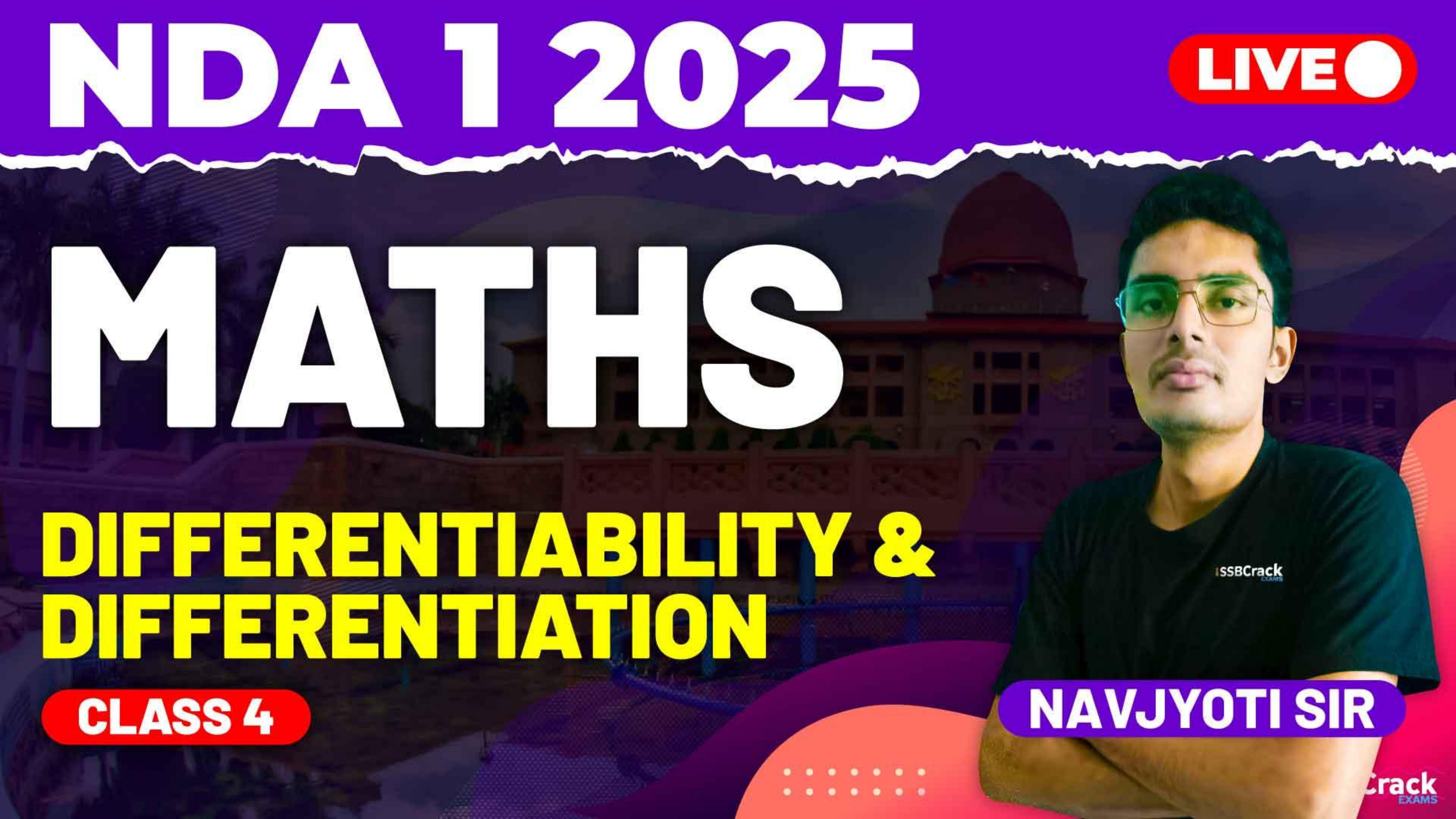

NDA 1 2025
LIVE
MATHS
DIFFERENTIABILITY &
DIFFERENTIATION
CLASS 4
NAVJYOTI SIR
SSBCrack EXAMS

09 Dec 2024 Live Classes Schedule

8:00AM	09 DEC 2024 DAILY CURRENT AFFAIRS	RUBY MA'AM
9:00AM	09 DEC 2024 DAILY DEFENCE UPDATES	DIVYANSHU SIR

NDA 1 2025 LIVE CLASSES

1:00PM	PHYSICS - SOUND	NAVJYOTI SIR
5:30PM	MATHS - DIFFERENTIABILITY & DIFFERENTIATION - CLASS 4	NAVJYOTI SIR

CDS 1 2025 LIVE CLASSES

1:00PM	PHYSICS - SOUND	NAVJYOTI SIR
7:00PM	MATHS - ALGEBRA - CLASS 4	NAVJYOTI SIR

Q) If $y = \sin(ax + b)$, then what is $\frac{d^2y}{dx^2}$ at $x = -\frac{b}{a}$, where a, b are constants and $a \neq 0$?

(a) 0 (b) –1

(c) $\sin(a - b)$ (d) $\sin(a + b)$

$y = \sin(ax + b)$

$\frac{dy}{dx} = \cos(ax+b) \cdot \frac{d}{dx}(ax+b)$

$= a\cos(ax+b)$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(a\cos(ax+b)\right) = a(-\sin(ax+b)) \cdot a$

$\frac{d^2y}{dx^2} = -a^2\sin(ax+b)$

$= -a^2\sin\left(a\left(\frac{-b}{a}\right) + b\right)$

$= -a^2\sin(-b+b) = -a^2\sin(0)$

$= -a^2(0) = 0$

Q) If $y = \sin(ax + b)$, then what is $\frac{d^2y}{dx^2}$ at $x = -\frac{b}{a}$, where a, b are constants and $a \neq 0$?

(a) 0 (b) –1

(c) $\sin(a - b)$ (d) $\sin(a + b)$

Ans: (a)

SSBCrack EXAMS

Q) Consider the following statements :

1. If $y = \ell n(\sec x + \tan x)$, then $\frac{dy}{dx} = \sec x$. ✓

2. If $y = \ell n(\text{cosec}\, x - \cot x)$, then $\frac{dy}{dx} = \text{cosec}\, x$. ✓

Which of the above is/are correct?

(a) 1 only (b) 2 only

(c) Both 1 and 2 ✓ (d) Neither 1 nor 2

① $\frac{dy}{dx} = \frac{1}{\sec x + \tan x}(\sec x \tan x + \sec^2 x) = \sec x$

② $\frac{dy}{dx} = \frac{1}{\text{cosec} x - \cot x}(-\text{cosec} x \cot x + \text{cosec}^2 x) = \text{cosec} x$

Q) Consider the following statements :

1. If $y = \ln(\sec x + \tan x)$, then $\frac{dy}{dx} = \sec x$.

2. If $y = \ln(\operatorname{cosec} x - \cot x)$, then $\frac{dy}{dx} = \operatorname{cosec} x$.

Which of the above is/are correct?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

Ans: (c)

SSBCrack EXAMS

Q) If $x^m + y^m = 1$ such that $\frac{dy}{dx} = -\frac{x}{y}$, then what should be the value of m ?

(a) 0 (b) 1

(c) 2 (d) None of the above

$$mx^{m-1} + my^{m-1}\left(\frac{dy}{dx}\right) = 0$$

$$\frac{dy}{dx} = -\frac{mx^{m-1}}{my^{m-1}} = -\frac{x^1}{y^1}$$

$$m-1 = 1$$

$$m = 2$$

Q) If $x^m + y^m = 1$ such that $\frac{dy}{dx} = -\frac{x}{y}$, then what should be the value of m ?

(a) 0 (b) 1

(c) 2 (d) None of the above

Ans: (c)

SSBCrack EXAMS

Q) If $y = x^x$, what is $\frac{dy}{dx}$ at $x = 1$ equal to ?

(a) 0 (b) 1

(c) -1 (d) 2

$$\frac{dy}{dx} = x^x \frac{d}{dx}(x \log x)$$

$$= x^x\left(\log x + x \cdot \frac{1}{x}\right)$$

$$= x^x(\log x + 1)$$

$x = 1$,

$$\frac{dy}{dx} = 1^1(\log(1) + 1)$$

$$= 1^1(0 + 1)$$

$$= 1$$

Q) If $y = x^x$, what is $\frac{dy}{dx}$ at $x = 1$ equal to ?

(a) 0
(b) 1
(c) -1
(d) 2

Ans: (b)

Q) What is the derivative of $\tan^{-1}\left(\frac{\sqrt{x}-x}{1+x^{3/2}}\right)$ at $x=1$?

(a) $-\frac{1}{4}$ (b) $\frac{1}{2}$

(c) $\frac{3}{2}$ (d) 1

$$\tan^{-1}\left(\frac{\sqrt{x}-x}{1+x\sqrt{x}}\right)$$

$$= \tan^{-1}\sqrt{x} - \tan x$$

derivative

$$\frac{1}{1+(\sqrt{x})^2}\cdot\frac{1}{2\sqrt{x}} - \frac{1}{1+x^2}$$

At $x=1$,

$$\frac{1}{2}\times\frac{1}{2} - \frac{1}{2} = \frac{1}{4} - \frac{1}{2}$$

$$= -\frac{1}{4}$$

Q) What is the derivative of $\tan^{-1}\left(\frac{\sqrt{x}-x}{1+x^{3/2}}\right)$ at $x=1$?

(a) $-\frac{1}{4}$

(b) $\frac{1}{2}$

(c) $\frac{3}{2}$

(d) 1

Ans: (a)

Q) If $y = f(x)$, $p = \frac{dy}{dx}$ and $q = \frac{d^2y}{dx^2}$, then what is $\frac{d^2x}{dy^2}$ equal to ?

(a) $-\frac{q}{p^2}$ (b) $-\frac{q}{p^3}$

(c) $\frac{1}{q}$ (d) $\frac{q}{p^2}$

$$\frac{d^2y}{dx^2} = \frac{dp}{dx} = q$$

$$\frac{dp}{dy} \times \frac{dy}{dx} = q$$

$$p\frac{dp}{dy} = q$$

$$\frac{dp}{dy} = \frac{q}{p}$$

$$\frac{dx}{dy} = \frac{1}{p}$$

$$\frac{d^2x}{dy^2} = \frac{d}{dy}\left(\frac{1}{p}\right)$$

$$\frac{d^2x}{dy^2} = -\frac{1}{p^2} \times \frac{dp}{dy}$$

$$= -\frac{1}{p^2} \times \frac{q}{p} = -\frac{q}{p^3}$$

SSBCrack EXAMS

Q) If $y = f(x)$, $p = \frac{dy}{dx}$ and $q = \frac{d^2y}{dx^2}$, then what is $\frac{d^2x}{dy^2}$ equal to ?

(a) $-\frac{q}{p^2}$

(b) $-\frac{q}{p^3}$

(c) $\frac{1}{q}$

(d) $\frac{q}{p^2}$

Ans: (b)

SSBCrack EXAMS

Q) If $y = \sin^{-1}x + \sin^{-1}\sqrt{1-x^2}$, what is $\frac{dy}{dx}$ equal to ?

(a) $\cos^{-1}x + \cos^{-1}\sqrt{1-x^2}$ (b) $\frac{1}{\cos x} + \frac{1}{\cos\sqrt{1-x^2}}$

(c) $\frac{\pi}{2}$ (d) 0

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-\left(\sqrt{1-x^2}\right)^2}}\frac{d}{dx}\left(\sqrt{1-x^2}\right)$$

$$= \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{x^2}} \times \frac{1}{2\sqrt{1-x^2}}(-2x)$$

$$= \frac{1}{\sqrt{1-x^2}} + \frac{1}{x} \times \frac{-x}{\sqrt{1-x^2}}$$

$$= \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}}$$

$$= 0$$

Q) If $y = \sin^{-1}x + \sin^{-1}\sqrt{1-x^2}$, what is $\frac{dy}{dx}$ equal to ?

(a) $\cos^{-1}x + \cos^{-1}\sqrt{1-x^2}$ (b) $\frac{1}{\cos x} + \frac{1}{\cos\sqrt{1-x^2}}$

(c) $\frac{\pi}{2}$ (d) 0

Ans: (d)

Q) If $f(x)=\sqrt{x+\sqrt{x+\sqrt{x+\sqrt{\ldots\infty}}}}$, then what is the value of $f'(x)$?

(a) $\dfrac{1}{1-2f(x)}$

(b) $\dfrac{1}{2f(x)-1}$

(c) $\dfrac{1}{1+2f(x)}$

(d) $\dfrac{1}{2+f(x)}$

$$f(x)=\sqrt{x+f(x)}$$

$$(f(x))^2 = x+f(x)$$

Taking derivative both sides,

$$2f(x)\cdot f'(x) = 1+f'(x)$$

$$f'(x)=\frac{1}{2f(x)-1}$$

Q) If $f(x) = \sqrt{x + \sqrt{x + \sqrt{x + \sqrt{\ldots \infty}}}}$, then what is the value of $f'(x)$?

(a) $\dfrac{1}{1 - 2f(x)}$

(b) $\dfrac{1}{2f(x) - 1}$

(c) $\dfrac{1}{1 + 2f(x)}$

(d) $\dfrac{1}{2 + f(x)}$

Ans: (b)

Q) If $y = \tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$, then $\frac{dy}{dx}$ is equal to

(a) 2 (b) -1 (c) $\frac{a}{b}$ (d) $\frac{b}{a}$

Divide by $b\cos x$,

$$y = \tan^{-1}\left(\frac{\frac{a}{b} - \tan x}{1 + \frac{a}{b}\tan x}\right)$$

$\frac{a}{b} = \tan\theta$

$$= \tan^{-1}\left(\frac{\tan\theta - \tan x}{1 + \tan\theta\tan x}\right) = \tan^{-1}(\tan(\theta - x))$$

$= \theta - x$

$= \tan^{-1}\left(\frac{a}{b}\right) - x$

derivative,

$= 0 - 1 = -1$

Q) If $y = \tan^{-1}\left(\dfrac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$, then $\dfrac{dy}{dx}$ is equal to

(a) 2 (b) -1 (c) $\dfrac{a}{b}$ (d) $\dfrac{b}{a}$

Ans: (b)

Q) If $f(x) = \cot^{-1}\left(\frac{x^x - x^{-x}}{2}\right)$, then $f'(1)$ is equal to

(a) -1 (b) 1

(c) $\log 2$ (d) $-\log 2$

$$f(x) = \tan^{-1}\left(\frac{2}{x^x - x^{-x}}\right)$$

$$= \tan^{-1}\left(\frac{2x^x}{x^{2x} - 1}\right) = \tan^{-1}\left(-\frac{2x^x}{1 - x^{2x}}\right)$$

Let $x^x = \tan\theta$

$$f(x) = \tan^{-1}\left(-\frac{2\tan\theta}{1-\tan^2\theta}\right)$$

$\tan^{-1}(-x) = -\tan^{-1}(x)$

$$= \tan^{-1}(-\tan 2\theta)$$

$$f(x) = -2\theta = -2\tan^{-1}(x^x)$$

$$f(x) = -2\theta = -2\tan^{-1}(x^x)$$

$$f'(x) = -2\left(\frac{1}{1+x^{2x}}\right)(x^x)(1+\log x)$$

$$f'(1) = -2\left(\frac{1}{1+1^2}\right)(1^1)(1+\log 1)$$

$$= -2 \times \frac{1}{2} \times 1$$

$$= -1$$

Q) If $f(x) = \cot^{-1}\left(\frac{x^x - x^{-x}}{2}\right)$, then $f'(1)$ is equal to

(a) -1

(b) 1

(c) $\log 2$

(d) $-\log 2$

Ans: (a)

Q) What is the derivative of $\sin^{-1}\left(\frac{t}{\sqrt{1+t^2}}\right)$ *wrt* $\cos^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right)$?

(a) 1 (b) –1

(c) 2 (d) –2

$u = f(t)$; $v = g(t)$

$$\frac{du}{dv} = \frac{\left(\frac{du}{dt}\right)}{\left(\frac{dv}{dt}\right)}$$

$$u = \sin^{-1}\left(\frac{t}{\sqrt{1+t^2}}\right)$$

$t = \tan\theta$

$$u = \sin^{-1}\left(\frac{\tan\theta}{\sec\theta}\right) = \sin^{-1}(\sin\theta)$$

$u = \theta = \tan^{-1}t$

$$\frac{du}{dt} = \frac{1}{1+t^2}$$

$$v = \cos^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right)$$

$$t = \tan\theta$$

$$v = \cos^{-1}(\cos\theta) = \theta = \tan^{-1} t$$

$$\frac{du}{dv} = \frac{\frac{d}{dt}(\tan^{-1} t)}{\frac{d}{dt}(\tan^{-1} t)} = 1$$

Q) What is the derivative of $\sin^{-1}\left(\frac{t}{\sqrt{1+t^2}}\right)$ *wrt* $\cos^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right)$?

(a) 1 (b) –1

(c) 2 (d) –2

Ans: (a)

Q) If $y = \log_{10} x + \log_x 10 + \log_x x + \log_{10} 10$ then what is $\left(\frac{dy}{dx}\right)_{x=10}$ equal to?

(a) 10 (b) 2

(c) 1 (d) 0

$\log_a a = 1$

$y = \log_{10} x + \log_x 10 + 1 + 1$

$y = \log_{10} x + \frac{1}{\log_{10} x} + 2$

$y = \log_{10} x + (\log_{10} x)^{-1} + 2 = \log_{10} x - \log_{10} x + 2 = 2$

$y = 2$

$\frac{dy}{dx} = 0$ (at $x = 10$ also.)

Q) If $y = \log_{10} x + \log_x 10 + \log_x x + \log_{10} 10$ then what is $\left(\frac{dy}{dx}\right)_{x=10}$ equal to?

(a) 10 (b) 2

(c) 1 (d) 0

Ans: (d)

Q) If $y = (\cos x)^{(\cos x)^{(\cos x)^{\infty}}}$, then $\frac{dy}{dx}$ is equal to

(a) $-\frac{y^2 \tan x}{1 - y\ln(\cos x)}$ ✓

(b) $\frac{y^2 \tan x}{1 + y\ln(\cos x)}$

(c) $\frac{y^2 \tan x}{1 - y\ln(\sin x)}$

(d) $\frac{y^2 \sin x}{1 + y\ln(\sin x)}$

$$y = (\cos x)^y$$

$$\log y = y \log \cos x$$

$$\frac{1}{y}\frac{dy}{dx} = y\left(\frac{1}{\cos x}\right)(-\sin x) + (\log \cos x)\frac{dy}{dx}$$

$$\frac{dy}{dx} = -y^2\frac{\sin x}{\cos x} + y\frac{dy}{dx}(\log \cos x)$$

$$\frac{dy}{dx} = \frac{-y^2 \tan x}{1 - y\ln \cos x}$$

Q) If $y = (\cos x)^{(\cos x)^{(\cos x)\infty}}$, then $\dfrac{dy}{dx}$ is equal to

(a) $-\dfrac{y^2 \tan x}{1 - y\ln(\cos x)}$

(b) $\dfrac{y^2 \tan x}{1 + y\ln(\cos x)}$

(c) $\dfrac{y^2 \tan x}{1 - y\ln(\sin x)}$

(d) $\dfrac{y^2 \sin x}{1 + y\ln(\sin x)}$

Ans: (a)

Q) Consider the following statements :

1. Derivative of $f(x)$ may not exist at some point.
2. Derivative of $f(x)$ may exist finitely at some point.
3. Derivative of $f(x)$ may be infinite (geometrically) at some point.

Which of the above statements are correct?

(a) 1 and 2 only
(b) 2 and 3 only
(c) 1 and 3 only
(d) 1, 2 and 3 ✓

SSBCrack EXAMS

Q) Consider the following statements :

1. Derivative of $f(x)$ may not exist at some point.
2. Derivative of $f(x)$ may exist finitely at some point.
3. Derivative of $f(x)$ may be infinite (geometrically) at some point.

Which of the above statements are correct?

(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

Ans: (d)

Q) The set of all points, where the function $f(x) = \sqrt{1 - e^{-x^2}}$ is differentiable, is

(a) $(0, \infty)$ (b) $(-\infty, \infty)$ (c) $(-\infty, 0) \cup (0, \infty)$ (d) $(-1, \infty)$

$$f'(x) = \frac{1}{2\sqrt{1-e^{-x^2}}}\left(0 - e^{-x^2}(-2x)\right)$$

$$= \frac{2e^{-x^2}}{2\sqrt{1-e^{-x^2}}} = \frac{e^{-x^2}}{\sqrt{1-e^{-x^2}}}$$

(Finding domain of $f'(x)$)

$1 - e^{-x^2} > 0 \ \forall \ x > 0$, and $x < 0$

For $x = 0$, $f'(x)$ is not defined $\left(\frac{1}{0}\right)$.

SSBCrack EXAMS

Q) The set of all points, where the function $f(x) = \sqrt{1 - e^{-x^2}}$ is differentiable, is

(a) $(0, \infty)$ (b) $(-\infty, \infty)$ (c) $(-\infty, 0) \cup (0, \infty)$ (d) $(-1, \infty)$

Ans: (c)

SSBCrack EXAMS

Q) If $u = \sin^{-1}(x - y), x = 3t, y = 4t^3$, then what is the derivative of u wrt t?

(a) $3(1 - t^2)$

(b) $3(1 - t^2)^{-\frac{1}{2}}$

(c) $5(1 - t^2)^{\frac{1}{2}}$

(d) $5(1 - t^2)$

$u = \sin^{-1}(3t - 4t^3)$

$= \sin^{-1}(3\sin\theta - 4\sin^3\theta)$

$= \sin^{-1}(\sin 3\theta)$

$u = 3\theta = 3\sin^{-1} t$

$t = \sin\theta$

$\theta = \sin^{-1} t$

$\frac{du}{dt} = 3 \cdot \frac{1}{\sqrt{1 - t^2}} = 3(1 - t^2)^{\frac{1}{2}}$

Q) If $u = \sin^{-1}(x - y)$, $x = 3t$, $y = 4t^3$, then what is the derivative of u wrt t?

(a) $3(1 - t^2)$

(b) $3(1 - t^2)^{-\frac{1}{2}}$

(c) $5(1 - t^2)^{\frac{1}{2}}$

(d) $5(1 - t^2)$

Ans: (b)

Q) What is the derivative of $\tan^{-1} x$ with respect to $\cot^{-1} x$?

(a) -1

(b) 1

(c) $\frac{1}{x^2+1}$

(d) $\frac{x}{x^2+1}$

$$\frac{\frac{1}{1+x^2}}{\frac{-1}{1+x^2}} = -1$$

Q) What is the derivative of $\tan^{-1} x$ with respect to $\cot^{-1} x$?

(a) -1

(b) 1

(c) $\frac{1}{x^2+1}$

(d) $\frac{x}{x^2+1}$

Ans: (a)

Q) If $x^2 + y^2 = t + \frac{1}{t}$, $x^4 + y^4 = t^2 + \frac{1}{t^2}$, then what is the value of $-x^3 y \frac{dy}{dx}$?

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) 1

$$\left(t + \frac{1}{t}\right)^2 - 2 = x^4 + y^4$$

$$(x^2+y^2)^2 - 2 = x^4 + y^4$$

$$2x^2y^2 - 2 = 0$$

$$x^2y^2 = 1$$

$$y^2 = \frac{1}{x^2}$$

$$x^2 = 1 \Rightarrow x = -1, 1$$

$$y^2 = 1 \Rightarrow y = -1, 1$$

$$2y\frac{dy}{dx} = \left(-\frac{1}{x^4}\right)(2x)$$

$$\frac{dy}{dx} = -\frac{1}{x^3 y}$$

$$-x^3 y \frac{dy}{dx} = 1$$

Q) If $x^2 + y^2 = t + \frac{1}{t}$, $x^4 + y^4 = t^2 + \frac{1}{t^2}$, then what is the value of $-x^3 y \frac{dy}{dx}$?

(a) $\frac{1}{4}$

(b) $\frac{1}{3}$

(c) $\frac{1}{2}$

(d) 1

Ans: (d)

Q) If $f(x) = e^x$ and $g(x) = \log x$, then what is the value of $(gof)'(x)$?

(a) 0 (b) 1

(c) e (d) None of these

Q) If $f(x) = e^x$ and $g(x) = \log x$, then what is the value of $(g o f)'(x)$?

(a) 0

(b) 1

(c) e

(d) None of these

Ans: (b)

SSBCrack EXAMS

PYQ – 2024 - I

Let $f(x)$ and $g(x)$ be two functions such that

$g(x) = x - \frac{1}{x}$ and $f \circ g(x) = x^3 - \frac{1}{x^3}$.

What is $g[f(x) - 3x]$ equal to ?

(a) $x^3 - \frac{1}{x^3}$ ✓

(b) $x^3 + \frac{1}{x^3}$

(c) $x^2 - \frac{1}{x^2}$

(d) $x^2 + \frac{1}{x^2}$

$$f\left(x - \frac{1}{x}\right) = x^3 - \frac{1}{x^3}$$

$$= \left(x - \frac{1}{x}\right)^3 + 3\left(x - \frac{1}{x}\right)$$

$$f(x) = x^3 + 3x$$

$$g\left[x^3 + 3x - 3x\right] = g\left[x^3\right] = x^3 - \frac{1}{x^3}$$

SSBCrack EXAMS

PYQ – 2024 - I

Let $f(x)$ and $g(x)$ be two functions such that

$g(x) = x - \frac{1}{x}$ and $f \circ g(x) = x^3 - \frac{1}{x^3}$.

What is $g[f(x) - 3x]$ equal to ?

(a) $x^3 - \frac{1}{x^3}$

(b) $x^3 + \frac{1}{x^3}$

(c) $x^2 - \frac{1}{x^2}$

(d) $x^2 + \frac{1}{x^2}$

Ans: (a)

Let $f(x)$ and $g(x)$ be two functions such that

$g(x) = x - \frac{1}{x}$ and $f \circ g(x) = x^3 - \frac{1}{x^3}$.

What is $f''(x)$ equal to ?

(a) $-\frac{2}{x^3}$

(b) $2x + \frac{2}{x^3}$

(c) $6x + 3$

(d) $6x$

PYQ – 2024 - I

$f(x) = x^3 + 3x$

$f'(x) = 3x^2 + 3$

$f''(x) = 6x + 0$

$f''(x) = 6x$

PYQ – 2024 - I

Let $f(x)$ and $g(x)$ be two functions such that

$g(x) = x - \frac{1}{x}$ and $f \circ g(x) = x^3 - \frac{1}{x^3}$.

What is $f''(x)$ equal to ?

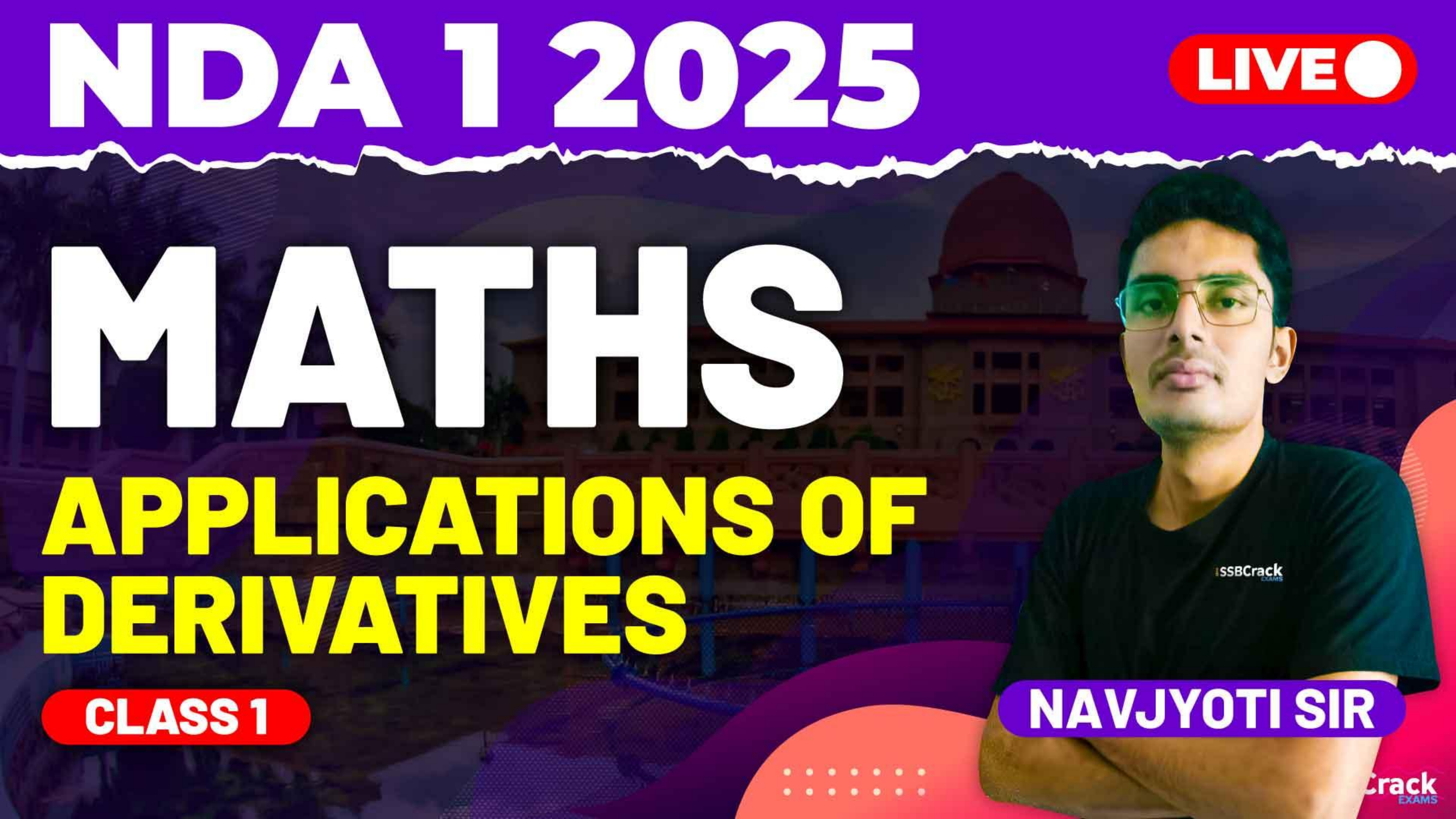
(a) $-\frac{2}{x^3}$

(b) $2x + \frac{2}{x^3}$

(c) $6x + 3$

(d) $6x$

Ans: (d)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CLASS 1
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