NDA 1 2025

LIVE

MATHS

DIFFERENTIABILITY & DIFFERENTIATION

CLASS 2

NAVJYOTI SIR

05 Dec 2024 Live Classes Schedule

8:00AM	05 DEC 2024 DAILY CURRENT AFFAIRS	RUBY MA'AM
9:00AM	05 DEC 2024 DAILY DEFENCE UPDATES	DIVYANSHU SIR

SSB INTERVIEW LIVE CLASSES

9:30AM	OVERVIEW OF OIR & PRACTICE	ANURADHA MA'AM

NDA 1 2025 LIVE CLASSES

1:00PM	PHYSICS - HUMAN EYE & THE COLOURFUL WORLD - CLASS 2	NAVJYOTI SIR
4:30PM	ENGLISH - SYNTHESIS OF SENTENCES - CLASS 1	ANURADHA MA'AM
5:30PM	MATHS - DIFFERENTIABILITY & DIFFERENTIATION - CLASS 2	NAVJYOTI SIR

CDS 1 2025 LIVE CLASSES

1:00PM	PHYSICS - HUMAN EYE & THE COLOURFUL WORLD - CLASS 2	NAVJYOTI SIR
4:30PM	ENGLISH - SYNTHESIS OF SENTENCES - CLASS 1	ANURADHA MA'AM
7:00PM	MATHS - ALGEBRA - CLASS 2	NAVJYOTI SIR

SSBCrack EXAMS

DIFFERENTIATING FUNCTIONS HAVING VARIABLE IN THEIR POWER

$x^x, \quad 4^{\sin x}, \quad 71^{\log x}$

$$\frac{d}{dx}\left(f(x)\right)^n = n\left(f(x)\right)^{n-1} f'(x)$$

Let $y = x^x$

$\log y = \log(x^x)$

$\log y = x \log x$ $\quad \left(\log(a^b) = b\log a\right)$

Differentiate both sides w.r.t. x,

$$\frac{1}{y}\cdot\frac{dy}{dx} = x\left(\frac{1}{x}\right) + \log x\,(1)$$

$$\frac{dy}{dx} = y\left[1 + \log x\right]$$

$$\frac{dy}{dx} = x^x(1+\log x)$$

LOGARITHM FUNCTION

Let $b > 1$ be a real number. Then we say logarithm of a to base b is x if $b^x = a$, Logarithm of a to the base b is denoted by $\log_b a$. If the base $b = 10$, we say it is common logarithm and if $b = e$, then we say it is natural logarithms. $\log x$ denotes the logarithm function to base e. The domain of logarithm function is $\mathbf{R}^+$, the set of all positive real numbers and the range is the set of all real numbers.

$b^x = a$

$\log_b a = x$

Base

$10 \longrightarrow$ common logarithm

$e \longrightarrow$ natural logarithm

1. $\log_b (xy) = \log_b x + \log_b y$

2. $\log_b \left(\dfrac{x}{y} \right) = \log_b x - \log_b y$

3. $\log_b x^n = n \log_b x$ #

4. $\log_b x = \dfrac{\log_c x}{\log_c b}$, where $c > 1$

5. $\log_b x = \dfrac{1}{\log_x b}$

6. $\log_b b = 1$ and $\log_b 1 = 0$

EXPONENTIAL FUNCTION

The exponential function with positive base $b > 1$ is the function $y = f(x) = b^x$. Its domain is **R**, the set of all real numbers and range is the set of all positive real numbers. Exponential function with base 10 is called the common exponential function and with base e is called the natural exponential function.

LOGARITHMIC DIFFERENTIATION

Logarithmic differentiation is a powerful technique to differentiate functions of the form $f(x) = (u(x))^{v(x)}$, where both f and u need to be positive functions for this technique to make sense.

SSBCrack EXAMS

QUESTION

Differentiate w.r.t. x : $2^{\cos^2 x}$

Let $y = 2^{\cos^2 x}$

$$\frac{dy}{dx} = 2^{\cos^2 x} \times \frac{d}{dx}\left(\log\left(2^{\cos^2 x}\right)\right)$$

$$= 2^{\cos^2 x} \times \frac{d}{dx}\left(\cos^2 x \log 2\right)$$

$$= 2^{\cos^2 x} \log 2 \frac{d}{dx}\left(\cos^2 x\right)$$

$$2^{\cos^2 x} \log 2 \left(2\cos x \cdot \frac{d}{dx}(\cos x)\right)$$

$$(\log 2) 2^{\cos^2 x}\left(2\cos x(-\sin x)\right)$$

$$= \underbrace{-2^{\cos^2 x} \log 2 \sin 2x}$$

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$$y = \left(u(x)\right)^{v(x)}$$

$$\boxed{\frac{dy}{dx} = \left(u(x)\right)^{v(x)} \cdot \frac{d}{dx}\left(v(x) \log u(x)\right)}$$

DIFFERENTIATING w.r.t ANOTHER FUNCTION

Let $u = f(x)$ and $v = g(x)$ be two functions of x, then to find derivative of $f(x)$ w.r.t. to $g(x)$, i.e., to find $\frac{du}{dv}$, we use the formula

$$\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}}.$$

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QUESTION

Differentiate $\log \sin x \; wrt \; \sqrt{\cos x}$

$u = \log \sin x$

$v = \sqrt{\cos x}$

$\frac{du}{dx} = \frac{1}{\sin x} \frac{d}{dx}(\sin x) = \frac{1}{\sin x} \cdot \cos x = \frac{\cos x}{\sin x}$

$\frac{dv}{dx} = \frac{1}{2\sqrt{\cos x}} \cdot (-\sin x) = -\frac{1}{2} \cdot \frac{\sin x}{\sqrt{\cos x}}$

$\frac{du}{dv} = \frac{\left(\frac{du}{dx}\right)}{\left(\frac{du}{dv}\right)}$

$= \frac{\frac{\cos x}{\sin x}}{-\frac{1}{2} \frac{\sin x}{\sqrt{\cos x}}}$

$= \frac{-2(\cos x)^{3/2}}{(\sin x)^2}$

PARAMETRIC DIFFERENTIATION

$$y = f(t)$$

$$x = g(t)$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

QUESTION

If $x = a\sec^3\theta$ and $y = a\tan^3\theta$, find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{3}$.

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{a(3\tan^2\theta)\sec^2\theta}{a(3\sec^2\theta)(\sec\theta\tan\theta)}$$

$$= \frac{\tan\theta}{\sec\theta} = \frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}} = \sin\theta$$

$$\frac{d}{dx}(c\,f(x)) = c\,\frac{d}{dx}(f(x))$$

where c is a constant.

$$\frac{dy}{dx} = \sin\theta \qquad \frac{dy}{dx}\left(\text{at } \theta = \pi/3\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$\frac{dx}{dy} = \frac{\left(\frac{dx}{d\theta}\right)}{\left(\frac{dy}{d\theta}\right)} = \frac{1}{\left(\frac{dy}{dx}\right)}$$

SSBCrack EXAMS

HIGHER ORDER DERIVATIVES

Let $y = f(x) \longrightarrow$ Differentiated one time $\longrightarrow f'(x)$ or $\frac{dy}{dx}$ or y_1

Second derivative,

$\neq$ $\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$ or $f''(x)$ or y_2.

Third derivative, $\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = \frac{d^3y}{dx^3}$ or $f'''(x)$ or y_3 and so on...

QUESTION

If $y = \tan x + \sec x$, prove that $\dfrac{d^2y}{dx^2} = \dfrac{\cos x}{(1-\sin x)^2}$.

$$\frac{dy}{dx} = \sec^2 x + \sec x \tan x = \sec x(\sec x + \tan x) = \frac{1}{\cos x}\left(\frac{1}{\cos x} + \frac{\sin x}{\cos x}\right)$$

$$= \frac{1+\sin x}{\cos^2 x}$$

$$= \frac{1+\sin x}{1-\sin^2 x} = \frac{1+\sin x}{(1+\sin x)(1-\sin x)}$$

$$\frac{dy}{dx} = \frac{1}{1-\sin x}$$

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$$\frac{dy}{dx} = \frac{1}{1-\sin x}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{(1-\sin x)^2}\left(0 - \cos x\right)$$

$$\boxed{\frac{d}{dx}\left(\frac{1}{f(x)}\right) = -\frac{1}{(f(x))^2}\frac{d}{dx}\left(f(x)\right)}$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{\cos x}{(1-\sin x)^2}}$$

QUESTION

Implicit function

If $y = \tan(x + y)$, find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \sec^2(x+y) \cdot \frac{d}{dx}(x+y)$$

$$= \sec^2(x+y)\left(1 + \frac{dy}{dx}\right)$$

$$\frac{dy}{dx} = \sec^2(x+y) + \sec^2(x+y)\frac{dy}{dx}$$

$$\frac{dy}{dx}\left(1 - \sec^2(x+y)\right) = \sec^2(x+y)$$

$$\frac{dy}{dx} = \frac{\sec^2(x+y)}{1 - \sec^2(x+y)}$$

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QUESTION

If $e^x + e^y = e^{x+y}$, prove that

$$\frac{dy}{dx} = -e^{y-x}.$$

$$e^x + e^y \frac{dy}{dx} = e^{x+y} \cdot \frac{d}{dx}(x+y)$$

$$'' = e^{x+y}\left(1 + \frac{dy}{dx}\right)$$

$$e^x - e^{x+y} = \frac{dy}{dx}\left(e^{x+y} - e^y\right)$$

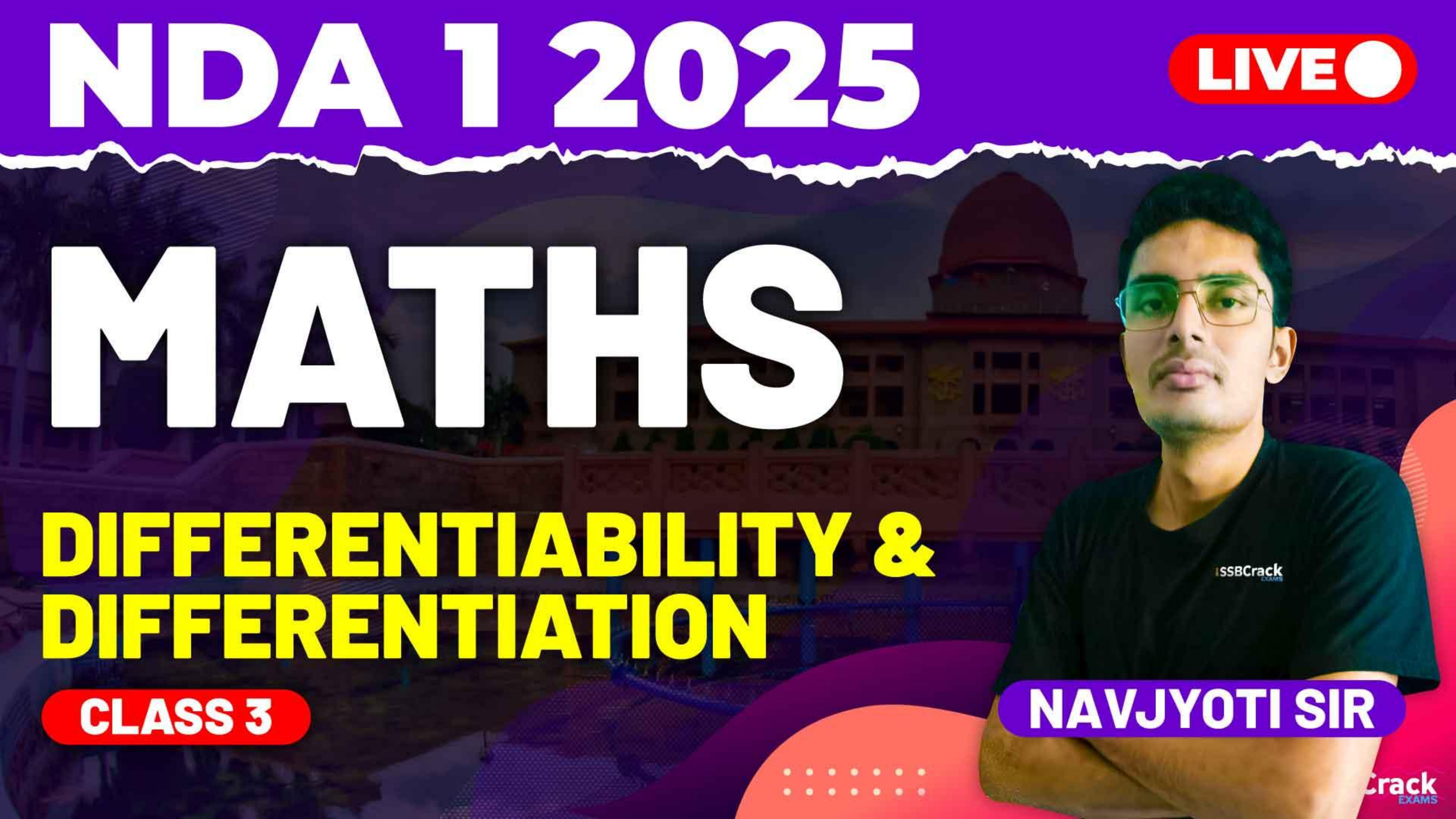
$$\frac{dy}{dx} = \frac{e^x - e^{x+y}}{e^{x+y} - e^y}$$

$$= -\left(\frac{e^{x+y} - e^x}{e^{x+y} - e^y}\right)$$

$$= -\left(\frac{e^x + e^y - e^x}{e^x + e^y - e^y}\right) = -\frac{e^y}{e^x}$$

$$\left(e^{x+y} = e^x + e^y\right)$$

$$= -e^{y-x}$$



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