NDA 1 2025

LIVE

MATHS

DIFFERENTIABILITY & DIFFERENTIATION

CLASS 3

NAVJYOTI SIR

06 Dec 2024 Live Classes Schedule

8:00AM	06 DEC 2024 DAILY CURRENT AFFAIRS	RUBY MA'AM
9:00AM	06 DEC 2024 DAILY DEFENCE UPDATES	DIVYANSHU SIR

SSB INTERVIEW LIVE CLASSES

9:30AM	OVERVIEW OF PPDT & PRACTICE	ANURADHA MA'AM

NDA 1 2025 LIVE CLASSES

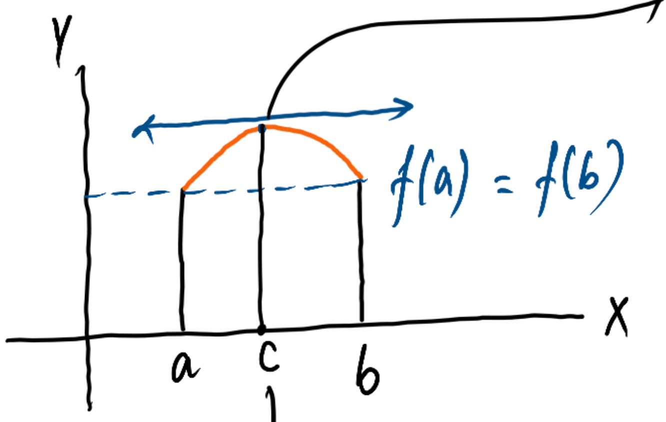
1:00PM	PHYSICS - WAVES	NAVJYOTI SIR
4:30PM	ENGLISH - SYNTHESIS OF SENTENCES - CLASS 2	ANURADHA MA'AM
5:30PM	MATHS - DIFFERENTIABILITY & DIFFERENTIATION - CLASS 3	NAVJYOTI SIR

CDS 1 2025 LIVE CLASSES

1:00PM	PHYSICS - WAVES	NAVJYOTI SIR
4:30PM	ENGLISH - SYNTHESIS OF SENTENCES - CLASS 2	ANURADHA MA'AM
7:00PM	MATHS - ALGEBRA - CLASS 3	NAVJYOTI SIR

ROLLE's THEOREM

Let $f : [a, b] \rightarrow \mathbf{R}$ be continuous on $[a, b]$ and differentiable on (a, b), such that $f(a) = f(b)$, where a and b are some real numbers. Then there exists at least one point c in (a, b) such that $f'(c) = 0$.



(tangent at a point c between a and b is parallel to x-axis)

$c \in [a, b]$

$f'(c)$ — slope of tangent at point c,

$f'(c) = 0$

QUESTION

Verify Rolle's theorem for the function, $f(x) = \sin 2x$ in $\left[0, \frac{\pi}{2}\right]$.

As $\sin x$ is continous function $\Rightarrow$ $\sin 2x$ is continous function.

$\sin 2x$ will be continous in $\left[0, \frac{\pi}{2}\right]$.

$\left(0, \frac{\pi}{2}\right)$ $\quad f'(x) = 2\cos 2x \Rightarrow$ differentiable in $\left(0, \frac{\pi}{2}\right)$

Rolle's Theorem is verified.

$f(0) = \sin 2(0) = 0$

$f\left(\frac{\pi}{2}\right) = \sin 2\left(\frac{\pi}{2}\right) = \sin \pi = 0$

$f'(c) = 2\cos 2c$

$2\cos 2c = 0$

$\cos 2c = 0$

$2c = \pi/2$

$c = \pi/4 \in \left[0, \frac{\pi}{2}\right]$

MEAN VALUE THEOREM

Let $f : [a, b] \to \mathbf{R}$ be a continuous function on $[a, b]$ and differentiable on (a, b). Then there exists at least one point c in (a, b) such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

QUESTION

Verify mean value theorem for the function f (x) = (x – 3) (x – 6) (x – 9) in [3, 5].

polynomial function is continous ⇒ f(x) is continous,

$$f(x) = (x-3)(x^2-15x+54) = x^3 - 15x^2 + 54x - 3x^2 + 45x - 162$$

$$= x^3 - 18x^2 + 99x - 162$$

$f'(x) = 3x^2 - 36x + 99$ ⇒ f(x) is differentiable in (3, 5)

$$f'(c) = \frac{f(5) - f(3)}{5-3} = \frac{8-0}{2} = 4$$

$$3c^2 - 36c + 99 = 4$$

$$3c^2 - 36c + 95 = 0$$

$$c = \frac{36 \pm \sqrt{(-36)^2 - 4 \times 3 \times 95}}{2 \times 3}$$

$$= \frac{36 \pm \sqrt{1296 + 1140}}{6}$$

$$= 6 \pm \sqrt{67}$$

$6 + \text{(+ve)} > 6$

$6 - \text{(-ve)} < 0$

900
36
360
1296

$\sqrt{2436}$ 406 203

36 6 3

As $c \notin [3,5]$

Does not follows Lagrange's mean value theorem.

SSBCrack EXAMS

Q) What is the derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1} x$?

(a) 0 (b) $\frac{1}{2}$

(c) 1 (d) x

$x = \tan\theta \Rightarrow \theta = \tan^{-1} x$,

$\tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right)$ $\frac{1-\cos\theta}{\sin\theta}$

$\tan^{-1}\left(\frac{2\sin^2\theta/2}{2\sin\theta/2\,\cos\theta/2}\right)$

$= \tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{\theta}{2} = \frac{1}{2}\left(\tan^{-1}x\right) = \frac{1}{2}u$

$u = \tan^{-1}x$

Ans. $\frac{1}{2}$

Q) What is the derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1} x$?

(a) 0 (b) $\frac{1}{2}$

(c) 1 (d) x

Ans: (b)

Q) Consider the curve $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$.

What is $\frac{dy}{dx}$ equal to ?

(a) $\tan\theta$ (b) $\cot\theta$

(c) $\sin 2\theta$ (d) $\cos 2\theta$

$$\frac{dy}{d\theta} = a\left(\cos\theta - (\theta(-\sin\theta) + \cos\theta(1))\right)$$

$$= a(\theta\sin\theta)$$

$$\frac{dx}{d\theta} = a\left(-\sin\theta + (\theta\cos\theta + \sin\theta)\right) = a\theta\cos\theta$$

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)}$$

$$= \frac{a\theta\sin\theta}{a\theta\cos\theta}$$

$$= \tan\theta$$

Q) Consider the curve $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$.

What is $\frac{dy}{dx}$ equal to ?

(a) $\tan\theta$ (b) $\cot\theta$

(c) $\sin 2\theta$ (d) $\cos 2\theta$

Ans: (a)

Q) What is $\dfrac{d^2y}{dx^2}$ equal to ?

(a) $\sec^2\theta$ (b) $-\operatorname{cosec}^2\theta$

(c) $\dfrac{\sec^3\theta}{a\theta}$ (d) None of these

$$\frac{dy}{dx} = \tan\theta$$

$$\frac{d^2y}{dx^2} = \frac{d}{d\theta}(\tan\theta)\cdot\frac{d\theta}{dx} = \sec^2\theta \times \frac{1}{\left(\frac{dx}{d\theta}\right)} = \sec^2\theta \times \frac{1}{a\theta\cos\theta}$$

$$\left(\frac{\sec^3\theta}{a\theta}\right)$$

Q) What is $\frac{d^2y}{dx^2}$ equal to ?

(a) $\sec^2\theta$

(b) $-\operatorname{cosec}^2\theta$

(c) $\frac{\sec^3\theta}{a\theta}$

(d) None of these

Ans: (c)

Q) If $y = \ell n\,(e^{mx} + e^{-mx})$, then what is $\frac{dy}{dx}$ at $x = 0$ equal to ?

(a) -1 (b) 0

(c) 1 (d) 2

$$y = \ln\left(e^{mx} + e^{-mx}\right)$$

$$\frac{dy}{dx} = \frac{1}{e^{mx} + e^{-mx}}\left(me^{mx} - me^{-mx}\right) = m\left(\frac{e^{mx} - e^{-mx}}{e^{mx} + e^{-mx}}\right)$$

$$(\text{For } x = 0) \Rightarrow m\left(\frac{0}{1+1}\right) = 0$$

Q) If $y = \ell n\,(e^{mx} + e^{-mx})$, then what is $\frac{dy}{dx}$ at $x = 0$ equal to ?

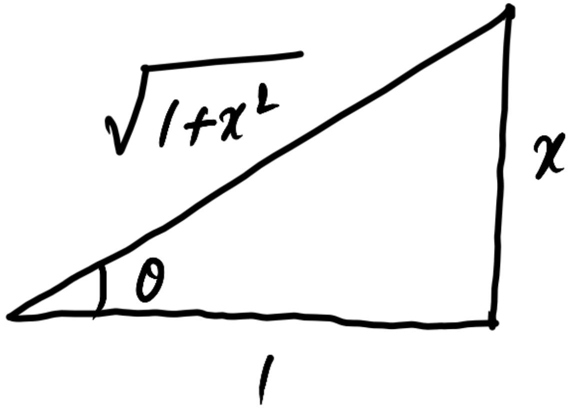
(a) -1	(b) 0
(c) 1	(d) 2

Ans: (b)

Q) If $y = \sec(\tan^{-1} x)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) 1 (d) $\sqrt{2}$

$$\sec\left(\sec^{-1}\left(\frac{\sqrt{1+x^2}}{1}\right)\right)$$



$$y = \sqrt{1+x^2}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1+x^2}} \frac{d}{dx}(1+x^2) = \frac{2x}{2\sqrt{1+x^2}} = \frac{x}{\sqrt{1+x^2}}$$

$$\frac{1}{\sqrt{1+1^2}} = \frac{1}{\sqrt{2}}$$

Q) If $y = \sec(\tan^{-1} x)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) 1 (d) $\sqrt{2}$

Ans: (a)

Q) What is the derivative of

$(\log_{\tan x} \cot x)(\log_{\cot x} \tan x)^{-1}$ at $x = \frac{\pi}{4}$?

(a) -1

(b) 0 ✓

(c) 1

(d) $\frac{1}{2}$

$$\left(\frac{\log_e \cot x}{\log_e \tan x}\right)\left(\frac{\log_e \tan x}{\log_e \cot x}\right)^{-1}$$

$$\frac{(\log_e \cot x)^2}{(\log_e \tan x)^2}$$

$$= \left(\frac{\log_e \frac{1}{\tan x}}{\log_e \tan x}\right)^2$$

$$= \left(\frac{-\log_e \tan x}{\log_e \tan x}\right)^2 = 1$$

differentiating $1 \longrightarrow 0$

$$\log_b a = \frac{\log_m a}{\log_m b}$$

$$\log \frac{1}{a} = -\log a$$

$$\log (a)^{-1}$$

$$(\log a^m = m \log a)$$

Q) What is the derivative of

$(\log_{\tan x} \cot x)(\log_{\cot x} \tan x)^{-1}$ at $x = \frac{\pi}{4}$?

(a) -1

(b) 0

(c) 1

(d) $\frac{1}{2}$

Ans: (b)

Q) If $f(1) = 1$, $f'(1) = 3$, then the derivative of $f(f(f(x))) + (f(x))^2$ at $x = 1$ is

(a) 12 (b) 9 (c) 15 (d) 33

$f'(f(f(x)))\, f'(f(x))\, f'(x) + 2f(x)f'(x)$

$3 \times 3 \times 3 + 2f(1)f'(1)$

$27 + 2 \times 1 \times 3$

$= 27 + 6 = 33$

Q) If $f(1) = 1$, $f'(1) = 3$, then the derivative of $f(f(f(x))) + (f(x))^2$ at $x = 1$ is

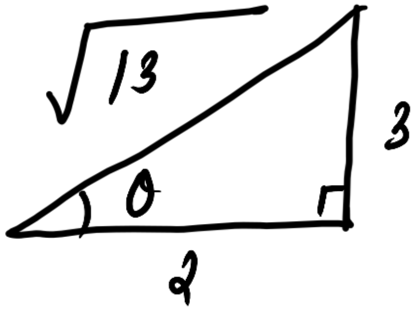
(a) 12 (b) 9 (c) 15 (d) 33

Ans: (d)

Q) What is the derivative of $\cos^{-1}\left(\dfrac{2\cos x + 3\sin x}{\sqrt{13}}\right)$?

(a) $\dfrac{1}{\sqrt{1-x^2}}$ (b) $-\dfrac{1}{\sqrt{1-x^2}}$

(c) 0 (d) 1 ✓



$$\cos^{-1}\left(\frac{2}{\sqrt{13}}\cos x + \frac{3}{\sqrt{13}}\sin x\right)$$

$$\cos\theta = \frac{2}{\sqrt{13}} \qquad \sin\theta = \frac{3}{\sqrt{13}}$$

$$\cos^{-1}(\cos\theta\cos x + \sin\theta\sin x) = \cos^{-1}(\cos(\theta - x)) = (\theta - x)$$

$$\frac{d}{dx}(\theta - x) = -1 \;/\; 1$$

$$= \cos^{-1}(\cos(x-\theta)) = x - \theta$$

Q) What is the derivative of $\cos^{-1}\left(\dfrac{2\cos x+3\sin x}{\sqrt{13}}\right)$?

(a) $\dfrac{1}{\sqrt{1-x^2}}$

(b) $-\dfrac{1}{\sqrt{1-x^2}}$

(c) 0

(d) 1

Ans: (d)

Q) If $f(x) = \cot^{-1}\left(\frac{x^x - x^{-x}}{2}\right)$, then $f'(1)$ is equal to

(a) -1 (b) 1

(c) $\log 2$ (d) $-\log 2$

$\cot^{-1} x = \tan^{-1}\left(\frac{1}{x}\right)$

$f(x) = \tan^{-1}\left(\frac{2}{x^x - x^{-x}}\right)$

$= \tan^{-1}\left(\frac{2x^x}{x^{2x} - 1}\right) = \tan^{-1}\left(\frac{-2x^x}{1 - x^{2x}}\right) = \tan^{-1}\left(\frac{-2\tan\theta}{1 - \tan^2\theta}\right)$

$x^x = \tan\theta$

$= \tan^{-1}(-\tan 2\theta)$

$= -\tan^{-1}(\tan 2\theta) = (-2\theta)$

NDA 1 2025 LIVE CLASS - MATHS - PART 3

$f(x) = -2\theta$

$\tan\theta = x^x$

$\theta = \tan^{-1}(x^x)$

$f(x) = -2\tan^{-1}(x^x)$

derivative

$\frac{d}{dx}(x^x) = x^x(1+\log x)$

$f'(x) = -2\left(\frac{1}{1+x^{2x}}\right)\frac{d}{dx}(x^x) = \frac{-2}{1+x^{2x}}\left(x^x(1+\log x)\right)$

$f'(1) = \frac{-2}{1+1^2}(1(1+0)) = \frac{-2}{2} = -1$

Q) If $f(x) = \cot^{-1}\left(\frac{x^x - x^{-x}}{2}\right)$, then $f'(1)$ is equal to

(a) -1 (b) 1

(c) $\log 2$ (d) $-\log 2$

Ans: (a)

SSBCrack EXAMS

Q) What is the derivative of $f(x) = x|x|$?

(a) $|x| + x$ (b) $2x$

(c) $2|x|$ (d) $-2|x|$

$|x| \xrightarrow{\text{derivative}} \frac{|x|}{x}$ or $\frac{x}{|x|}$

$$f'(x) = x\left(\frac{|x|}{x}\right) + |x|(1)$$

$$= 2|x|$$

Q) What is the derivative of $f(x) = x|x|$?

(a) $|x| + x$ (b) $2x$

(c) $2|x|$ (d) $-2|x|$

Ans: (c)

Q) If y is a function of x and $\log(x + y) = 2xy$, then the value of $y'(0)$ is

(a) 1 (b) -1 (c) 2 (d) 0

Q) If y is a function of x and $\log(x + y) = 2xy$, then the value of $y'(0)$ is

(a) 1 (b) –1 (c) 2 (d) 0

Ans: (a)

Q) If $x^2 + y^2 = 1$, then

(a) $yy'' - 2(y')^2 + 1 = 0$

(b) $yy'' + (y')^2 + 1 = 0$

(c) $yy'' + (y')^2 - 1 = 0$

(d) $yy'' + 2(y')^2 + 1 = 0$

SSBCrack EXAMS

Q) If $x^2 + y^2 = 1$, then

(a) $yy'' - 2(y')^2 + 1 = 0$

(b) $yy'' + (y')^2 + 1 = 0$

(c) $yy'' + (y')^2 - 1 = 0$

(d) $yy'' + 2(y')^2 + 1 = 0$

Ans: (d)

Q) What is the derivative of $\tan^{-1}\left(\dfrac{\sqrt{x}-x}{1+x^{3/2}}\right)$ at $x=1$?

(a) $-\dfrac{1}{4}$

(b) $\dfrac{1}{2}$

(c) $\dfrac{3}{2}$

(d) 1

Q) What is the derivative of $\tan^{-1}\left(\frac{\sqrt{x}-x}{1+x^{3/2}}\right)$ at $x = 1$?

(a) $-\frac{1}{4}$

(b) $\frac{1}{2}$

(c) $\frac{3}{2}$

(d) 1

Ans: (a)

Q) The derivative of $y = a^{x \log_a \sin x}$ is equal to

(a) $\log \sin x + x \tan x$

(b) $\log \sin x + x \cot x$

(c) $y \log (\sin x \, e^{x \cot x})$

(d) $y \log (\sin x \, e^{x \tan x})$

Q) The derivative of $y = a^{x \log_a \sin x}$ is equal to

(a) $\log \sin x + x \tan x$ (b) $\log \sin x + x \cot x$

(c) $y \log (\sin x \, e^{x \cot x})$ (d) $y \log (\sin x \, e^{x \tan x})$

Ans: (c)

Q) What is the derivative of $2^{(\sin x)^2}$ with respect to $\sin x$?

(a) $\sin x\, 2^{(\sin x)^2} \ln 4$

(b) $2 \sin x\, 2^{(\sin x)^2} \ln 4$

(c) $\ln (\sin x)\, 2^{(\sin x)^2}$

(d) $2 \sin x \cos x\, 2^{(\sin x)^2}$

Q) What is the derivative of $2^{(\sin x)^2}$ with respect to sin x?

(a) $\sin x \, 2^{(\sin x)^2} \ln 4$

(b) $2 \sin x \, 2^{(\sin x)^2} \ln 4$

(c) $\ln (\sin x) \, 2^{(\sin x)^2}$

(d) $2 \sin x \cos x \, 2^{(\sin x)^2}$

Ans: (a)

Q) The derivative of ln(x+ sin x) with respect to (x+ cos x) is

(a) $\dfrac{1+\cos x}{(x+\sin x)(1-\sin x)}$

(b) $\dfrac{1-\cos x}{(x+\sin x)(1+\sin x)}$

(c) $\dfrac{1-\cos x}{(x-\sin x)(1+\cos x)}$

(d) $\dfrac{1+\cos x}{(x-\sin x)(1-\cos x)}$

Q) The derivative of ln(x+ sin x) with respect to (x+ cos x) is

(a) $\dfrac{1+\cos x}{(x+\sin x)(1-\sin x)}$

(b) $\dfrac{1-\cos x}{(x+\sin x)(1+\sin x)}$

(c) $\dfrac{1-\cos x}{(x-\sin x)(1+\cos x)}$

(d) $\dfrac{1+\cos x}{(x-\sin x)(1-\cos x)}$

Ans: (a)

Q) If $y = \cot^{-1}\left[\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$, where $0 < x < \dfrac{\pi}{2}$, then $\dfrac{dy}{dx}$ is equal to

(a) $\dfrac{1}{2}$

(b) 2

(c) $\sin x + \cos x$

(d) $\sin x - \cos x$

Q) If $y = \cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$, where $0 < x < \frac{\pi}{2}$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{1}{2}$ (b) 2

(c) $\sin x + \cos x$ (d) $\sin x - \cos x$

Ans: (a)

Q) If $y = \tan^{-1}\left(\frac{5 - 2\tan\sqrt{x}}{2 + 5\tan\sqrt{x}}\right)$, then what is $\frac{dy}{dx}$ equal to?

(a) $-\frac{1}{2\sqrt{x}}$

(b) 1

(c) -1

(d) $\frac{1}{2\sqrt{x}}$

Q) If $y = \tan^{-1}\left(\frac{5 - 2\tan\sqrt{x}}{2 + 5\tan\sqrt{x}}\right)$, then what is $\frac{dy}{dx}$ equal to?

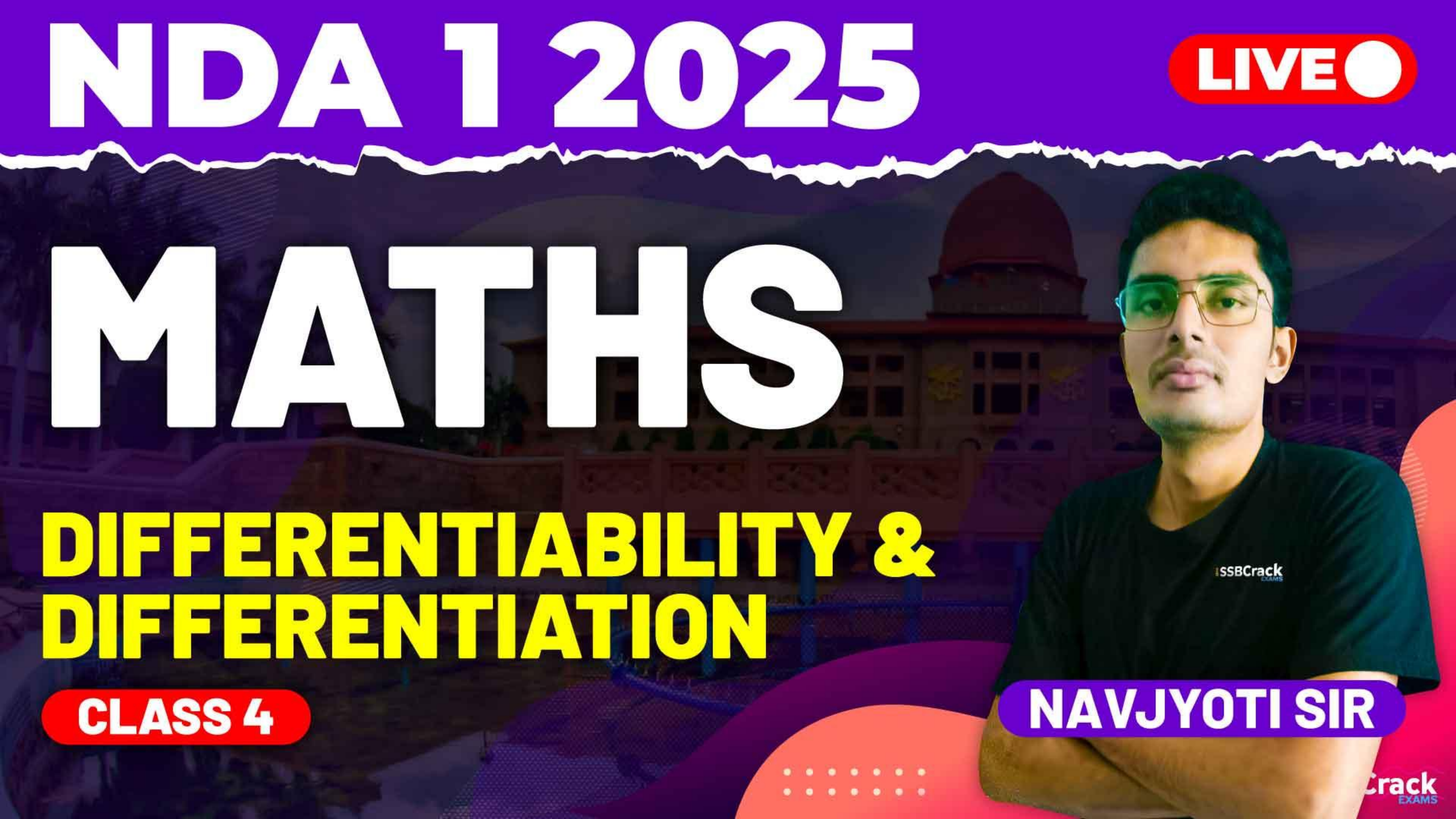
(a) $-\frac{1}{2\sqrt{x}}$

(b) 1

(c) -1

(d) $\frac{1}{2\sqrt{x}}$

Ans: (a)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