
NDA 1 2025
LIVE
MATHS
INTEGRATION
CLASS 1
SSBCrack EXAMS
NAVJYOTI SIR
Crack EXAMS

17 Dec 2024 Live Classes Schedule

8:00AM	17 DEC 2024 DAILY CURRENT AFFAIRS	RUBY MA'AM
9:00AM	17 DEC 2024 DAILY DEFENCE UPDATES	DIVYANSHU SIR

SSB INTERVIEW LIVE CLASSES

9:30AM	OVERVIEW OF GPE & PRACTICE	ANURADHA MA'AM

NDA 1 2025 LIVE CLASSES

1:00PM	PHYSICS - WORK ENERGY POWER - CLASS 1	NAVJYOTI SIR
4:30PM	ENGLISH - REPORTED SPEECH - CLASS 1	ANURADHA MA'AM
5:30PM	MATHS - INTEGRATION - CLASS 1	NAVJYOTI SIR

CDS 1 2025 LIVE CLASSES

1:00PM	PHYSICS - WORK ENERGY POWER - CLASS 1	NAVJYOTI SIR
4:30PM	ENGLISH - REPORTED SPEECH - CLASS 1	ANURADHA MA'AM
7:00PM	MATHS - STATISTICS - CLASS 3	NAVJYOTI SIR

SSBCrack EXAMS

INDEFINITE INTEGRALS

Let $\frac{d}{dx} F(x) = f(x)$. Then, we write $\int f(x)\,dx = F(x) + C$.

integral sign

constant of integration,

$f(x) \longrightarrow$ integrand

$dx \longrightarrow$ gives the variable w.r.t. which $f(x)$ is to be integrated

family of curves

$x^2 = 2x$

$(x^2 + 1)' = 2x$

$(x^2 + 2)' = 2x$

$\vdots$

all these constants are denoted by c.

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INDEFINITE INTEGRALS

If two functions differ by a constant, they have the same derivative.

$$(\sin^2 x + a)' \longrightarrow 2\sin x\cos x + 0 = 2\sin x\cos x$$

$$(\sin^2 x + b)' \longrightarrow 2\sin x\cos x + 0 = 2\sin x\cos x$$

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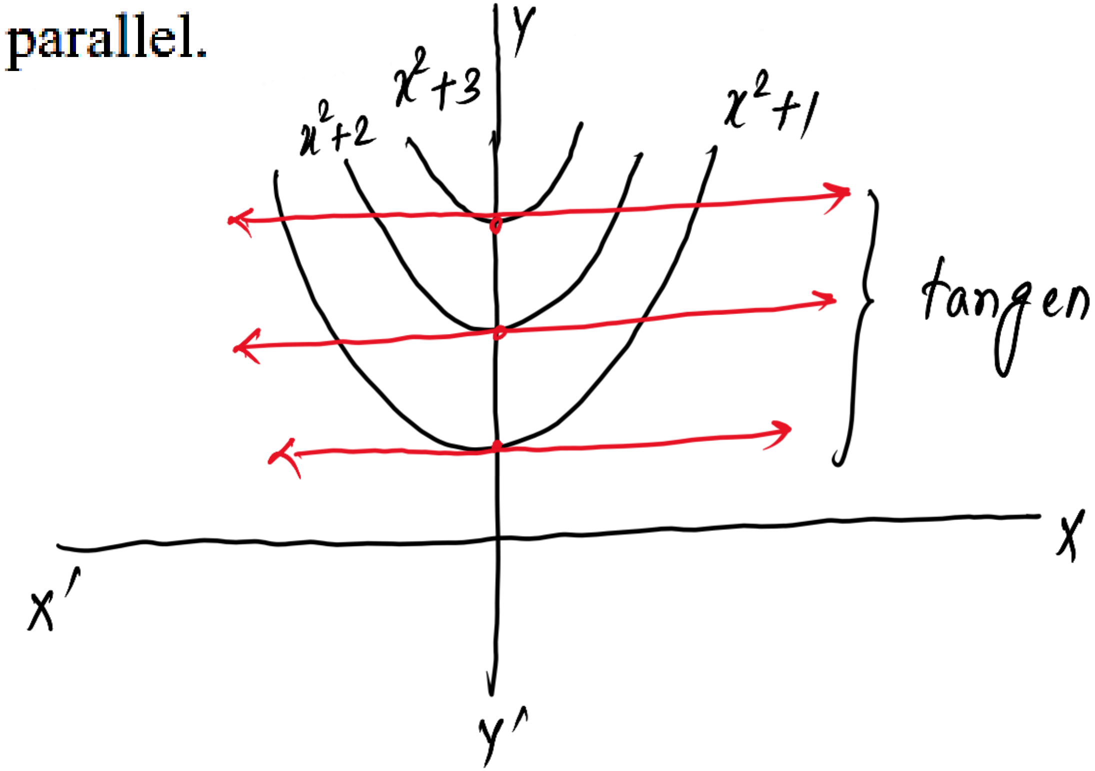
INDEFINITE INTEGRALS

Geometrically, the statement $\int f(x)\,dx = F(x) + C = y$ (say) represents a family of curves. The different values of C correspond to different members of this family and these members can be obtained by shifting any one of the curves parallel to itself. Further, the tangents to the curves at the points of intersection of a line $x = a$ with the curves are parallel.

x^2+1

x^2+2

x^2+3



tangents $\longrightarrow$ slope $= 0$ (parallel to x-axis)

PROPERTIES

The process of differentiation and integration are inverse of each other, i.e., $\frac{d}{dx}\int f(x)dx = f(x)$ and $\int f'(x)dx = f(x) + C$, where C is any arbitrary constant.

$$\frac{d}{dx}(x^n) = nx^{n-1} \Rightarrow \frac{d}{dx}(x^{n+1}) = (n+1)x^n$$

* $$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$$\frac{d}{dx}(x) = 1 \Rightarrow \int 1 \cdot dx = \int dx = x + C$$

* $$\int \sin x\,dx = -\cos x + C$$

* $$\int \cos x\,dx = \sin x + C$$

* $$\int \sec^2 x\,dx = \tan x + C$$

* $$\int \sec x \tan x\,dx = \sec x + C$$

* $\int \operatorname{cosec}^2 x \, dx = -\cot x + c$

* $\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + c$

* $\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1} x + c = -\cos^{-1} x + c$

* $\int \frac{1}{|x|\sqrt{x^2-1}} \, dx = \sec^{-1} x + c = -\operatorname{cosec}^{-1} x + c$

* # $\int \frac{1}{1+x^2} \, dx = \tan^{-1} x + c = -\cot^{-1} x + c$

* $\int e^x \, dx = e^x + c$

* $\int \frac{1}{x} \, dx = \log|x| + c$

* $\int a^x \, dx = \frac{a^x}{\log a} + c$ $\quad \left(\frac{d}{dx}(a^x) = a^x \cdot \log a \right.$

* $\int f(ax+b) \, dx = \frac{F(ax+b)}{\frac{d}{dx}(ax+b)} + c = \frac{F(ax+b)}{a} + c$

$$\int \tan x \, dx = \log|\sec x| + C = -\log|\cos x| + C$$

$$\log\left|\frac{1}{\cos x}\right| = \log|\cos x|^{-1}$$

$$= -\log|\cos x|$$

$$\int \cot x \, dx = \log|\sin x| + C$$

$$\int \sec x \, dx = \log|\sec x + \tan x| + C = \log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) + C$$

$$\int \operatorname{cosec} x \, dx = \log|\operatorname{cosec} x - \cot x| + C$$

PROPERTIES

Two indefinite integrals with the same derivative lead to the same family of curves and so they are equivalent. So if f and g are two functions such that

$\frac{d}{dx}\int f(x)\,dx = \frac{d}{dx}\int g(x)\,dx$, then $\int f(x)\,dx$ and $\int g(x)\,dx$ are equivalent.

PROPERTIES

The integral of the sum of two functions equals the sum of the integrals of the functions i.e., $\int (f(x) + g(x))\, dx = \int f(x)dx + \int g(x)dx$.

PROPERTIES

A constant factor may be written either before or after the integral sign, i.e.,

$\int a\, f(x)\, dx = a \int f(x)\, dx$, where 'a' is a constant.

$$\int \left(k_1 f_1(x) + k_2\, f_2(x) + \ldots +, k_n f_n(x)\right) dx = k_1 \int f_1(x) dx + k_2 \int f_2(x) dx + \ldots + k_n \int f_n(x) dx$$

QUESTION

Integrate $\left(\frac{2a}{\sqrt{x}} - \frac{b}{x^2} + 3c\sqrt[3]{x^2}\right)$ w.r.t. x

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$$

$$I = \int \frac{2a}{\sqrt{x}} - \frac{b}{x^2} + 3c\sqrt[3]{x^2} \; dx$$

$$= \int \frac{2a}{\sqrt{x}} dx - \int \frac{b}{x^2} dx + \int 3c\sqrt[3]{x^2} \, dx$$

$$= 2a \int x^{-1/2} dx - b \int x^{-2} dx + 3c \int x^{2/3} dx$$

$$= 2a \frac{x^{-1/2+1}}{-1/2+1} - b \frac{x^{-2+1}}{-2+1} + 3c \frac{x^{2/3+1}}{2/3+1}$$

$$= 2a \left(\frac{x^{1/2}}{1/2}\right) + b\left(x^{-1}\right) + 3c\left(\frac{x^{5/3}}{5/3}\right)$$

$$= \underbrace{4a\sqrt{x} + \frac{b}{x} + \frac{9c}{5} x^{5/3}}$$

$$\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2} \Rightarrow \int \frac{-1}{x^2}\,dx = \frac{1}{x} + C$$

$$\frac{d}{dx}\left(\sqrt{x}\right) = \frac{1}{2\sqrt{x}} \Rightarrow \int \frac{1}{\sqrt{x}}\,dx = 2\sqrt{x} + C$$

METHODS OF INTEGRATION

There are some methods or techniques for finding the integral where we can not directly select the antiderivative of function f by reducing them into standard forms. Some of these methods are based on

1. Integration by substitution ✓
2. Integration using partial fractions ✓
3. Integration by parts ✓

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INTEGRATION BY SUBSTITUTION

$$I = \int f(x) \cdot f'(x)\, dx$$

$$f(x) = t \Rightarrow \frac{d}{dx}(f(x)) = \frac{dt}{dx} \Rightarrow f'(x) = \frac{dt}{dx}$$

$$f'(x)\, dx = dt$$

$$I = \int t\, dt = \frac{t^2}{2} + c$$

$$= \frac{1}{2}(f(x))^2 + c$$

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IMPORTANT SUBSTITUTIONS

Important Substitution

Expression	Substitution
1. $a^2 + x^2, \sqrt{x^2 + a^2}, \dfrac{1}{\sqrt{x^2 + a^2}}$	$x = a \tan \theta$ or $a \cot \theta$
2. $a^2 - x^2, \sqrt{a^2 - x^2}, \dfrac{1}{\sqrt{a^2 - x^2}}$	$x = a \sin \theta$ or $a \cos \theta$
3. $x^2 - a^2, \sqrt{x^2 - a^2}$	$x = a \sec \theta$ or $a \operatorname{cosec} \theta$
4. $\sqrt{\dfrac{a - x}{a + x}}$ or $\sqrt{\dfrac{a + x}{a - x}}$	$x = a \cos 2\theta$
5. $\sqrt{\dfrac{x - \alpha}{\beta - x}}$ or $\sqrt{(x - \alpha)(\beta - x)}, (\beta > \alpha)$	$x = \alpha \cos^2\theta + \beta \sin^2\theta$

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QUESTION

Evaluate $\int \frac{3ax}{b^2 + c^2x^2}\,dx$

$f'(x).dx$ — numerator,

$$I = \int \frac{3ax}{b^2+c^2x^2}\,dx$$

$$= 3a\int \frac{x}{b^2+c^2x^2}\,dx$$

Let $b^2 + c^2x^2 = t$

$$c^2(2x)\,dx = dt$$

$$x\,dx = \frac{dt}{2c^2}$$

$$= 3a\int \frac{dt}{2c^2(t)} = \frac{3a}{2c^2}\int \frac{dt}{t}$$

$$= \frac{3a}{2c^2}\log t + c$$

$$\frac{3a}{2c^2}\log\left(b^2+c^2x^2\right) + c$$

QUESTION

Evaluate $\int \sqrt{\frac{1+x}{1-x}}\, dx$, $x \neq 1$.

$$\frac{1+x}{1-x} \times \frac{1+x}{1+x} = \frac{(1+x)^2}{1-x^2}$$

$$I = \int \sqrt{\frac{(1+x)^2}{1-x^2}}\, dx$$

$$= \int \frac{1+x}{\sqrt{1-x^2}}\, dx = \int \frac{1}{\sqrt{1-x^2}}\, dx + \int \frac{x}{\sqrt{1-x^2}}\, dx$$

$$= \sin^{-1} x + I_1 + c$$

$$I_1 = \int \frac{x}{\sqrt{1-x^2}}\,dx$$

$$1-x^2 = t$$

$$-2x\,dx = dt$$

$$x\,dx = \frac{-1}{2}dt$$

$$I_1 = \int \frac{\frac{-1}{2}dt}{\sqrt{t}}$$

$$= \frac{-1}{2}\int \frac{1}{\sqrt{t}}dt = \frac{-1}{2}\left(2\sqrt{t}\right)$$

$$= -\sqrt{t} = -\sqrt{1-x^2}$$

$$I = \sin^{-1}x + I_1 + C = \sin^{-1}x - \sqrt{1-x^2} + C$$

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QUESTION

HW

Evaluate the integral $\int \frac{dx}{x\cos^2(1+\log x)}$.

a. $\log|1+\tan x|+C$

b. $\log|1-\tan x|+C$

c. $\tan(1+\log x)+C$

d. $\tan(1-\log x)+C$


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