NDA 1 2025

LIVE

MATHS

APPLICATIONS OF DERIVATIVES

CLASS 4

NAVJYOTI SIR

Crack
EXAMS

Let $f(x) = \frac{x}{\ln x}$; $(x > 1)$

Consider the following statements :

1. $f(x)$ is increasing in the interval (e, ∞) ✓
2. $f(x)$ is decreasing in the interval $(1, e)$ ✓
3. $9\ln 7 > 7\ln 9$

Which of the statements given above are correct ?

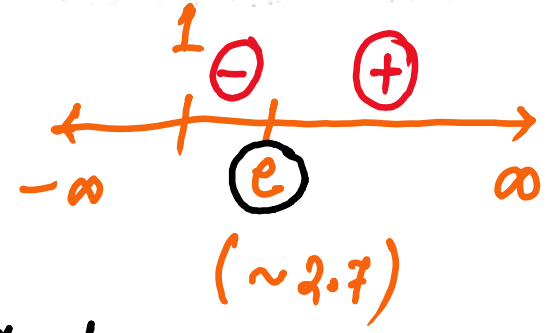
(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

PYQ – 24 - I



$$f'(x) = \frac{\ln x\,(1) - x\left(\frac{1}{x}\right)}{(\ln x)^2} = \frac{\ln x - 1}{(\ln x)^2}$$

At $x = 1$, $f'(x)$ is not defined

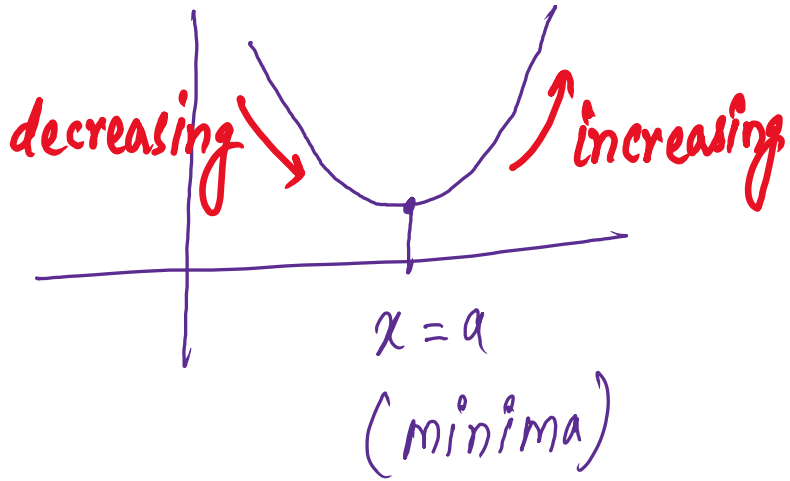
$f'(x) = 0$,

$\ln x - 1 = 0 \Rightarrow \ln x = 1 \Rightarrow x = e$ → critical points $\Rightarrow x = 1, e$

$$f(x) = \frac{x}{\ln x}$$

$$f'(x) = \frac{\ln x\,(1) - x\left(\frac{1}{x}\right)}{(\ln x)^2} = \frac{\ln x - 1}{(\ln x)^2}$$

$$f''(x) = \frac{(\ln x)^2\left(\frac{1}{x}\right) - (\ln x - 1)\left(2\ln x \cdot \frac{1}{x}\right)}{(\ln x)^4}$$



$$f''(e) = \frac{1^2\left(\frac{1}{e}\right) - (1-1)(\quad)}{(1)^4} = \frac{1}{e} > 0 \Rightarrow e \text{ is a point of minima}$$

$9 \ln 7 \qquad 7 \ln 9$

$7^9 > 9^7$

Let $f(x) = \frac{x}{\ln x}$; $(x > 1)$

Consider the following statements :

1. $f(x)$ is increasing in the interval (e, ∞)
2. $f(x)$ is decreasing in the interval $(1, e)$
3. $9\ln 7 > 7\ln 9$

Which of the statements given above are correct ?

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

PYQ – 24 - I

Ans : (d)

PYQ – 24 - I

A differentiable function $f(x)$ has a local maximum at $x = 0$. Let $y = 2f(x) + ax - b$.

Which of the following is/are correct ?

1. $f'(0) = 0$ ✓
2. $f''(0) < 0$

Select the correct answer using the code given below :

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

$x = 0$ is a point of maxima,

$f'(0) = 0$ —

$f''(0) < 0$ —

(solving for maxima)

PYQ – 24 - I

A differentiable function $f(x)$ has a local maximum at $x = 0$. Let $y = 2f(x) + ax - b$.

Which of the following is/are correct ?

1. $f'(0) = 0$

2. $f''(0) < 0$

Select the correct answer using the code given below :

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

Ans : (c)

PYQ – 24 - I

The function y has a relative maxima at $x = 0$ for

(a) $a > 0, b = 0$

(b) for all b and $a = 0$ ✓

(c) for all $b > 0$ only

(d) for all a and $b = 0$

$$y = 2f(x) + ax - b$$

$$y' = 2f'(x) + a$$

At $x = 0$,

$$y' = 0$$

$$2f'(0) + a = 0$$

$$0 + a = 0 \Rightarrow a = 0$$

PYQ – 24 - I

The function y has a relative maxima at $x = 0$ for

(a) $a > 0, b = 0$

(b) for all b and $a = 0$

(c) for all $b > 0$ only

(d) for all a and $b = 0$

Ans : (b)

PYQ – 24 - II

For what value of the angle between the vectors $\vec{a}$ and $\vec{b}$ is the quantity $|\vec{a} \times \vec{b}| + \sqrt{3}|\vec{a} \cdot \vec{b}|$ maximum?

(a) 0°
(b) 30°
(c) 45°
(d) 60°

$|\vec{a} \times \vec{b}| = ab\sin\theta$

$|\vec{a} \cdot \vec{b}| = ab\cos\theta$

$y = ab\sin\theta + \sqrt{3}ab\cos\theta$

For max. value of y,

$\frac{dy}{d\theta} = 0$

$ab\cos\theta - \sqrt{3}ab\sin\theta = 0$

$\cos\theta - \sqrt{3}\sin\theta = 0$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\theta = 30°$

$\frac{d^2y}{d\theta^2} = ab(-\sin\theta - \sqrt{3}\cos\theta)$

$ab\left(-\frac{1}{2} - \frac{3}{2}\right)$

$ab(-2) = -2ab < 0$

$\theta = 30°$ is a point of maxima.

PYQ – 24 - II

For what value of the angle between the vectors $\vec{a}$ and $\vec{b}$ is the quantity $|\vec{a} \times \vec{b}| + \sqrt{3}|\vec{a} \cdot \vec{b}|$ maximum?

(a) 0°

(b) 30°

(c) 45°

(d) 60°

Ans : (b)

PYQ – 24 - II

Let $f(x) = \cos 2x + x$ on $[-\pi/2, \pi/2]$.

What is the greatest value of $f(x)$?

(a) $\frac{\sqrt{3}}{2} - \frac{\pi}{12}$

(b) $\frac{\sqrt{3}}{2} + \frac{\pi}{12}$ ✓

(c) $\frac{\sqrt{3}}{2} + \frac{\pi}{9}$

(d) $\frac{\sqrt{3}}{2} + \frac{\pi}{6}$

$f'(x) = -\sin 2x \cdot 2 + 1$

critical points,

$-2\sin 2x + 1 = 0$

$\sin 2x = \frac{1}{2}$

$2x = \frac{\pi}{6} \Rightarrow x = \frac{\pi}{12}$

$2x = \pi - \frac{\pi}{6} \Rightarrow 2x = \frac{5\pi}{6} \Rightarrow x = \frac{5\pi}{12}$

$x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$2x \in [-\pi, \pi]$

$f\left(\frac{\pi}{12}\right) = \frac{\sqrt{3}}{2} + \frac{\pi}{12}$ ✓

$f\left(\frac{5\pi}{12}\right) = \cos\frac{5\pi}{6} + \frac{5\pi}{12} = -\frac{\sqrt{3}}{2} + \frac{5\pi}{12}$

$f\left(\frac{-\pi}{2}\right) = -1 - \frac{\pi}{2}$

$f\left(\frac{\pi}{2}\right) = -1 + \frac{\pi}{2}$

PYQ – 24 - II

Let $f(x) = \cos 2x + x$ on $[-\pi/2, \pi/2]$.

What is the greatest value of $f(x)$?

(a) $\frac{\sqrt{3}}{2} - \frac{\pi}{12}$

(b) $\frac{\sqrt{3}}{2} + \frac{\pi}{12}$

(c) $\frac{\sqrt{3}}{2} + \frac{\pi}{9}$

(d) $\frac{\sqrt{3}}{2} + \frac{\pi}{6}$

Ans : (b)

PYQ – 24 - II

What is the least value of $f(x)$?

(a) $-\left(1+\frac{\pi}{2}\right)$ ✓

(b) $-\left(\frac{1}{2}+\frac{\pi}{2}\right)$

(c) $-\left(1+\frac{\pi}{4}\right)$

(d) $-2\left(\frac{1}{2}-\frac{\pi}{4}\right)$

$f\left(\frac{\pi}{12}\right) = \frac{\sqrt{3}}{2} + \frac{\pi}{12}$

$f\left(\frac{5\pi}{12}\right) = \cos\frac{5\pi}{6} + \frac{5\pi}{12} = -\frac{\sqrt{3}}{2} + \frac{5\pi}{12}$

$f\left(\frac{-\pi}{2}\right) = -1 - \frac{\pi}{2}$ ✓

$f\left(\frac{\pi}{2}\right) = -1 + \frac{\pi}{2}$

PYQ – 24 - II

What is the least value of $f(x)$?

(a) $-\left(1+\frac{\pi}{2}\right)$

(b) $-\left(\frac{1}{2}+\frac{\pi}{2}\right)$

(c) $-\left(1+\frac{\pi}{4}\right)$

(d) $-2\left(\frac{1}{2}-\frac{\pi}{4}\right)$

Ans : (a)

PYQ – 24 - II

Let $f : (-1, 1) \to R$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$. Let $h(x) = f(2f(x) + 2)$ and $g(x) = (h(x))^2$.

What is $h'(0)$ equal to?

(a) −2

(b) −1

(c) 0

(d) 2

$$h'(x) = f'(2f(x)+2) \cdot (2f'(x) + 0)$$

$$= 2f'(x)\, f'(2f(x)+2)$$

$$h'(0) = 2f'(0)\, f'(2f(0)+2)$$

$$= 2f'(2(-1)+2)$$

$$= 2f'(0)$$

$$= 2 \times 1 = 2$$

PYQ – 24 - II

Let $f : (-1, 1) \to R$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$. Let $h(x) = f(2f(x) + 2)$ and $g(x) = (h(x))^2$.

What is $h'(0)$ equal to?

(a) -2

(b) -1

(c) 0

(d) 2

Ans : (d)

PYQ – 24 - II

What is $g'(0)$ equal to?

(a) –4

(b) –2

(c) 0

(d) 4

$$g(x) = h(x)^2$$

$$g'(x) = 2h(x)h'(x)$$

$$g'(0) = 2h(0)h'(0)$$

$$= 2 \times -1 \times 2$$

$$= -4$$

$$h(x) = f(2f(x)+2)$$

$$= f(2f(0)+2)$$

$$= f(2(-1)+2)$$

$$= f(0) = -1$$

PYQ – 24 - II

What is $g'(0)$ equal to?

(a) -4

(b) -2

(c) 0

(d) 4

Ans : (a)

Consider the curve $y = e^{2x}$.

What is the slope of the tangent to the curve at (0, 1) ?

(a) 0 (b) 1

(c) 2 (d) 4

Consider the curve $y = e^{2x}$.

What is the slope of the tangent to the curve at (0, 1) ?

(a) 0 (b) 1

(c) 2 (d) 4

Ans : (c)

Where does the tangent to the curve at (0, 1) meet the x-axis?

(a) (1, 0)
(b) (2, 0)
(c) (–1/2, 0)
(d) (1/2, 0)

eqn of tangent

Let the point be $(x_1, 0)$.

$x_1 =$ ____

Where does the tangent to the curve at (0, 1) meet the x-axis?

(a) $(1, 0)$ (b) $(2, 0)$

(c) $(-1/2, 0)$ (d) $(1/2, 0)$

Ans : (c)

Consider the function $f(x)=\dfrac{x^2-x+1}{x^2+x+1}$

What is the maximum value of the function ?

(a) 1/2 (b) 1/3

(c) 2 (d) 3

$$f'(x)=\frac{(x^2+x+1)(2x-1)-(x^2-x+1)(2x+1)}{(x^2+x+1)^2}$$

$$\Rightarrow \frac{-2x^2+4x^2-2}{(x^2+x+1)^2}=0 \quad \Rightarrow 2x^2-2=0$$

$$x^2=1$$

$$x=1,-1$$

$x=-1,$

$$f(-1)=\frac{1+1+1}{1-1+1}$$

$$=\frac{3}{1}=3$$

$$f'(x) = \frac{2x^2 - 2}{(x^2+x+1)^2}$$

$$f''(x) = \frac{(x^2+x+1)^2(4x) - (2x^2-2)(2)(x^2+x+1)(2x+1)}{(x^2+x+1)^2}$$

$f''(1) = (3(4) - 0)/(3)^2 = 12/9 = 4/3 > 0 \Rightarrow 1$ is a point of minima

$f''(-1) = (1(-4) - 0)/(1) = (-4) < 0 \Rightarrow -1$ is a point of maxima,

Consider the function $f(x)=\dfrac{x^2-x+1}{x^2+x+1}$

What is the maximum value of the function ?

(a) 1/2 (b) 1/3

(c) 2 (d) 3

Ans : (d)

What is the minimum value of the function ?

(a) 1/2 (b) 1/3

(c) 2 (d) 3

$f(1) = \_\_$

What is the minimum value of the function ?

(a) 1/2 (b) 1/3

(c) 2 (d) 3

Ans : (b)

How many tangents are parallel to x-axis for the curve $y = x^2 - 4x + 3$?

(a) 1
(b) 2
(c) 3
(d) No tangent is parallel to x-axis

$$\frac{dy}{dx} = 2x - 4$$

Slope $= 0$

$2x - 4 = 0 \Rightarrow (x = 2)$ → one value of $x \Rightarrow$ one value of $(2x-4)$ $\Rightarrow$ only 1 slope

only 1 tangent

How many tangents are parallel to x-axis for the curve $y = x^2 - 4x + 3$?

(a) 1

(b) 2

(c) 3

(d) No tangent is parallel to x-axis

Ans : (a)

What is the minimum value of $\frac{a^2}{\cos^2 x} + \frac{b^2}{\sin^2 x}$ where $a > 0$ and $b > 0$?

(a) $(a + b)^2$
(b) $(a - b)^2$
(c) $a^2 + b^2$
(d) $|a^2 + b^2|$

$$\frac{a^2\sin^2x + b^2\cos^2x + 2ab\sin x\cos x - 2ab\sin x\cos x}{(\sin x\cos x)^2}$$

$$= \frac{4(a\sin x + b\cos x)^2 - ab\sin 2x}{(\sin 2x)^2}$$

What is the minimum value of $\frac{a^2}{\cos^2 x} + \frac{b^2}{\sin^2 x}$ where $a > 0$ and $b > 0$?

(a) $(a + b)^2$
(b) $(a - b)^2$
(c) $a^2 + b^2$
(d) $|a^2 + b^2|$

Ans : (a)

Under what conditions is the tangent to a given curve at a point perpendicular to x-axis ?

(a) $\frac{dy}{dx} = 0$

(b) $\frac{dy}{dx} = 1$

(c) $\frac{dx}{dy} = 0$

(d) $\frac{d^2y}{dx^2} = 1$

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(a) $\frac{dy}{dx} = 0$

(b) $\frac{dy}{dx} = 1$

(c) $\frac{dx}{dy} = 0$

(d) $\frac{d^2y}{dx^2} = 1$

Ans : (c)

The tangent to the curve $y = x^2 - 5x + 5$, parallel to the line $2y = 4x + 1$, also passes through the point

(a) $\left(\frac{1}{4}, \frac{7}{2}\right)$

(b) $\left(\frac{7}{2}, \frac{1}{4}\right)$

(c) $\left(-\frac{1}{8}, 7\right)$

(d) $\left(\frac{1}{8}, -7\right)$

$2x - 5 = 2$

$2x = 7$

$x = \frac{7}{2}$

$y = -\frac{1}{4}$

$\left(\frac{7}{2}, -\frac{1}{4}\right)$

eqn of tangent

$y + \frac{1}{4} = 2\left(x - \frac{7}{2}\right)$

$\frac{4y+1}{4} = 2x - 7$

$4y + 1 = 8x - 28$ ✓

put options and check, (d) ✓

The tangent to the curve $y = x^2 - 5x + 5$, parallel to the line $2y = 4x + 1$, also passes through the point

(a) $\left(\frac{1}{4}, \frac{7}{2}\right)$

(b) $\left(\frac{7}{2}, \frac{1}{4}\right)$

(c) $\left(-\frac{1}{8}, 7\right)$

(d) $\left(\frac{1}{8}, -7\right)$

Ans : (d)

If the tangent to the curve $y = \frac{x}{x^2 - 3}$, $x \in R$, $(x \neq \pm \sqrt{3})$, at a point $(\alpha, \beta) \neq (0, 0)$ on it is parallel to the line $2x + 6y - 11 = 0$, then

(a) $|6\alpha + 2\beta| = 19$

(b) $|6\alpha + 2\beta| = 9$

(c) $|2\alpha + 6\beta| = 19$

(d) $|2\alpha + 6\beta| = 11$

If the tangent to the curve $y = \frac{x}{x^2 - 3}$, $x \in R$, $(x \neq \pm \sqrt{3})$, at a point $(\alpha, \beta) \neq (0, 0)$ on it is parallel to the line $2x + 6y - 11 = 0$, then

(a) $|6\alpha + 2\beta| = 19$

(b) $|6\alpha + 2\beta| = 9$

(c) $|2\alpha + 6\beta| = 19$

(d) $|2\alpha + 6\beta| = 11$

Ans : (a)

If θ denotes the acute angle between the curves, $y = 10 - x^2$ and $y = 2 + x^2$ at a point of their intersection, then $|\tan\theta|$ is equal to

(a) $\frac{7}{17}$ (b) $\frac{8}{15}$ (c) $\frac{4}{9}$ (d) $\frac{8}{17}$

If θ denotes the acute angle between the curves, $y = 10 - x^2$ and $y = 2 + x^2$ at a point of their intersection, then $|\tan\theta|$ is equal to

(a) $\frac{7}{17}$ (b) $\frac{8}{15}$ (c) $\frac{4}{9}$ (d) $\frac{8}{17}$

Ans : (b)

Function $f(x) = \dfrac{\lambda \sin x + 6 \cos x}{2 \sin x + 3 \cos x}$ is monotonic

increasing, if

(a) $\lambda > 1$

(b) $\lambda < 1$

(c) $\lambda < 4$

(d) $\lambda > 4$

Function $f(x)=\dfrac{\lambda \sin x+6\cos x}{2\sin x+3\cos x}$ is monotonic increasing, if

(a) $\lambda > 1$
(b) $\lambda < 1$
(c) $\lambda < 4$
(d) $\lambda > 4$

Ans : (d)

What is the minimum value of the function $f(x) = log_{10}(x^2 + 2x + 11)$?

(a) 0

(b) 1

(c) 2

(d) 10

What is the minimum value of the function $f(x) = log_{10}(x^2 + 2x + 11)$?

(a) 0

(b) 1

(c) 2

(d) 10

Ans : (b)

The angle of intersection of the curves $y = x^2$ and $x = y^2$ at (1, 1) is

(a) $\tan^{-1}\left(\frac{4}{3}\right)$

(b) $\tan^{-1}(1)$

(c) $90°$

(d) $\tan^{-1}\left(\frac{3}{4}\right)$

The angle of intersection of the curves $y = x^2$ and $x = y^2$ at (1, 1) is

(a) $\tan^{-1}\left(\frac{4}{3}\right)$

(b) $\tan^{-1}(1)$

(c) $90°$

(d) $\tan^{-1}\left(\frac{3}{4}\right)$

Ans : (d)

The length of subtangent to the curve $x^2y^2 = a^4$ at the point $(-a, a)$ is

(a) $3a$ (b) $2a$ (c) a (d) $4a$

The length of subtangent to the curve $x^2y^2 = a^4$ at the point $(-a, a)$ is

(a) $3a$ (b) $2a$ (c) a (d) $4a$

Ans : (c)

If $f(x) = kx^3 - 9x^2 + 9x + 3$ is monotonically increasing in every interval, which one of the following is correct?

(a) $k < 3$
(b) $k \leq 3$
(c) $k > 3$
(d) $k \geq 3$

If $f(x) = kx^3 - 9x^2 + 9x + 3$ is monotonically increasing in every interval, which one of the following is correct?

(a) $k < 3$
(b) $k \leq 3$
(c) $k > 3$
(d) $k \geq 3$

Ans : (c)

If the rate of change in volume of spherical soap bubble is uniform, then the rate of change of surface area varies as

(a) square of radius
(b) square root of radius
(c) inversely proportional to radius
(d) cube of the radius

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(a) square of radius
(b) square root of radius
(c) inversely proportional to radius
(d) cube of the radius

Ans : (c)

Match List I (Function) with List II (Property) and select the correct answer using the codes given below the lists.

List I (Function)		List II (Property)	
A.	$f(x) = \frac{\tan x}{x}$	1.	Increasing for every $x > 1$
B.	$f(x) = (x - 1) - \log\ x$	2.	Decreasing for every $x > 0$
C.	$f(x) = \frac{\sin x}{x}$	3.	Neither increasing nor decreasing for $x > 0$
D.	$f(x) = \frac{\log\ (1 + x)}{x}$	4.	Decreasing for x in $(0, \pi/2)$
		5.	Increasing for x in $(0, \pi/2)$

Codes

	A	B	C	D
(a)	2	4	3	5
(b)	5	3	1	2
(c)	5	1	4	2
(d)	2	4	1	5

Match List I (Function) with List II (Property) and select the correct answer using the codes given below the lists.

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C.	$f(x) = \frac{\sin x}{x}$	3.	Neither increasing nor decreasing for $x > 0$
D.	$f(x) = \frac{\log\ (1 + x)}{x}$	4.	Decreasing for x in $(0, \pi/2)$
		5.	Increasing for x in $(0, \pi/2)$

Codes

	A	B	C	D
(a)	2	4	3	5
(b)	5	3	1	2
(c)	5	1	4	2
(d)	2	4	1	5

Ans : (c)

In which one of the following intervals is the function $f(x) = x^2 - 5x + 6$ decreasing?

(a) $(-\infty, 2]$ (b) $[3, \infty)$ (c) $(-\infty, \infty)$ (d) $(2, 3)$

In which one of the following intervals is the function $f(x) = x^2 - 5x + 6$ decreasing?

(a) $(-\infty, 2]$ (b) $[3, \infty)$ (c) $(-\infty, \infty)$ (d) $(2, 3)$

Ans : (a)

Let x, y be real numbers such that $-4 \leq x \leq 4, -5 \leq y \leq 5$. Let $\theta \in R$ and let $A = x\cos\theta - y\sin\theta$, $B = x\cos\theta + y\sin\theta$. What is the maximum value of $A^2 - B^2$?

(a) 32
(b) 40
(c) 50
(d) 80

Let x, y be real numbers such that $-4 \leq x \leq 4, -5 \leq y \leq 5$. Let $\theta \in R$ and let $A = x\cos\theta - y\sin\theta$, $B = x\cos\theta + y\sin\theta$. What is the maximum value of $A^2 - B^2$?

(a) 32
(b) 40
(c) 50
(d) 80

Ans : (b)

Let the slope of the curve $y = \cos^{-1}(\sin x)$ be $\tan\theta$. Then the value of θ in the interval $(0, \pi)$ is

(a) $\frac{\pi}{6}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

Let the slope of the curve $y = \cos^{-1}(\sin x)$ be $\tan\theta$. Then the value of θ in the interval $(0, \pi)$ is

(a) $\frac{\pi}{6}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

Ans : (b)

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