
NDA 1 2025
LIVE
MATHS
INTEGRATION
CLASS 3
NAVJYOTI SIR
SSBCrack EXAMS

19 Dec 2024 Live Classes Schedule

9:00AM	19 DEC 2024 DAILY DEFENCE UPDATES	DIVYANSHU SIR

SSB INTERVIEW LIVE CLASSES

9:30AM	COMPLETE SCREENING TESTS	ANURADHA MA'AM

NDA 1 2025 LIVE CLASSES

1:00PM	PHYSICS - ROTATIONAL MOTION	NAVJYOTI SIR
✓ 4:30PM	ENGLISH - SENTENCE IMPROVEMENT - CLASS 1	ANURADHA MA'AM
✓ 5:30PM	MATHS - INTEGRATION - CLASS 3	NAVJYOTI SIR

CDS 1 2025 LIVE CLASSES

1:00PM	PHYSICS - ROTATIONAL MOTION	NAVJYOTI SIR
✓ 4:30PM	ENGLISH - SENTENCE IMPROVEMENT - CLASS 1	ANURADHA MA'AM
✓ 7:00PM	MATHS - SET THEORY	NAVJYOTI SIR

INTEGRATION BY USING PARTIAL FRACTIONS

$$\frac{P(x)}{Q(x)} = T(x) + \frac{P_1(x)}{Q(x)}$$

higher degree

lower degree

Remainder

lower degree

Quotient

Divisor

comparatively higher degree

$$\int \frac{P(x)}{Q(x)}\,dx = \int T(x)\,dx + \int \frac{P_1(x)}{Q(x)}\,dx$$

partial fractions,

FORMS OF PARTIAL FRACTIONS

S. No.	Form of the proper rational function	Form of the partial fraction
# 1.	$\dfrac{px \pm q}{(x \pm a)\,(x \pm b)}$, $a \neq b$	$\dfrac{A}{x \pm a} + \dfrac{B}{x \pm b}$
2.	$\dfrac{px \pm q}{(x \pm a)^2}$	$\dfrac{A}{x \pm a} + \dfrac{B}{(x \pm a)^2}$
3.	$\dfrac{px^2 \pm qx \pm r}{(x \pm a)\,(x \pm b)\,(x \pm c)}$	$\dfrac{A}{x \pm a} + \dfrac{B}{x \pm b} + \dfrac{C}{x \pm c}$
4.	$\dfrac{px^2 \pm qx \pm r}{(x \pm a)^2 (x \pm b)}$	$\dfrac{A}{x \pm a} + \dfrac{B}{(x \pm a)^2} + \dfrac{C}{x \pm b}$
5.	$\dfrac{px^2 \pm qx \pm r}{(x \pm a)\,(x^2 \pm bx \pm c)}$	$\dfrac{A}{x \pm a} + \dfrac{Bx + C}{x^2 \pm bx \pm c}$
6.	$\dfrac{Px^2 \pm qx \pm r}{(x \pm a)^3}$	$\dfrac{A}{(x \pm a)} + \dfrac{B}{(x \pm a)^2} + \dfrac{C}{(x \pm a)^3}$

Where $x^2 + bx + c$ can not be factorised further

degree 1 less than Dr.

$\dfrac{\text{Numerator (Nr)}}{\text{Denominator (Dr)}}$

$Nr = A(Dr)' + B$

$\displaystyle \frac{f'(x)}{f(x)}\,dx$

QUESTION

Find $\int \frac{x^3}{x^4+3x^2+2}\,dx$

$$= \int \frac{x^2 \cdot x\,dx}{x^4+3x^2+2}$$

Let $x^2 = t$.

$$2x\,dx = dt$$

$$I = \frac{1}{2}\int \frac{t}{t^2+3t+2}\,dt$$

$$= \frac{1}{2}\int \frac{t}{(t+1)(t+2)}$$

$$\frac{t}{(t+1)(t+2)} = \frac{A}{\underbrace{t+1}} + \frac{B}{\underbrace{t+2}}$$

$$t = A(t+2) + B(t+1)$$

$$t = t(A+B) + (2A+B)$$

comparing coefficients,

$A + B = 1$

$2A + B = 0$

$2A + 1 - A = 0$

$A + 1 = 0$

$A = -1$

$B = 2$

$$A = -1 \quad ; \quad B = 2$$

$$I = \frac{1}{2}\left[\int \frac{-1}{t+1}\,dt + \int \frac{2}{t+2}\,dt\right]$$

$$= -\frac{1}{2}\log|t+1| + \log|t+2| + c$$

$$= -\log|t+1|^{1/2} + \log|t+2| + c$$

$$= \log\left|\frac{t+2}{\sqrt{t+1}}\right| + c \Rightarrow \log\left|\frac{x^2+2}{\sqrt{x^2+1}}\right| + c$$

QUESTION

Evaluate $\int \frac{x^2\,dx}{x^4+x^2-2}$.

Let $t = x^2$

$$\frac{x^2}{x^4+x^2-2} = \frac{t}{t^2+t-2} = \frac{t}{(t+2)(t-1)}$$

$$\frac{t}{(t+2)(t-1)} = \frac{A}{t+2} + \frac{B}{t-1}$$

comparing coefficients,

$$t = A(t-1) + B(t+2)$$

$A + B = 1$

$-A + 2B = 0$

$A = 2B$

$B = \frac{1}{3}$

$A = \frac{2}{3}$

$$\frac{2}{3}\left(\frac{1}{x^2+2}\right) + \frac{1}{3}\left(\frac{1}{x^2-1}\right)$$

$$\frac{2}{3}\left(\frac{1}{x^2+2}\right) + \frac{1}{3}\left(\frac{1}{x^2-1}\right)$$

$$I = \frac{2}{3}\int \frac{1}{x^2+2}\,dx + \frac{1}{3}\int \frac{1}{x^2-1}\,dx$$

$$= \frac{2}{3}\int \frac{1}{x^2+(\sqrt{2})^2}\,dx + \frac{1}{3}\int \frac{1}{x^2-(1)^2}\,dx$$

$$\boxed{\begin{aligned} &\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C \\ &\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C \end{aligned}}$$

$$= \frac{2}{3}\left(\frac{1}{\sqrt{2}}\right)\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + \frac{1}{3}\times\frac{1}{2\times 1}\log\left|\frac{x-1}{x+1}\right| + C = \frac{\sqrt{2}}{3}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + \frac{1}{6}\log\left|\frac{x-1}{x+1}\right| + C$$

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DEFINITE INTEGRAL

The definite integral is denoted by $\int_a^b f(x)\,dx$, where a is the lower limit of the integral and b is the upper limit of the integral.

$\int_a^b f(x)\,dx = F(b) - F(a)$, if F is an antiderivative of $f(x)$.

$$\int f(x)\,dx = F(x) + c$$

removed

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

PROPERTIES

$$\int_a^b f(x)\,dx = \int_{a'}^{b'} f(t)\,dt$$

$x = t^2$,

$a = t^2 \Rightarrow t = \sqrt{a}$

$b = t^2 \Rightarrow t = \sqrt{b}$

$$\int_{\sqrt{a}}^{\sqrt{b}} f(t)\,dt$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx \text{, in particular, } \int_a^a f(x)\,dx = 0$$

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PROPERTIES

considering,

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

$a < c < b$

(lower limit – number in between)

(number in between – upper limit)

PROPERTIES

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

PROPERTIES

$$\int_0^{2a} f(x)\,dx = \int_0^{a} f(x)\,dx + \int_0^{a} f(2a-x)\,dx$$

$$\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^{a} f(x)\,dx, \text{if } f(2a-x)=f(x), \\ 0, \text{if } f(2a-x)=-f(x). \end{cases}$$

PROPERTIES

#

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx \text{, if } f \text{ is an even function i.e., } f(-x) = f(x)$$

#

$$\int_{-a}^{a} f(x)\,dx = 0 \text{, if } f \text{ is an odd function i.e., } f(-x) = -f(x)$$

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QUESTION

Find $\int_{0}^{\frac{\pi}{4}} \sqrt{1+\sin 2x}\, dx$

$$I = \int_{0}^{\pi/4} \sqrt{1+\sin 2x}\; dx$$

$$= \int_{0}^{\pi/4} \sqrt{(\cos x + \sin x)^2}\; dx$$

$$= \int_{0}^{\pi/4} \cos x + \sin x \; dx$$

$$1 + \sin 2x = \underbrace{\cos^2 x + \sin^2 x}_{} + \underbrace{2 \sin x \cos x}_{}$$

$$= (\cos x + \sin x)^2$$

$$I = \int_{0}^{\pi/4} \cos x\, dx + \int_{0}^{\pi/4} \sin x\, dx$$

$$\left[\sin x\right]_{0}^{\pi/4} + \left[-\cos x\right]_{0}^{\pi/4}$$

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$$I = \left[\sin x\right]_0^{\pi/4} + \left[\cos x\right]_{\pi/4}^{0}$$

$$= \left(\sin\frac{\pi}{4} - \sin 0^\circ\right) + \left(\cos 0^\circ - \cos\frac{\pi}{4}\right)$$

$$= \left(\frac{1}{\sqrt{2}} - 0\right) + \left(1 - \frac{1}{\sqrt{2}}\right)$$

$$= 1$$

$$\int_a^b f(x)\,dx = \left[F(x)\right]_a^b = -\int_b^a f(x)\,dx$$

$$= -\left[F(x)\right]_a^b = \left[F(x)\right]_b^a$$

QUESTION

Find $\displaystyle\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}\,dx$

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$I = \int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}\,dx \quad \text{—— (1)}$$

$$I = \int_2^8 \frac{\sqrt{10-(8+2-x)}}{\sqrt{(8+2)-x}+\sqrt{10-(8+2-x)}}\,dx$$

$$I = \int_2^8 \frac{\sqrt{x}}{\sqrt{10-x}+\sqrt{x}} \quad \text{—— (2)}$$

(1) + (2),

$$\int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

$$2I = \int_2^8 \frac{\sqrt{10-x}+\sqrt{x}}{\sqrt{10-x}+\sqrt{x}}\,dx = \int_a^b (f(x)+g(x))\,dx$$

$$2I = \int_2^8 1 \, dx$$

$$2I = \left[x\right]_2^8$$

$$2I = 8 - 2 = 6$$

$$I = 3$$

QUESTION

Evaluate $\int_0^{\frac{\pi}{2}} \frac{\tan^7 x}{\cot^7 x + \tan^7 x} dx$

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

$$I = \int_0^{\pi/2} \frac{\tan^7 x}{\cot^7 x + \tan^7 x} dx \quad \text{— } (1)$$

$$I = \int_0^{\pi/2} \frac{\tan^7 (\pi/2 - x)}{\cot^7 (\pi/2 - x) + \tan^7\left(\frac{\pi}{2} - x\right)} dx$$

$$I = \int_0^{\pi/2} \frac{\cot^7 x}{\tan^7 x + \cot^7 x} \quad \text{— } (2)$$

$(1) + (2),$

$$2I = \int_0^{\pi/2} \frac{\tan^7 x + \cot^7 x}{\tan^7 x + \cot^7 x} dx$$

$$2I = \int_0^{\pi/2} 1 \cdot dx$$

$$2I = \left[x\right]_0^{\pi/2}$$

$$2I = \frac{\pi}{2}$$

$$\boxed{I = \frac{\pi}{4}}$$


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