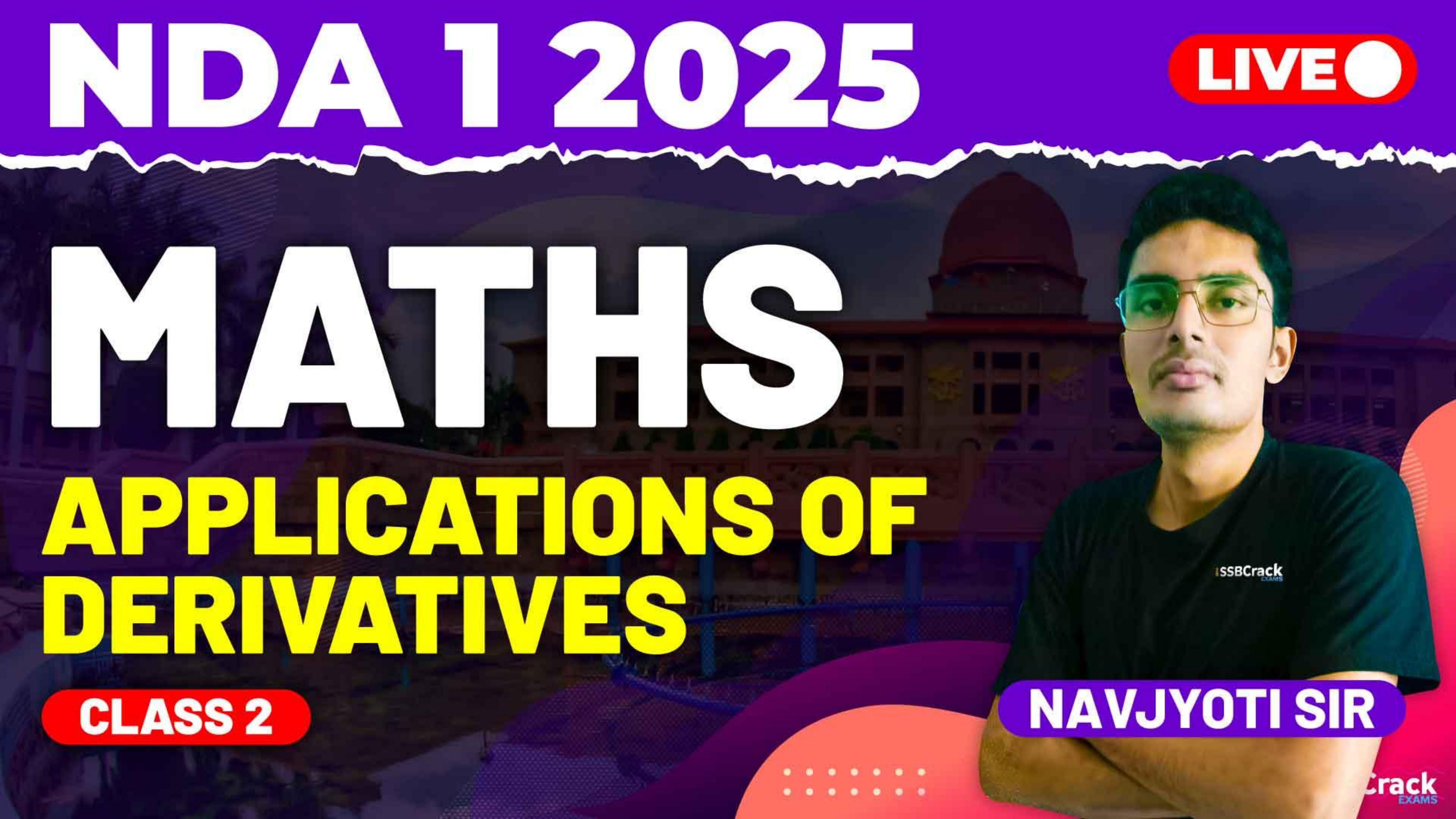
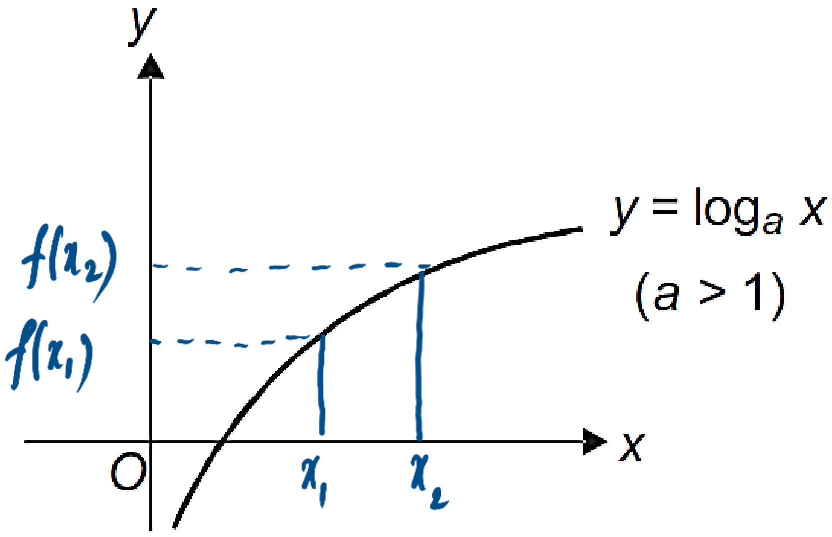

NDA 1 2025
LIVE
MATHS
APPLICATIONS OF DERIVATIVES
CLASS 2
NAVJYOTI SIR
SSBCrack EXAMS

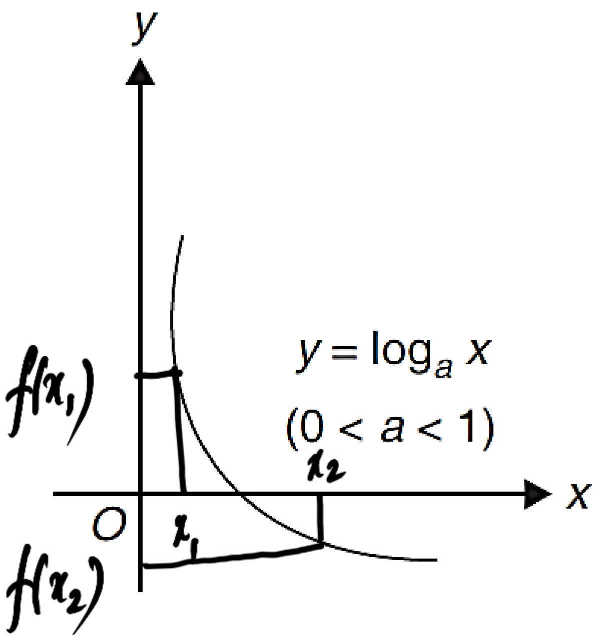
INCREASING AND DECREASING FUNCTIONS

A continuous function in an interval (a, b) is :

(i) strictly increasing if for all $x_1, x_2 \in (a, b)$, $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$ or for all $x \in (a, b)$, $f'(x) > 0$

(ii) strictly decreasing if for all $x_1, x_2 \in (a, b)$, $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$ or for all $x \in (a, b)$, $f'(x) < 0$





$x_2 > x_1$

$f(x_2) < f(x_1)$

$f'(x) < 0$

Let f be a continuous function on $[a, b]$ and differentiable in (a, b) then

(i) f is increasing in $[a, b]$ if $f'(x) > 0$ for each $x \in (a, b)$
(ii) f is decreasing in $[a, b]$ if $f'(x) < 0$ for each $x \in (a, b)$
(iii) f is a constant function in $[a, b]$ if $f'(x) = 0$ for each $x \in (a, b)$.

EXAMPLE

The intervals in which the function $f(x) = 2x^2 - \log|x|,\ x \neq 0$ is increasing in

(a) $(-\infty, +\infty)$ ✗

(b) $\left(-\frac{1}{2}, 0\right) \cup \left(\frac{1}{2}, \infty\right)$ ✓

(c) $\left[-\frac{1}{2}, 1\right]$

(d) None of the above

Values of x at which $f'(x) = 0$ or $f'(x)$ does not exist

Critical points

$x = 0, \frac{1}{2}, -\frac{1}{2}$

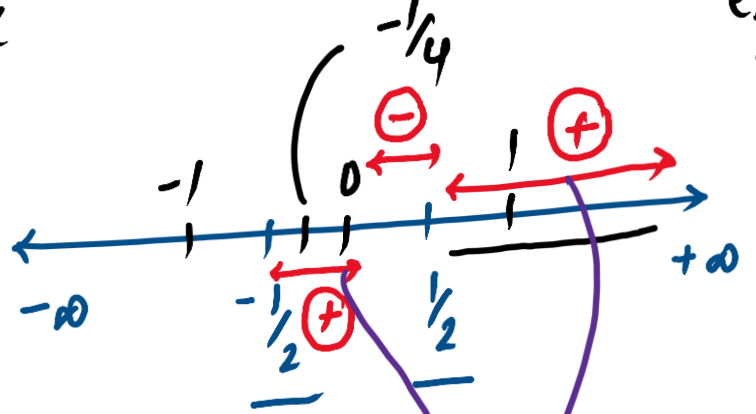
$f'(x) = 4x - \frac{1}{x}$

$f'(x) = 0$

$4x^2 - 1 = 0$

$x^2 = \frac{1}{4}$

$x = \pm\frac{1}{2}$



$f'(x)$ ⊖ decreasing

$\left(-\frac{1}{2}, 0\right) \cup \left(\frac{1}{2}, \infty\right)$

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EXAMPLE

The intervals in which the function $f(x) = 2x^2 - \log|x|,\ x \neq 0$ is increasing in

(a) $(-\infty, +\infty)$

(b) $\left(-\frac{1}{2}, 0\right) \cup \left(\frac{1}{2}, \infty\right)$

(c) $\left[-\frac{1}{2}, 1\right]$

(d) None of the above

Ans: (b)

EXAMPLE

The interval in which the function $f(x) = (x+2)\, e^{-x}$ is decreasing in

(a) $(-1, \infty)$ (b) $(-1, 1)$

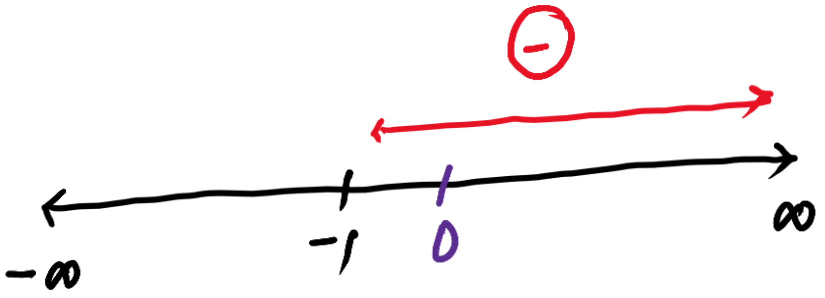
(c) $(-1, 2)$ (d) $(1, 2)$

$$f'(x) = (x+2)(-e^{-x}) + e^{-x}(1)$$

$$= -e^{-x}(x+2-1)$$

$$f'(x) = -e^{-x}(x+1)$$

$$f'(x) = 0 \Rightarrow \underbrace{x = -1}$$



$$f'(x) = -e^{-x}(x+1)$$

$$f'(0) = (-1)(1) = -1$$

$(-1, \infty) \longrightarrow$ <u>decreasing</u>

EXAMPLE

The interval in which the function $f(x) = (x + 2)\, e^{-x}$ is decreasing in

(a) $(-1, \infty)$

(b) $(-1, 1)$

(c) $(-1, 2)$

(d) $(1, 2)$

Ans: (a)

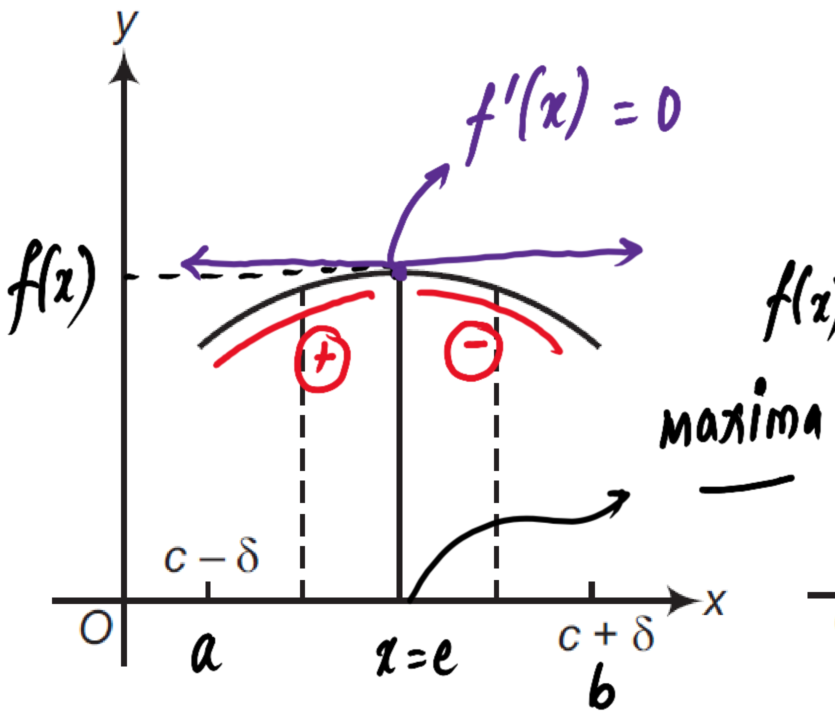
MAXIMA AND MINIMA

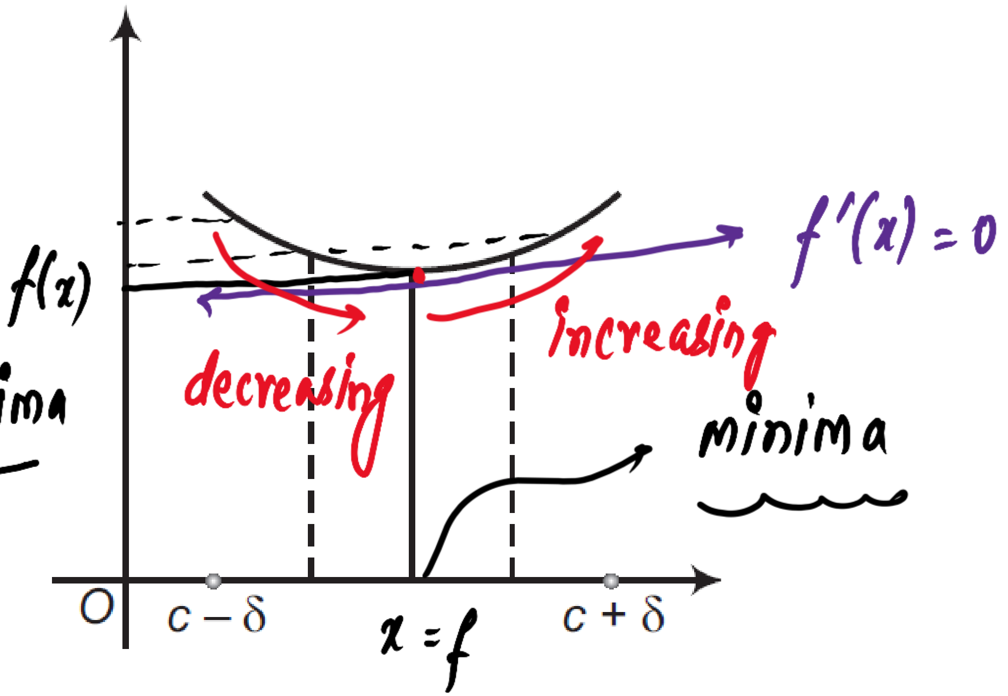
Local (interval)

Absolute (domain)

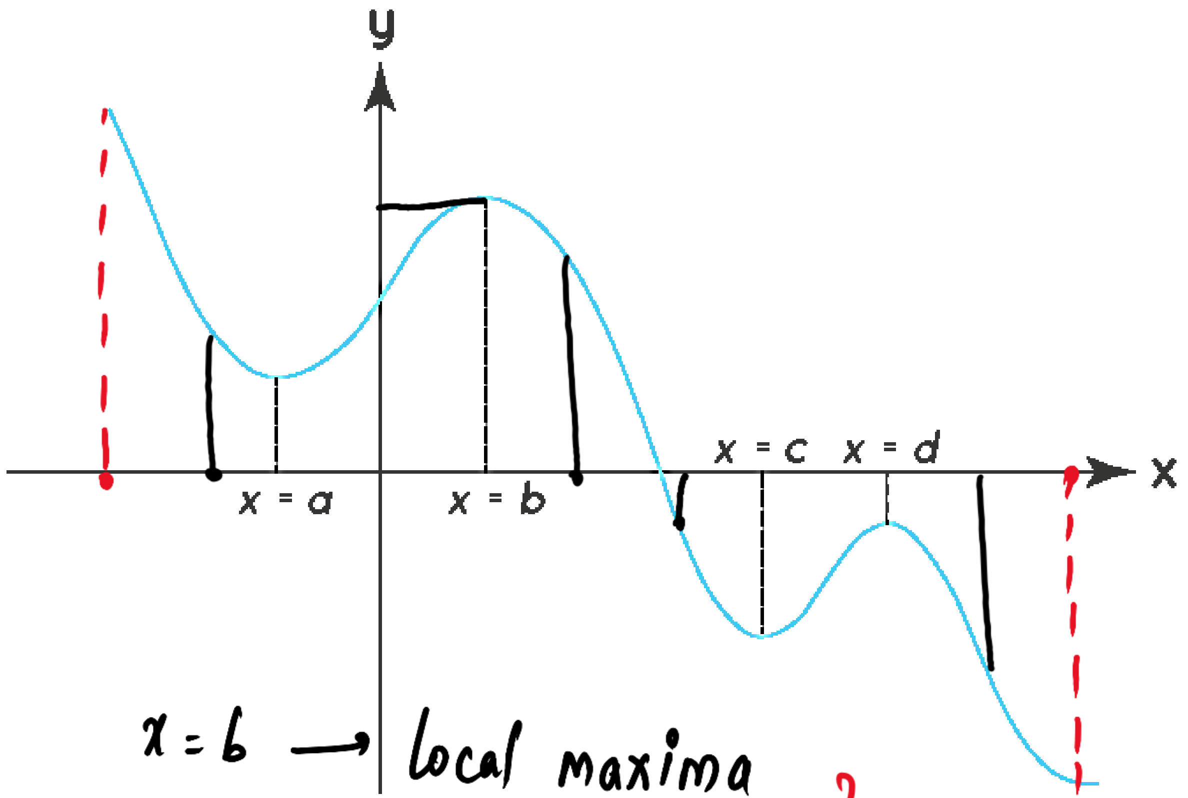
At the point of maxima and minima,

$f'(x) = 0$





LOCAL MAXIMA AND MINIMA



$x = b \rightarrow$ local maxima

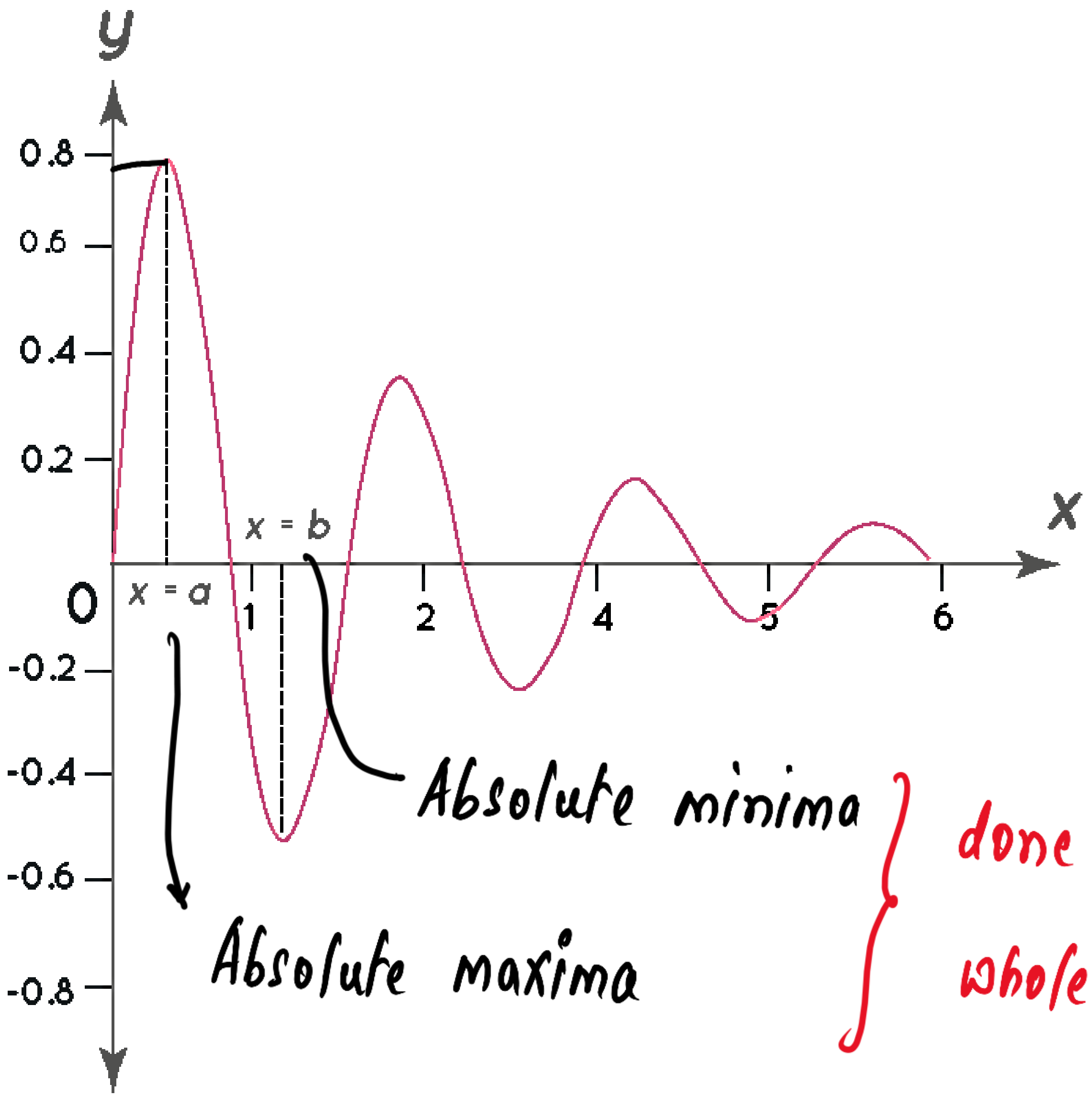
$x = a \rightarrow$ local minima

$x = c \rightarrow$ local minima

$x = d \rightarrow$ local maxima

done on intervals in the domain.

ABSOLUTE MAXIMA AND MINIMA



Absolute minima

Absolute maxima

done on whole domain

CRITICAL POINT

A point c in the domain of a function f at which either $f'(c) = 0$ or f is not differentiable.

WORKING RULE FOR FINDING POINTS OF MAXIMA AND MINIMA

First derivative test:

(i) If $f'(x)$ changes sign from positive to negative as x increases through c, then c is a point of local maxima, and $f(c)$ is local maximum value.

(ii) If $f'(x)$ changes sign from negative to positive as x increases through c, then c is a point of local minima, and $f(c)$ is local minimum value.

(iii) If $f'(x)$ does not change sign as x increases through c, then c is neither a point of local minima nor a point of local maxima. Such a point is called a point of inflection.

$x = a$ (Maxima)

$f(a) \rightarrow$ max. value of function

$x = b$ (minima)

$f(b)$ – min. value of f

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WORKING RULE FOR FINDING POINTS OF MAXIMA AND MINIMA

SECOND DERIVATIVE TEST :

Let f be a function defined on an interval I and c ∈ I. Let f be twice differentiable at c. Then

(i) $x = c$ is a point of local maxima if $f'(c) = 0$ and $f''(c) < 0$. In this case $f(c)$ is then the local maximum value.

(ii) $x = c$ is a point of local minima if $f'(c) = 0$ and $f''(c) > 0$. In this case $f(c)$ is the local minimum value.

(iii) The test fails if $f'(c) = 0$ and $f''(c) = 0$. In this case, we go back to first derivative test.

QUESTION

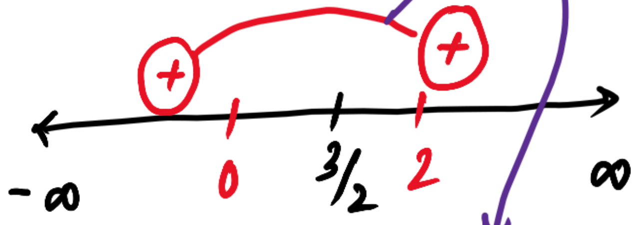
Show that the function f (x) = $4x^3 - 18x^2 + 27x - 7$ has neither maxima nor minima.

$$f'(x) = 12x^2 - 36x + 27 = 3(4x^2 - 12x + 9)$$

$$= 3(4x^2 - 6x - 6x + 9)$$

$$= 3(2x(2x-3) - 3(2x-3))$$

$$= \underbrace{3(2x-3)^2}$$



sign didn't change.

3/2 is neither a point of maxima nor minima

$$f'(x) = 0$$

$$3(2x-3)^2 = 0 \Rightarrow x = \frac{3}{2}$$

$$f''(x) = 6(2x-3)(2) = 12(2x-3)$$

$$f''(3/2) = 12(0) = 0$$

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QUESTION

Find all the points of local maxima and local minima of the function

$$f(x) = -\frac{3}{4}x^4 - 8x^3 - \frac{45}{2}x^2 + 105$$

$\boxed{x=0}$; $x=-3$; $\boxed{x=-5}$

$$f'(x) = -\frac{3}{4}(4x^3) - 8(3x^2) - \frac{45}{2}(2x) = 0$$

$$-3x^3 - 24x^2 - 45x = 0$$

$$-3x(x^2 + 8x + 15) = 0$$

$$(-3x)(x+3)(x+5) = 0$$

$$f''(x) = -9x^2 - 48x - 45$$

$f''(0) = -45 < 0$ $\boxed{\text{(local maxima)}}$

$f''(-3) = -81 + 144 - 45 > 0$ (local minima)

$f''(-5) = -225 + 240 - 45$

$= -60 < 0$ $\boxed{\text{(local maxima)}}$

QUESTION

Find the maximum and minimum values of

$f(x) = \sec x + \log \cos^2 x,\ 0 < x < 2\pi$

$f(x) = \sec x + \log(\cos x)^2$

$f'(x) = \sec x \tan x + 2\frac{1}{\cos x}(-\sin x)$

$= \tan x(\sec x - 2)$

$f'(x) = 0$

$\tan x(\sec x - 2) = 0$

$\tan x = 0$

$x = 0, \pi$

$\sec x - 2 = 0$

$x = \frac{\pi}{3},\ 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$x = 0$ (max.) ; $x = \pi$ (max.) ; $x = \frac{\pi}{3}$ (min.) ; $x = \frac{5\pi}{3}$ (min.)

$$f'(x) = \tan x(\sec x - 2)$$

$$f''(x) = \sec^2 x(\sec x - 2) + \tan x(\sec x \tan x)$$

$$= \sec x(\sec^2 x - 2\sec x + \tan^2 x)$$

$$= \sec x(1 + 2\tan^2 x - 2\sec x)$$

$$f''\left(\frac{\pi}{3}\right) = +ve \longrightarrow$$

$$f''(0) = 1(-1) = -1 < 0$$

$$f''\left(\frac{5\pi}{3}\right) = 2(1 + 6 - 2(2)) = 6 > 0$$

$$f''(\pi) = -1(1 + 0 - 2(-1)) = -3 < 0$$

$f(x) = \sec x + \log \cos^2 x$

$x = 0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}$

$f(0) = 1 + 0 = 1$

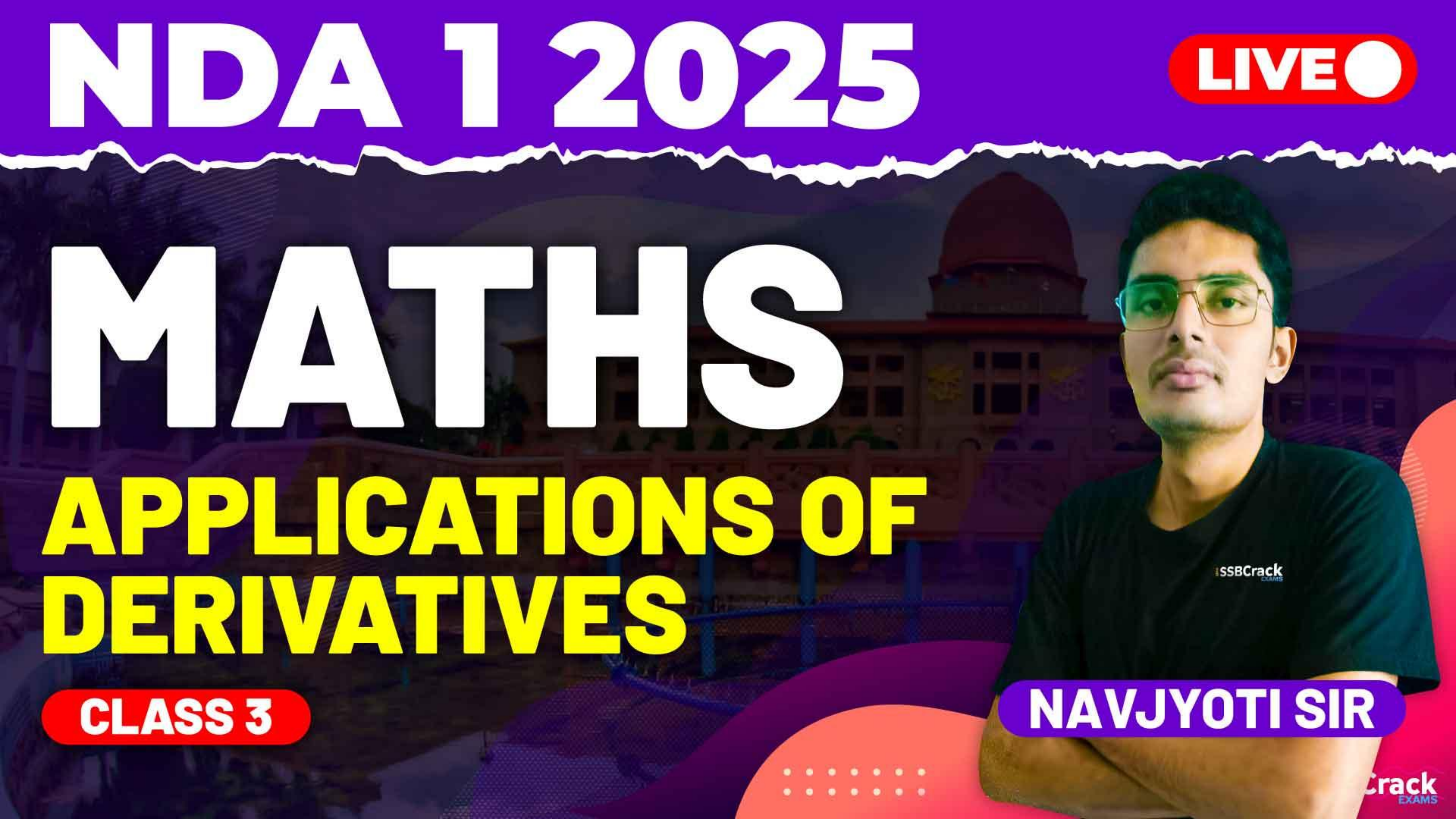
$f(\pi) = -1 + 0 = -1$

maximum values of $f(x)$ (local)

$f\left(\frac{\pi}{3}\right) = 2 + \log \frac{1}{4}$

$f\left(\frac{5\pi}{3}\right) = 2 + \log \frac{1}{4}$

minimum values of $f(x)$ (local)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