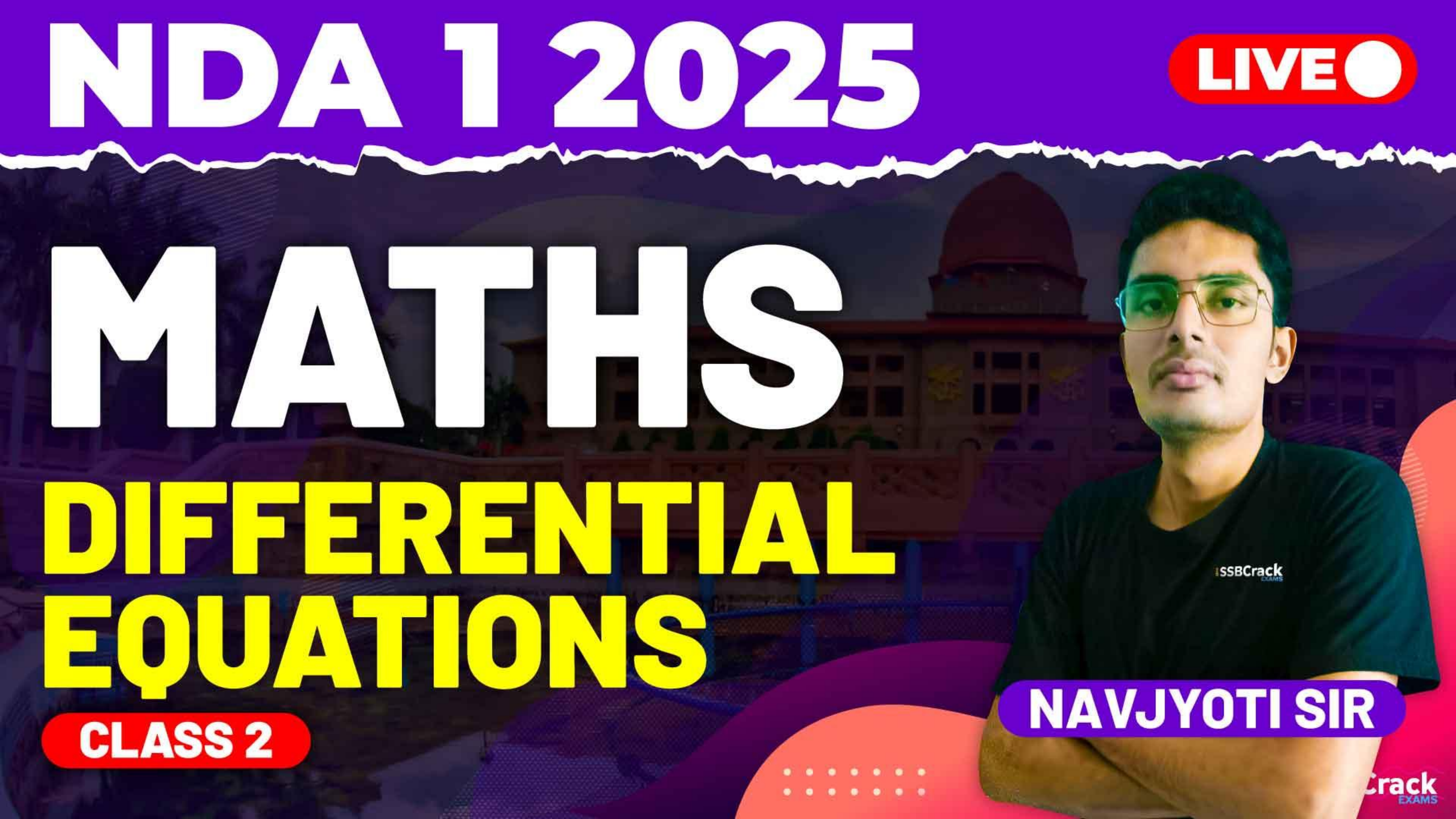
NDA 1 2025
LIVE
MATHS
DIFFERENTIAL
EQUATIONS
CLASS 2
SSBCrack EXAMS
NAVJYOTI SIR
Crack EXAMS

METHODS TO SOLVE DIFFERENTIAL EQUATIONS

Variable seperable

Homogeneous

Linear diff. eqn.

VARIABLE SEPERABLE METHOD

'Variable separable method' is used to solve such an equation in which variables can be separated completely, i.e., terms containing x should remain with dx and terms containing y should remain with dy.

$$\int f(y)\,dy = \int f(x)\,dx + c$$

QUESTION

The solution of the differential equation $2x.\frac{dy}{dx} - y = 3$ represents a family of

(A) straight lines (B) circles (C) parabolas (D) ellipses

$$2x\frac{dy}{dx} - y = 3$$

$$\frac{dy}{dx} = \frac{3+y}{2x}$$

$$\frac{1}{3+y}dy = \frac{1}{2x}dx$$

$$\frac{2}{3+y}dy = \frac{1}{x}dx$$

$$2\int\frac{1}{3+y}dy = \int\frac{1}{x}dx$$

$$2\log(3+y) = \log x + \log c$$

$$2\log(3+y) = \log x + \log c$$

$$\log(3+y)^2 = \log(cx)$$

$$(3+y)^2 = Cx$$

$$(y-(-3))^2 = c(x-0)$$

$$y^2 = 4ax$$

eqn. of parabola,

ANS. (c)

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EQUATIONS REDUCIBLE TO VARIABLE SEPERABLE

Differential equation of the form $\frac{dy}{dx} = f(ax + by + c)$ can be reduced to variable seperable form by the substitution ax + by + c = v.

$$a + b\frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{1}{b}\left(\frac{dv}{dx} - a\right) = f(v)$$

$$\int F(v)\,dv = \int f(x)\,dx + C \quad (\text{variable - seperable})$$

HOMOGENEOUS FUNCTION

A function F (x, y) is said to be a homogeneous function of degree n if

F $(\lambda x, \lambda y) = \lambda^n$ F (x, y) for some non-zero constant λ.

Example : $f(x, y) = x^2 - 2xy$

degree = 2

$$f(\lambda x, \lambda y) = (\lambda x)^2 - 2(\lambda x)(\lambda y) = \lambda^2(x^2 - 2xy) = \lambda^2 f(x, y)$$

$f(x, y) = \tan\left(\frac{x}{y}\right)$

degree = 1

$$f(\lambda x, \lambda y) = \tan\left(\frac{\lambda x}{\lambda y}\right) = \tan\left(\frac{x}{y}\right) = \lambda^0 f(x, y)$$

QUESTION

Which of the following is not a homogeneous function of x and y.

(A) $x^2 + 2xy$ (B) $2x - y$ (C) $\cos^2\left(\frac{y}{x}\right)+\frac{y}{x}$ (D) $\sin x - \cos y$

(A) $(\lambda x)^2 + 2(\lambda x)(\lambda y) = \lambda^2(x^2 + 2xy)$ ✓

(B) $2(\lambda x) - \lambda y = \lambda(2x - y)$

(C) ✓

(D) $\sin(\lambda x) - \cos(\lambda y) =$ λ cannot be taken out

HOMOGENEOUS DIFFERENTIAL EQUATION

A differential equation which can be expressed in the form $\frac{dy}{dx} = F(x, y)$ or $\frac{dx}{dy} = G(x, y)$, where $F(x, y)$ and $G(x, y)$ are homogeneous functions of degree zero, is called a homogeneous differential equation.

$$\frac{dy}{dx} = F(\lambda x, \lambda y) = \lambda^0 F(x, y) = F(x, y)$$

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SOLVING HOMOGENEOUS DIFFERENTIAL EQUATION

To solve a homogeneous differential equation of the type $\frac{dy}{dx} = \mathrm{F}(x, y)$, we make substitution $y = vx$ and to solve a homogeneous differential equation of the type $\frac{dx}{dy} = \mathrm{G}(x, y)$, we make substitution $x = vy$.

① $\frac{dy}{dx} = F(x,y) \Rightarrow y = vx \rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = F(x, vx) \Rightarrow \int f(v)\,dv = \int f(x)\,dx + c$ (variable - seperable)

(2) $$\frac{dx}{dy} = G(x, y)$$

$$x = vy$$

$$\frac{dx}{dy} = v + \frac{dv}{dy}$$

$$v + \frac{dv}{dy} = G(vy, y)$$

$$\left\{ \int g(v)\, dv = \int f(y)\, dy + c \right.$$

(variable - seperable)

QUESTION

State the type of the differential equation for the equation.

$xdy - ydx = \sqrt{x^2 + y^2}\, dx$ and solve it.

$$x\,dy = \left(\sqrt{x^2+y^2} + y\right) dx$$

$$\frac{dy}{dx} = \frac{\sqrt{x^2+y^2} + y}{x} = F(x,y)$$

$x \to \lambda x \; ; \; y \to \lambda y$

$$\frac{\lambda\sqrt{x^2+y^2} + \lambda y}{\lambda x} = \frac{\sqrt{x^2+y^2} + y}{x} = F(x,y)$$

It is a homogeneous differential eqn.

$$\frac{dy}{dx} = \frac{\sqrt{x^2+y^2} + y}{x}$$

$y = vx,$

$$v + x\frac{dv}{dx} = \frac{\sqrt{x^2 + v^2x^2} + vx}{x}$$

$$x\frac{dv}{dx} = \frac{x\sqrt{1+v^2} + vx}{x} - v$$

$$x\frac{dv}{dx} = \frac{x\sqrt{1+v^2}}{x}$$

$$x\frac{dv}{dx} = \sqrt{1+v^2}$$

variable seperable

$$\frac{1}{\sqrt{1+v^2}}dv = \frac{1}{x}dx$$

$$\int \frac{1}{\sqrt{1+v^2}}dv = \int \frac{1}{x}dx$$

$$\log\left|v + \sqrt{1+v^2}\right| = \log x + \log c$$

$$\log\left|v+\sqrt{1+v^2}\right| = \log x + \log c$$

$$\left(y = vx \qquad v = \frac{y}{x}\right)$$

$$\log\left|\frac{y}{x}+\sqrt{1+\frac{y^2}{x^2}}\right| = \log(cx)$$

removed

$$\frac{y}{x}+\frac{1}{x}\sqrt{x^2+y^2} = cx$$

$$cx^2 = y+\sqrt{x^2+y^2}$$

LINEAR DIFFERENTIAL EQUATION

2 formats :

→ $\frac{dy}{dx} + Py = Q$] $P, Q \longrightarrow$ constants or functions in x.

→ $\frac{dx}{dy} + Px = Q$] P, Q — constants or functions of y.

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SOLVING - LINEAR DIFFERENTIAL EQUATION

(1) $\frac{dy}{dx} + Py = Q,$ $\quad P, Q \longrightarrow f(x)$

Integrating Factor (I.F.) $= e^{\int P\,dx}$

solution,

$$\boxed{y\,(IF) = \int Q \times (IF)\,dx + c}$$

② $\dfrac{dx}{dy} + Px = Q$ $\quad P, Q \longrightarrow f(y)$

$I.F. = e^{\int P\,dy}$

Solution,

$$\boxed{x\,(I.F.) = \int (Q)(I.F.)\,dy + C}$$

QUESTION

Solve the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$.

of the form, $\frac{dy}{dx} + Py = Q$; $P = \frac{1}{x}$ and $Q = x^2$

$f(x)$

$$I.F. = e^{\int P dx} = e^{\int \frac{1}{x} dx} = e^{\log x} = x$$

Solution $\longrightarrow$ $y(IF) = \int Q(IF) + C \Rightarrow y(x) = \int x^2 . x \, dx + c$

$$y \cdot x = \int x^2 \cdot x \, dx + c$$

$$xy = \frac{x^4}{4} + c$$

QUESTION

The integrating factor of the differential equation

$$\frac{dy}{dx}(x \log x) + y = 2\log x \text{ is}$$

(A) e^x (B) $\log x$ (C) $\log(\log x)$ (D) x

$$\frac{dy}{dx}(x \log x) + y = 2\log x$$

Divide by $x \log x$,

$$\frac{dy}{dx} + \frac{y}{x \log x} = \frac{2\log x}{x \log x}$$

$$\frac{dy}{dx} + \frac{y}{x \log x} = \frac{2}{x}$$

of the form $\frac{dy}{dx} + Py = Q$

$P = \frac{1}{x \log x}$; $Q = \frac{2}{x}$

$$IF = e^{\int P\,dx}$$

$$= e^{\int \frac{1}{x\log x}dx}$$

$$= e^{\log(\log x)}$$

$$IF = \log x$$

$t = \log x$

$dt = \frac{1}{x}dx$

$$\int \frac{dt}{t} = \log t = \log(\log x)$$

Ans. (b) $\log x$

QUESTION

The solution of the differential equation $\frac{dy}{dx}+\frac{2xy}{1+x^2}=\frac{1}{(1+x^2)^2}$ is :

(A) $y(1+x^2)=c+\tan^{-1}x$

(B) $\frac{y}{1+x^2}=c+\tan^{-1}x$

(C) $y\log(1+x^2)=c+\tan^{-1}x$

(D) $y(1+x^2)=c+\sin^{-1}x$

It is a LDE of form $\frac{dy}{dx}+Py=Q$

$P=\frac{2x}{1+x^2}$

$Q=\frac{1}{(1+x^2)^2}$

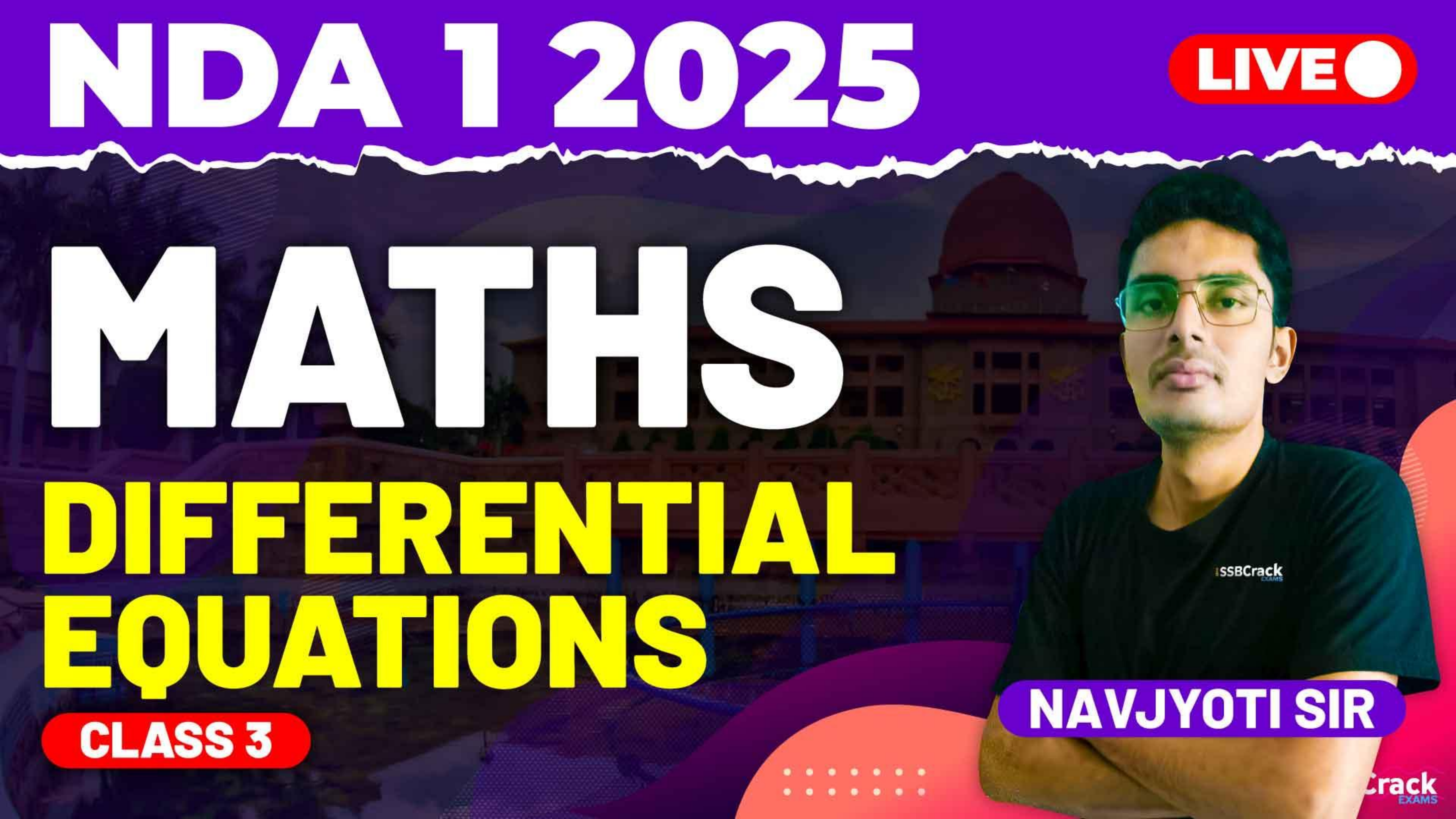
$IF = e^{\int P\,dx} = e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = \underline{1+x^2}$

$$y(1+x^2) = \int \frac{1}{(1+x^2)^2}(1+x^2)\,dx + C$$

$$y(1+x^2) = \int \frac{1}{1+x^2}\,dx$$

$$y(1+x^2) = \tan^{-1}(x) + C$$

Ans. (A)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