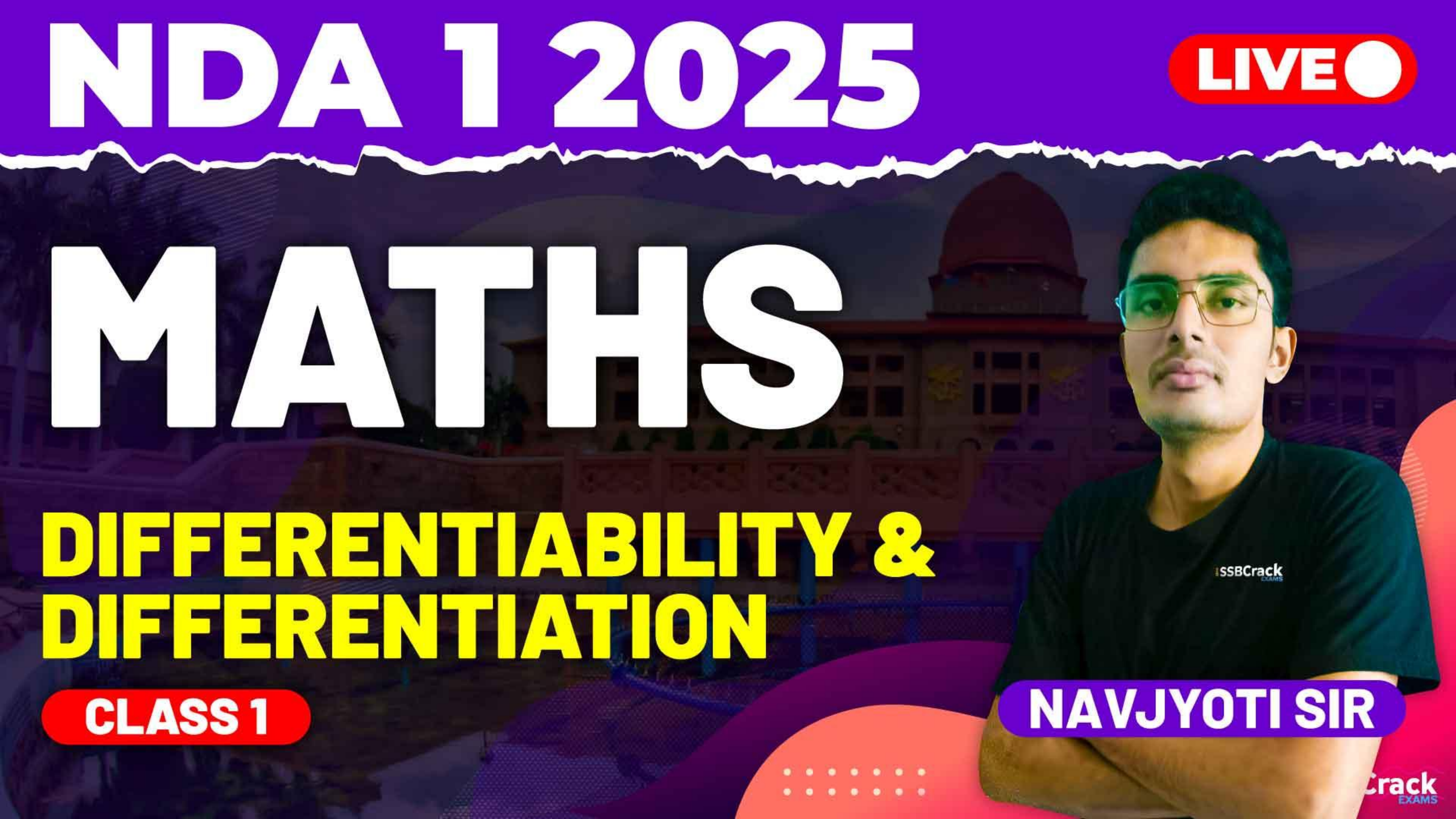

NDA 1 2025
LIVE
MATHS
DIFFERENTIABILITY &
DIFFERENTIATION
CLASS 1
NAVJYOTI SIR
SSBCrack
Crack
EXAMS

04 Dec 2024 Live Classes Schedule

8:00AM	04 DEC 2024 DAILY CURRENT AFFAIRS	RUBY MA'AM
9:00AM	04 DEC 2024 DAILY DEFENCE UPDATES	DIVYANSHU SIR

SSB INTERVIEW LIVE CLASSES

9:30AM	OVERVIEW OF GD & LECTURETTE	ANURADHA MA'AM

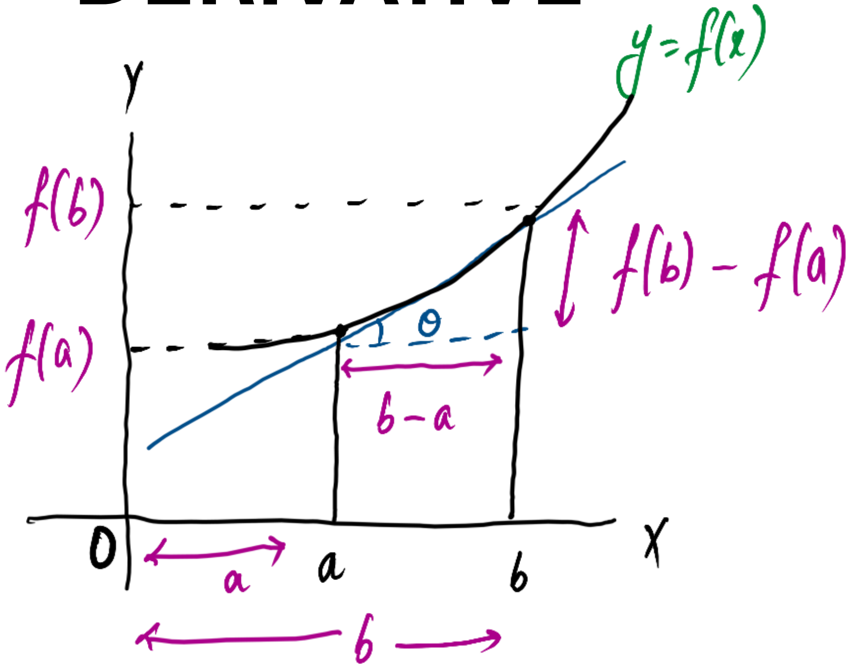
NDA 1 2025 LIVE CLASSES

1:00PM	PHYSICS - HUMAN EYE & THE COLOURFUL WORLD - CLASS 1	NAVJYOTI SIR
4:30PM	ENGLISH - ADAPTATION OF BORROWED WORDS - CLASS 2	ANURADHA MA'AM
5:30PM	MATHS - DIFFERENTIABILITY & DIFFERENTIATION - CLASS 1	NAVJYOTI SIR

CDS 1 2025 LIVE CLASSES

1:00PM	PHYSICS - HUMAN EYE & THE COLOURFUL WORLD - CLASS 1	NAVJYOTI SIR
4:30PM	ENGLISH - ADAPTATION OF BORROWED WORDS - CLASS 2	ANURADHA MA'AM
7:00PM	MATHS - ALGEBRA - CLASS 1	NAVJYOTI SIR

DERIVATIVE



$$\text{slope} = \tan\theta = \frac{f(b) - f(a)}{b - a} = \frac{\Delta y}{\Delta x} \;\text{—}\; \frac{\text{change in } y}{\text{change in } x}$$

change

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = f'(x) = \text{derivative of function } f,$$

$\frac{dy}{dx}$

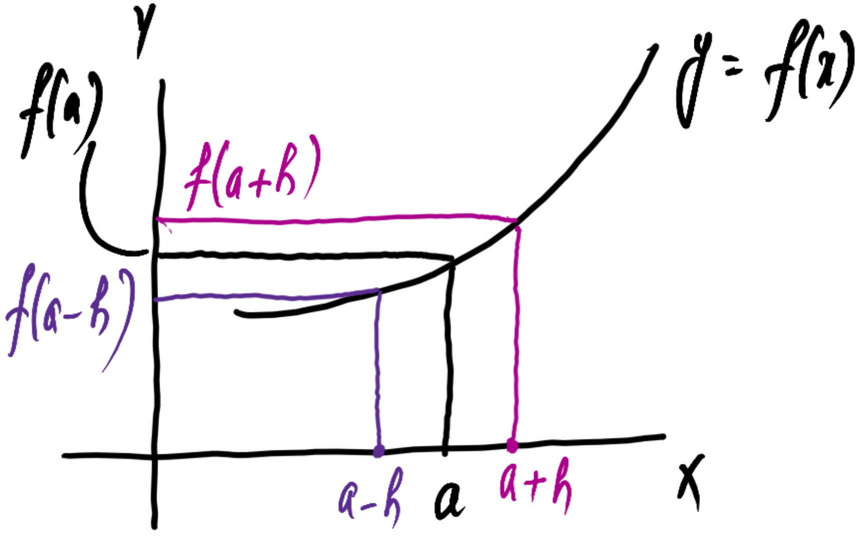
differential coefficient in y,

differential coefficient in x.

y — dependent variable

x — independent variable

DIFFERENTIABILITY OF A FUNCTION



For a point a in domain of f(x),

* **Left hand derivative at x = a,**

$$\lim_{h\to 0} \frac{f(a-h)-f(a)}{a-h-a} = \lim_{h\to 0} \frac{f(a-h)-f(a)}{-h}$$

* **Right hand Derivative at x = a,**

$$\lim_{h\to 0} \frac{f(a+h)-f(a)}{a+h-a} = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

If LHD = RHD, we say f(x) is differentiable at x = a.

DIFFERENTIABILITY OF A FUNCTION

PROPERTIES

(i) The function $y = f(x)$ is said to be differentiable in an open interval (a, b) if it is differentiable at every point of (a, b)

(ii) The function $y = f(x)$ is said to be differentiable in the closed interval $[a, b]$ if $\mathrm{R}f'(a)$ and $\mathrm{L}f'(b)$ exist and $f'(x)$ exists for every point of (a, b).

(iii) Every differentiable function is continuous, but the converse is not true

QUESTION

Let $f(x) = x|x|$, for all $x \in \mathbf{R}$. Discuss the derivability of $f(x)$ at $x = 0$

same as differentiability,

$$f(x) = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$$

LHD at $x = 0$

$$\lim_{h \to 0} \frac{f(0-h) - f(0)}{-h} = \frac{-h^2 - 0}{-h} = \lim_{h \to 0} h = 0$$

RHD at $x = 0$

$$\lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \frac{h^2 - 0}{h} = \lim_{h \to 0} h = 0$$

As LHD = RHD at $x = 0$,

$f(x)$ is derivable at $x = 0$.

DIFFERENTIATION OF IMPORTANT FUNCTIONS

→ finding the derivative,

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(c\,f(x)) = c\frac{d}{dx}(f(x))$$

(where c is a constant)

$$\frac{d}{dx}(x) = 1$$

$$\frac{d}{dx}(x^3) = 3x^2$$

$$\frac{d}{dx}(\sqrt{x}) = \left(x^{1/2}\right)' = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-1/2}$$

$$= \frac{1}{2x^{1/2}} = \frac{1}{2\sqrt{x}}$$

$$\frac{d}{dx}(5x^4) = 5\frac{d}{dx}(x^4)$$

$$= 5 \times 4x^3 = 20x^3$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$$

$$\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$$

(function starting with 'c' has negative derivative)

$$\frac{d}{dx}\left(\sin^{-1}x\right) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\left(\cos^{-1}x\right) = \frac{-1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}\left(\sec^{-1}x\right) = \frac{1}{x\sqrt{x^2-1}} \qquad \frac{d}{dx}\left(\operatorname{cosec}^{-1}x\right) = \frac{-1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}\left(\tan^{-1}x\right) = \frac{1}{1+x^2} \qquad \frac{d}{dx}\left(\cot^{-1}x\right) = \frac{-1}{1+x^2}$$

DIFFERENTIATION OF IMPORTANT FUNCTIONS

$\frac{d}{dx}(e^x) = e^x$ ✓

$\frac{d}{dx}(\log_e x) = \frac{1}{x}, (x > 0)$ ✓

$$\frac{d}{dx}(\log_a x) = \frac{1}{x \log_e a}$$

$$\frac{d}{dx}|x| = \frac{x}{|x|} \text{ or } \frac{|x|}{x}, \{x \neq 0\}$$

$$\frac{d}{dx}(a^x) = a^x \log_e a$$

ALGEBRA OF DERIVATIVES

(i) $\frac{d(u \pm v)}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$ ✓

(ii) $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

(iii) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ $\left(u \frac{d}{dx}\left(\frac{1}{v}\right) + \frac{1}{v}\frac{d}{dx}(u) \right)$

CHAIN RULE

Chain rule is a rule to differentiate composition of functions. Let $f = vou$. If $t = u(x)$ and both $\frac{dt}{dx}$ and $\frac{dv}{dt}$ exist then $\frac{df}{dx} = \frac{dv}{dt} \cdot \frac{dt}{dx}$

QUESTION

Differentiate $\sqrt{\tan\sqrt{x}}$ w.r.t. x

of form $(\sqrt{x})$

$$\frac{d}{dx}\left(\sqrt{\underline{\tan\sqrt{x}}}\right) \cdot \frac{d}{dx}\left(\tan\sqrt{x}\right) \cdot \frac{d}{dx}\left(\sqrt{x}\right)$$

$$\frac{1}{2\sqrt{\tan\sqrt{x}}} \times \sec^2\sqrt{x} \times \frac{1}{2\sqrt{x}} = \frac{\sec^2\sqrt{x}}{4\sqrt{x\tan\sqrt{x}}}$$

QUESTION

If $y = \sin^{-1}\left\{x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right\}$ and $0 < x < 1$, then find $\frac{dy}{dx}$.

$x = \sin A \qquad \sqrt{x} = \sin B$

$y = \sin^{-1}\left\{\sin A\sqrt{1-\sin^2 B} - \sin B\sqrt{1-\sin^2 A}\right\}$

$= \sin^{-1}\left\{\sin A\cos B - \sin B\cos A\right\}$

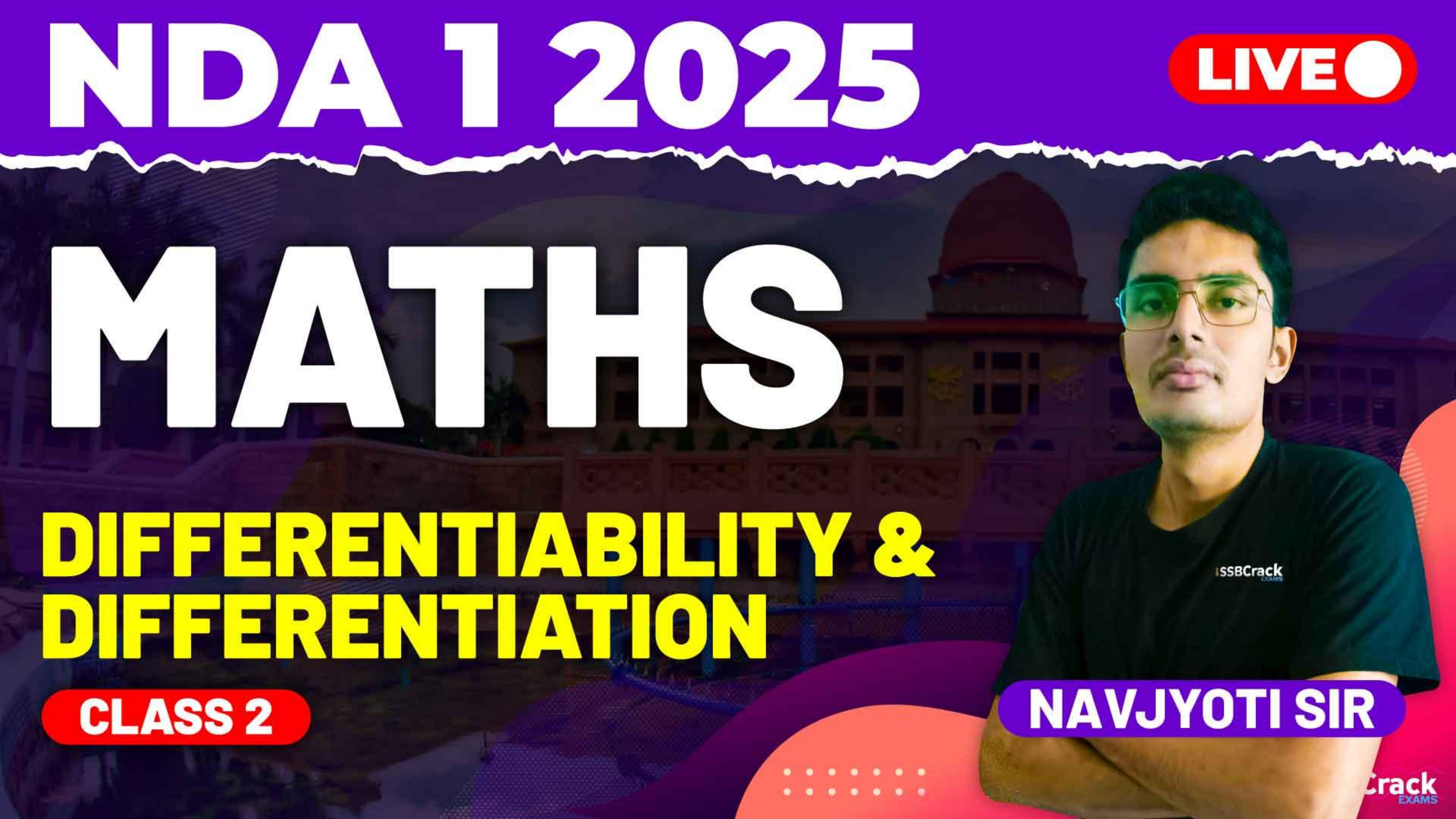
$= \sin^{-1}\left\{\sin(A-B)\right\} = A - B = \sin^{-1}x - \sin^{-1}\sqrt{x}$

$$y = \sin^{-1} x - \sin^{-1}\sqrt{x}$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sin^{-1} x\right) - \frac{d}{dx}\left(\sin^{-1}\sqrt{x}\right)$$

$$= \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-(\sqrt{x})^2}}\frac{d}{dx}\left(\sqrt{x}\right)$$

$$= \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x}} \times \frac{1}{2\sqrt{x}} = \underbrace{\frac{1}{\sqrt{1-x^2}} - \frac{1}{2\sqrt{x(1-x)}}}$$


NDA 1 2025
LIVE
MATHS
DIFFERENTIABILITY &
DIFFERENTIATION
CLASS 2
NAVJYOTI SIR
SSBCrack
EXAMS
Crack
EXAMS