NDA 1 2025

LIVE

MATHS

DIFFERENTIAL EQUATIONS

CLASS 1

NAVJYOTI SIR

Crack
EXAMS

DIFFERENTIAL EQUATION

An equation involving derivative (derivatives) of the dependent variable (y) with respect to independent variable (variables) (x) is called a differential equation.

$$\frac{dy}{dx} + 2xy = x^3$$

Order = 1

$$\left(\frac{d^2y}{dx^2}\right) - 5\frac{dy}{dx} + 6y = x^2$$

Order = 2

dy – differential coefficient of y

dx – " " " x

ORDER OF DIFFERENTIAL EQUATION

Order of a differential equation is the order of the highest order derivative occurring in the differential equation.

$$\left(\frac{dy}{dx}\right)^2 = 1 + \frac{d^2y}{dx^2} \longrightarrow \text{Order} = 2$$

maximum 2 times y is being taken derivative.

DEGREE OF DIFFERENTIAL EQUATION

Degree of a differential equation is defined if it is a polynomial equation in its derivatives.

Degree (when defined) of a differential equation is the highest power (positive integer only) of the highest order derivative in it.

$\left(\frac{dy}{dx}\right)^2 = 1 + \left(\frac{d^2y}{dx^2}\right)^1$ → degree = 1

powers are whole numbers ⇒ it is polynomial eqn.

$(a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots) = 0$

(powers of x are whole numbers)

$\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$, $\frac{d^3y}{dx^3}$ etc.

x → derivative term

for a diff. eqn.,

(*) $\left(\frac{dy}{dx}\right)^{5/2} - 2x = 3\frac{d^2y}{dx^2}$

$\left(\frac{dy}{dx}\right)^{5/2} = 3\frac{d^2y}{dx^2} + 2x$

squaring both sides,

$\left(\frac{dy}{dx}\right)^{5} = \left(3\frac{d^2y}{dx^2}\right)^2 + 2(2x)3\frac{d^2y}{dx^2} + (2x)^2$

degree = 2

$\left(\frac{dy}{dx}\right)^{5} = 9\left(\frac{d^2y}{dx^2}\right)^2 + 12x\frac{d^2y}{dx^2} + 4x^2$

polynomial in derivative terms

QUESTION

The order and degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^2=\frac{d^2y}{dx^2}$ respectively, are

(A) 1, 2 (B) 2, 2 (C) 2, 1 (D) 4, 2

Order = 2

Ans. (C)

QUESTION

The degree of the differential equation

$$\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2 \log\left(\frac{d^2y}{dx^2}\right) \text{ is}$$

(A) 1 (B) 2 (C) 3 (D) not defined

degree is not defined.

$\log x$ is a series, which contains different powers of x,

FORMING A DIFFERENTIAL EQUATION

To form a differential equation from a given function, we differentiate the function successively as many times as the number of arbitrary constants in the given function and then eliminate the arbitrary constants.

{ Order of diff. eqn. = number of arbitrary constants }

QUESTION

Find the differential equation of the family of curves $y = Ae^{2x} + B.e^{-2x}$.

2 arbitrary constants.

$$\frac{dy}{dx} = Ae^{2x}(2) + Be^{-2x}(-2)$$

$$= 2Ae^{2x} - 2Be^{-2x}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = 2A(2e^{2x}) - 2B(-2Be^{-2x})$$

$$\frac{d^2y}{dx^2} = 4Ae^{2x} + 4Be^{-2x}$$

$$\frac{d^2y}{dx^2} = 4(Ae^{2x} + Be^{-2x}) = 4y$$

$$\frac{d^2y}{dx^2} - 4y = 0$$

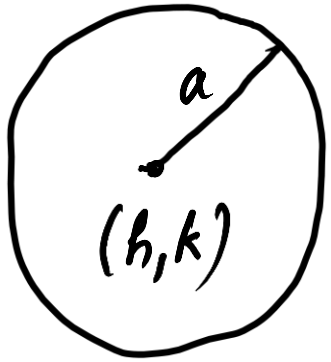
order = 2 = number of (A, B) arbitrary constants

QUESTION

The order of the differential equation of all circles of given radius a is:

(A) 1 (B) 2 (C) 3 (D) 4

eqn of circle having radius 'a' $\Rightarrow (x-h)^2 + (y-k)^2 = a^2$



arbitrary constant.

$= 2$

Order of diff. eqn. = (2)

QUESTION

Order of the differential equation representing the family of parabolas $y^2 = 4ax$ is _____ .

$y^2 = 4ax$

arbitrary constant = 1 = order of diff. eqn.

Ans. 1 (one)

SOLUTION OF A DIFFERENTIAL EQUATION

A relation between involved variables, which satisfy the given differential equation is called its solution.

→ The solution is a function containing y and x.

General Solution

→ contains arbitrary constants (ac)

→

Particular Solution

→ does not contains ac.

→ we find the values of ac.

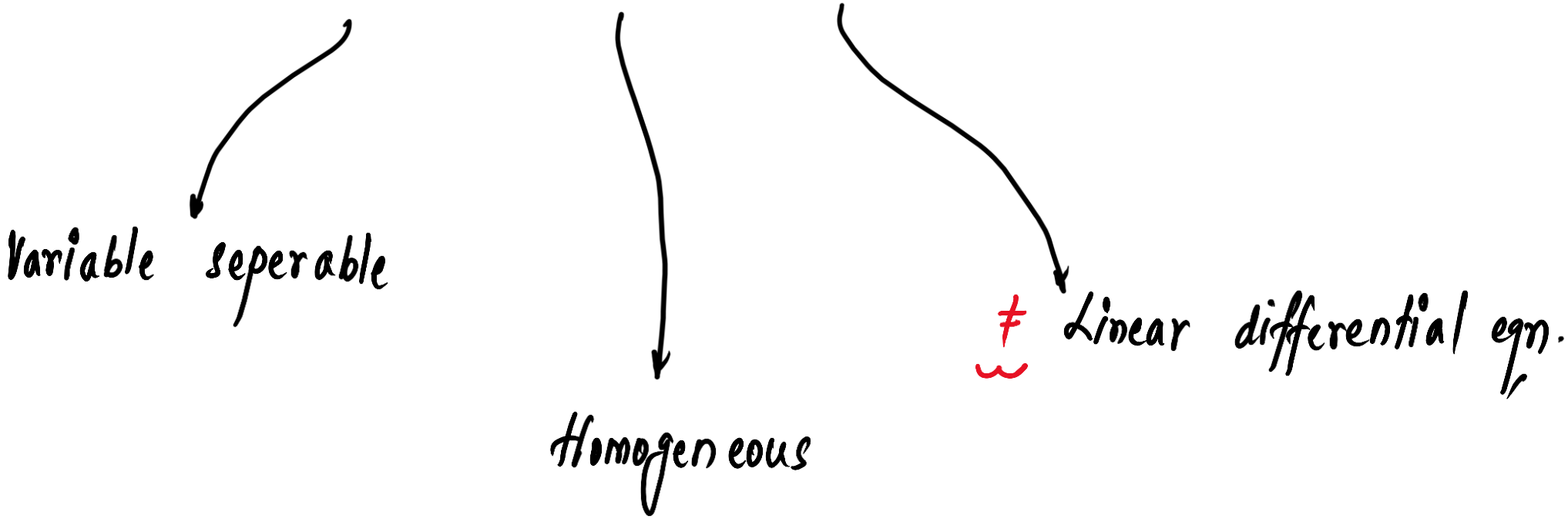
QUESTION

The number of arbitrary constants in a particular solution of the differential equation $\tan x\, dx + \tan y\, dy = 0$ is ____0____ .

no arbitrary constant,

Ans. Zero (0)

METHODS TO SOLVE DIFFERENTIAL EQUATION



QUESTION

Given that $\frac{dy}{dx} = ye^x$ and $x = 0, y = e$. Find the value of y when $x = 1$.

$$\frac{dy}{dx} = ye^x$$

$$\frac{1}{y}dy = e^x dx$$

$$\int \frac{1}{y}dy = \int e^x dx$$

$$\log y = e^x + \log c$$

$$\log y - \log c = e^x$$

$$\log\left(\frac{y}{c}\right) = e^x$$

$$\frac{y}{c} = e^{e^x} \quad \text{(General solution)}$$

$$y = ce^{e^x}$$

$x = 0$; $y = e$

$$e = ce^{e^0} \Rightarrow c = 1$$

$$y = 1 \cdot e^{e^x} \Rightarrow y = e^{e^x}$$

$y = e^{e^x}$ ⟶ particular solution

$x = 1$; $y = ?$

$y = e^{e^1}$

$y = e^e$

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