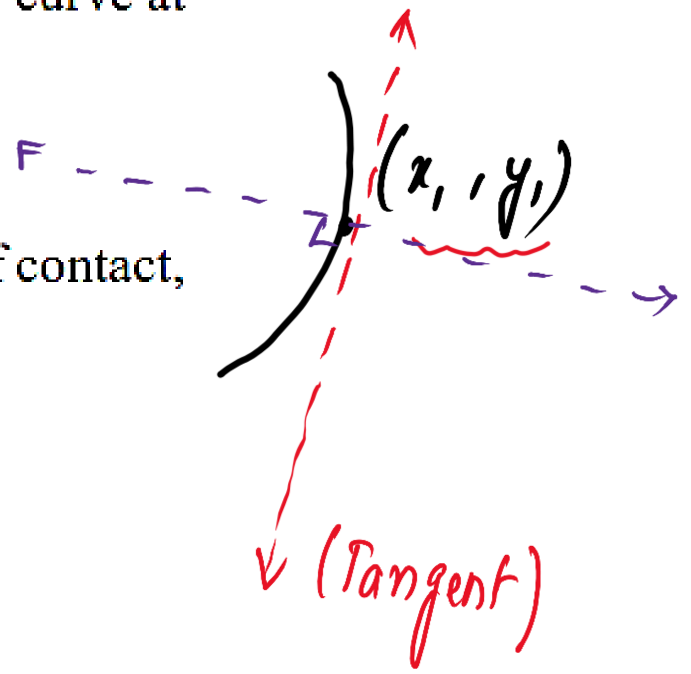
A line touching a curve $y = f(x)$ at a point (x_1, y_1) is called the tangent to the curve at that point and its equation is given $y - y_1 = \left(\frac{dy}{dx}\right)_{(x_1, y_1)}(x - x_1)$. ✓

m

The normal to the curve is the line perpendicular to the tangent at the point of contact, and its equation is given as:

$$y - y_1 = \frac{-1}{\left(\frac{dy}{dx}\right)_{(x_1, y_1)}}(x - x_1)$$

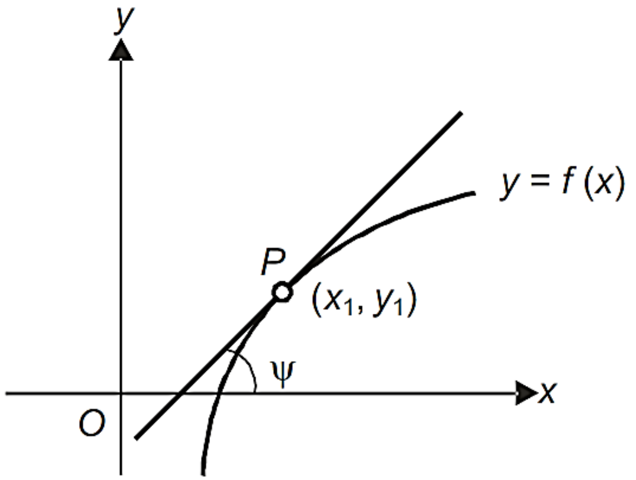
$\left(-\frac{1}{m}\right)$



The angle of intersection between two curves is the angle between the tangents to the curves at the point of intersection.

TANGENTS AND NORMALS

Slope of tangent at $P = \left(\frac{dy}{dx}\right)_{(x_1,\, y_1)} = \tan\psi = m$



A line touching a curve $y = f(x)$ at a point (x_1, y_1) is called the tangent to the curve at that point and its equation is given $y - y_1 = \left(\frac{dy}{dx}\right)_{(x_1, y_1)}(x - x_1)$.

IMPORTANT RESULTS

If tangent is parallel to x-axis, then

$$\left(\frac{dy}{dx}\right)_{(x_1,\, y_1)} = 0$$

If tangent is parallel to y-axis or perpendicular to x-axis, then

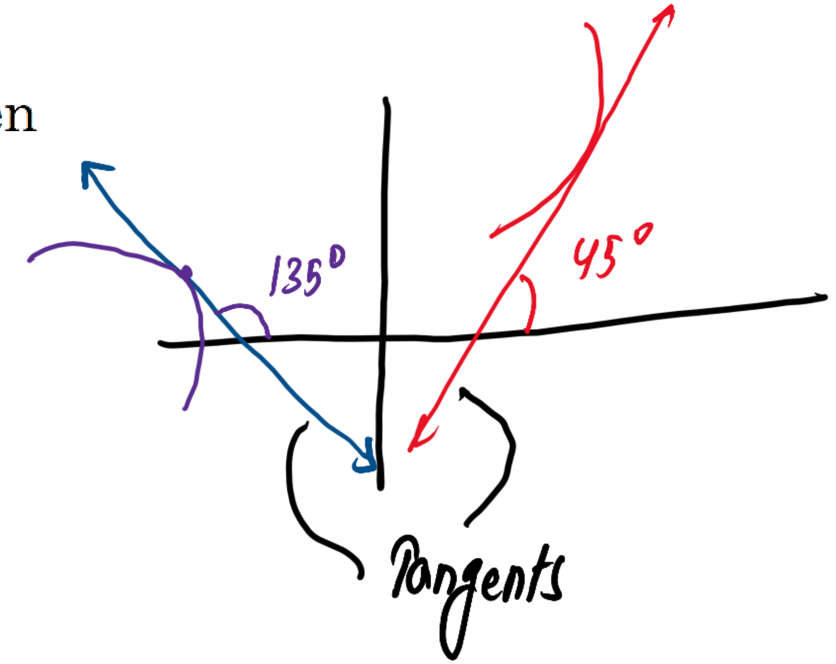
$$\left(\frac{dx}{dy}\right)_{(x_1,\, y_1)} = 0$$

$$\frac{dy}{dx} = \frac{1}{0} \quad (\infty)$$

If the tangent is equally inclined to the axes, then

$$\frac{dy}{dx} = \tan 45^\circ \text{ or } \tan 135^\circ$$

$$= \pm 1$$



EXAMPLE

The point on the curve $3y = 6x - 5x^3$ in which the normal is passing through the origin is

(a) $\left(1, \frac{1}{3}\right)$

(b) $(2, 3)$

(c) $(1, 2)$

(d) $(-3, 3)$

$\underline{3y} = 6x - 5x^3$

Only (a) $\left(1, \frac{1}{3}\right)$ satisfies this curve.

EXAMPLE

The point on the curve $3y = 6x - 5x^3$ in which the normal is passing through the origin is

(a) $\left(1, \frac{1}{3}\right)$ (b) $(2, 3)$

(c) $(1, 2)$ (d) $(-3, 3)$

Ans: (a)

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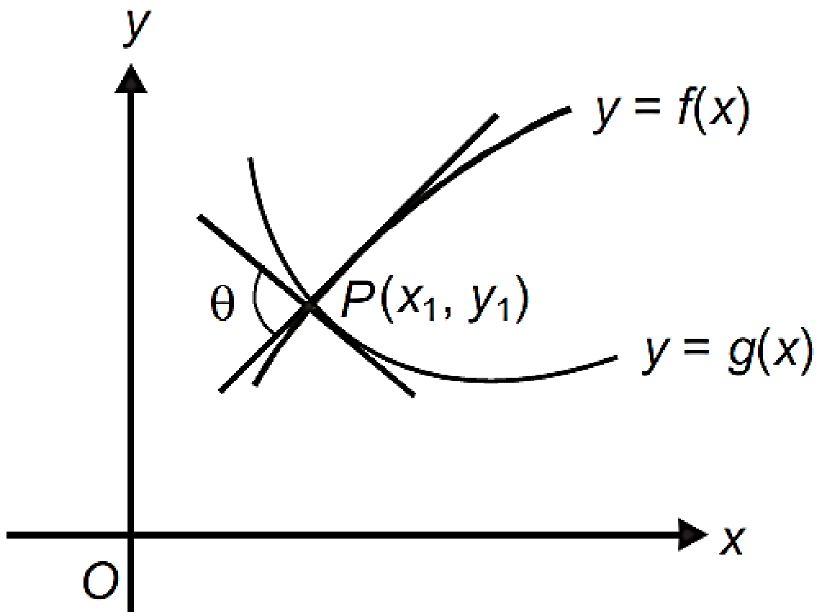
ANGLE OF INTERSECTION

Angle of intersection is angle between tangents at intersection points of two curves.
Let they intersect at $P(x_1, y_1)$,
then

$$m_1 = \left[\frac{df(x)}{dx}\right]_{(x_1,\, y_1)} \qquad m_2 = \left[\frac{dg(x)}{dx}\right]_{(x_1,\, y_1)} \qquad \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$



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EXAMPLE

The angle of intersection of the curves $y = x^2, 6y = 7 - x^3$ at point (1, 1) is

(a) π (b) $\frac{\pi}{2}$ (c) 2π (d) 4π

$$m_1 = \left.\frac{dy}{dx}\right|_{at\,(1,1)} = 2x = 2 \times 1 = \underbrace{2}$$

$$m_2 = \frac{-3x^2}{6} = \left.-\frac{x^2}{2}\right|_{at\,(1,1)} = -\frac{1}{2}$$

product of slopes = -1

angle of intersection = $90° = \underbrace{\frac{\pi}{2}}$

EXAMPLE

The angle of intersection of the curves $y = x^2, 6y = 7 - x^3$ at point (1, 1) is

(a) π (b) $\frac{\pi}{2}$ (c) 2π (d) 4π

Ans: (b)

QUESTION

Find the angle of intersection of the curves $y^2 = x$ and $x^2 = y$.

$y^2 = x$ $\qquad$ $x^2 = y$

$y^4 = x^2$ $\qquad$ $y^4 = y$

$y^4 - y = 0 \Rightarrow y(y^3 - 1) = 0$

$y = 0, \quad y = 1$

$x = 0, \quad x = 1$

Two intersection points $\longrightarrow$ $(0,0)$ $\quad$ $(1,1)$

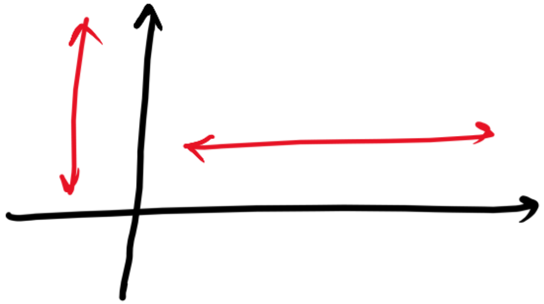
$y^2 = x$

$2y \dfrac{dy}{dx} = 1$

$\dfrac{dy}{dx} = \dfrac{-1}{2y} = m_1$

$x^2 = y$

$2x = \dfrac{dy}{dx} = m_2$



parallel to y-axis

parallel to x-axis

At $(0,0) \longrightarrow m_1 = \infty \; ; \; m_2 = 0$

$\theta = 90^\circ$

At $(1,1) \longrightarrow m_1 = \dfrac{-1}{2} \; ; \; m_2 = 2$

$m_1 m_2 = 90^\circ$

$\theta = 90^\circ$

ORTHOGONAL AND TOUCHING CURVES

Two curves are said to be orthogonal curves, if the angle of intersection of two curves is right angle *i.e.*, if $m_1 m_2 = -1$.

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QUESTION

Find the condition for the curves $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$; $xy = c^2$ to intersect orthogonally.

Let the point of intersection be (x_1, y_1).

$$\frac{1}{a^2}(2x) - \frac{1}{b^2}(2y)\frac{dy}{dx} = 0$$

$$xy = c^2$$

$$y + x\frac{dy}{dx} = 0$$

$$m_1 = \frac{dy}{dx} = \frac{2x}{a^2} \times \frac{b^2}{2y} = \frac{b^2x}{a^2y}$$

$$m_2 = \frac{dy}{dx} = -\frac{y}{x}$$

$m_1 m_2 = -1$

At point (x_1, y_1) $\quad \frac{b^2x_1}{a^2y_1} \times -\frac{y_1}{x_1} = -1$

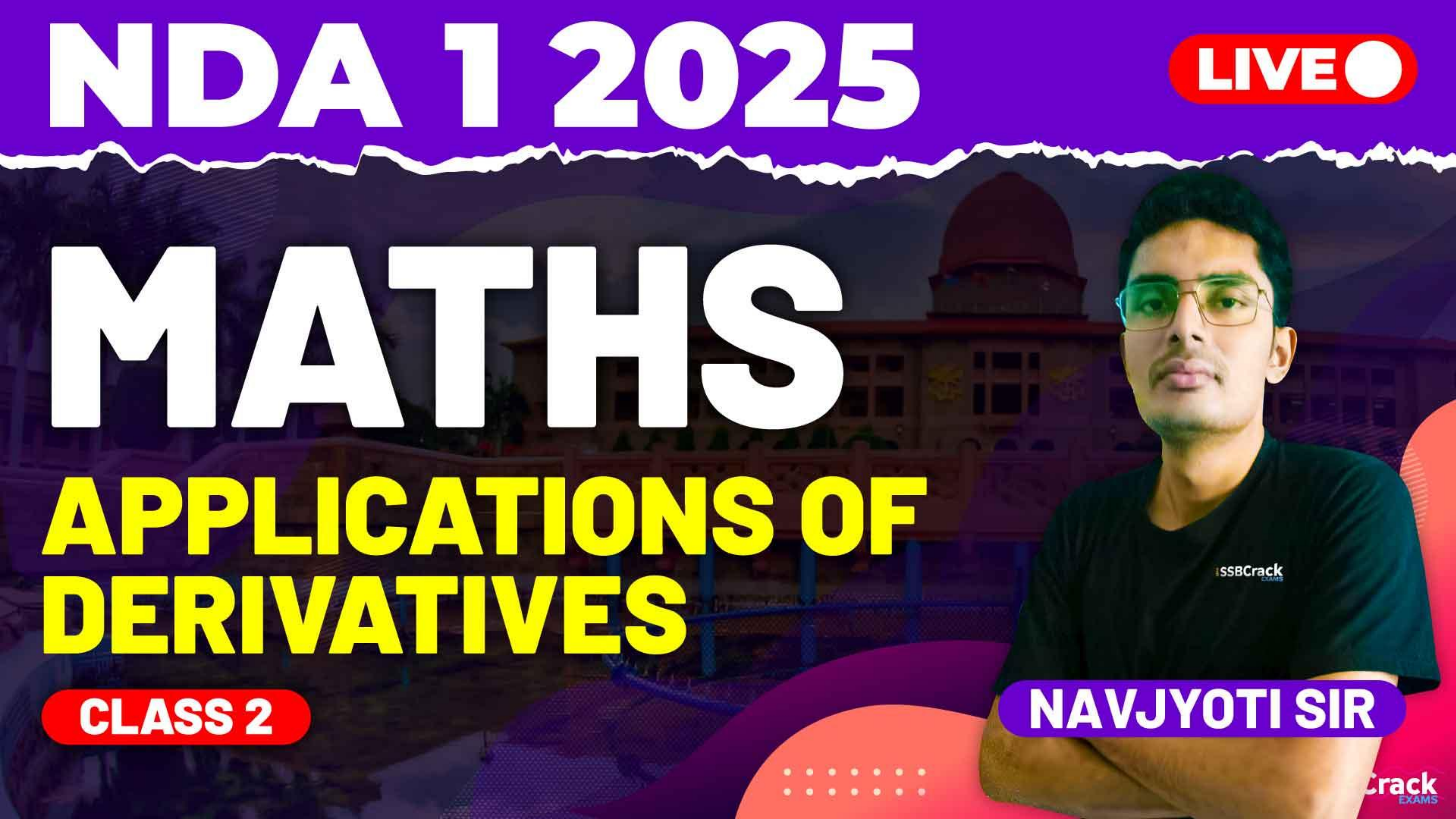
(For orthogonally intersecting curves

At point (x_1, y_1)

$$\frac{b^2 x_1}{a^2 y_1} \times \frac{-y_1}{x_1} = -1$$

$$\frac{b^2}{a^2} = 1$$

$$b^2 = a^2$$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