
NDA 1 2025
LIVE
MATHS
INTEGRATION
CLASS 4
SSBCrack EXAMS
NAVJYOTI SIR
Crack EXAMS

20 Dec 2024 Live Classes Schedule

8:00AM	20 DEC 2024 DAILY CURRENT AFFAIRS	RUBY MA'AM
9:00AM	20 DEC 2024 DAILY DEFENCE UPDATES	DIVYANSHU SIR

SSB INTERVIEW LIVE CLASSES

9:30AM	SSB 'LECTURETTE TEST'	ANURADHA MA'AM

NDA 1 2025 LIVE CLASSES

1:00PM	PHYSICS - GRAVITATION	NAVJYOTI SIR
4:30PM	ENGLISH - SENTENCE IMPROVEMENT - CLASS 2	ANURADHA MA'AM
5:30PM	MATHS - INTEGRATION - CLASS 4	NAVJYOTI SIR

CDS 1 2025 LIVE CLASSES

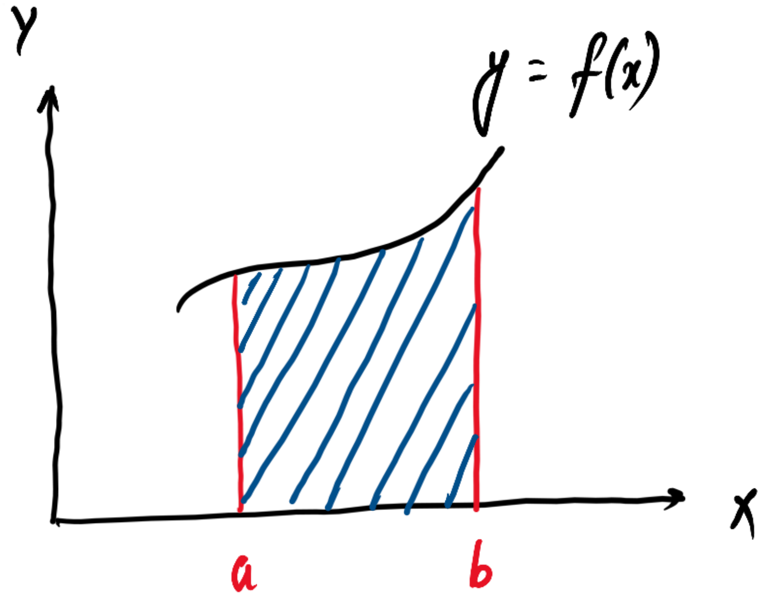
1:00PM	PHYSICS - GRAVITATION	NAVJYOTI SIR
4:30PM	ENGLISH - SENTENCE IMPROVEMENT - CLASS 2	ANURADHA MA'AM

SSBCrack EXAMS

AREA BOUNDED BY CURVES

The area of the region bounded by the curve $y = f(x)$, x-axis and the lines $x = a$ and $x = b$ $(b > a)$ is given by the formula:

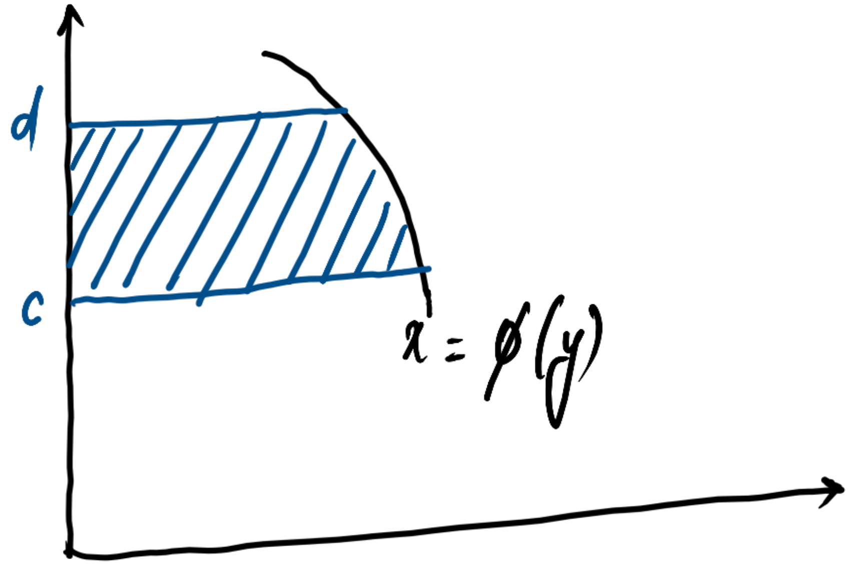
$$\text{Area} = \int_a^b y\,dx = \int_a^b f(x)\,dx$$



AREA BOUNDED BY CURVES

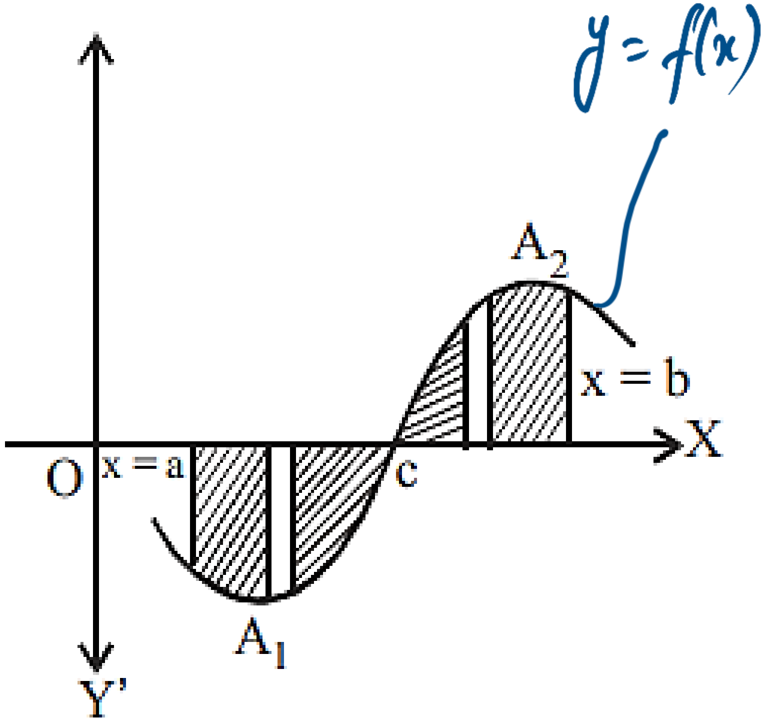
The area of the region bounded by the curve $x = \phi(y)$, y-axis and the lines $y = c$, $y = d$ is given by the formula:

$$\text{Area} = \int_c^d x\,dy = \int_c^d \phi(y)\,dy$$



SSBCrack EXAMS

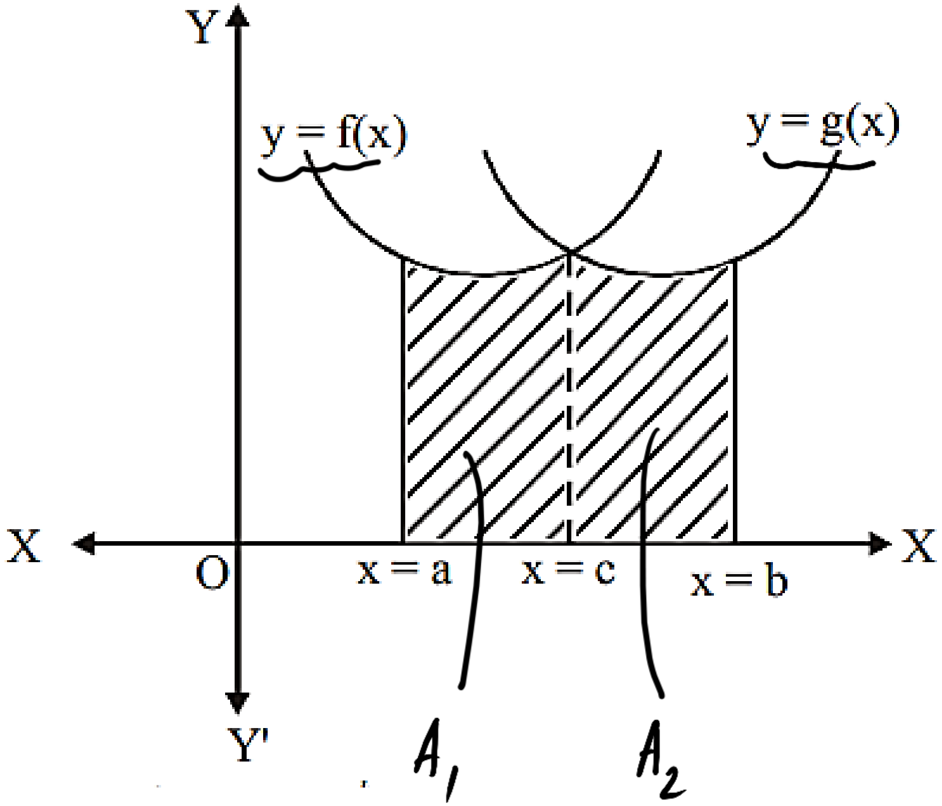
AREA BOUNDED BY CURVES



Shaded area $= A_1 + A_2$

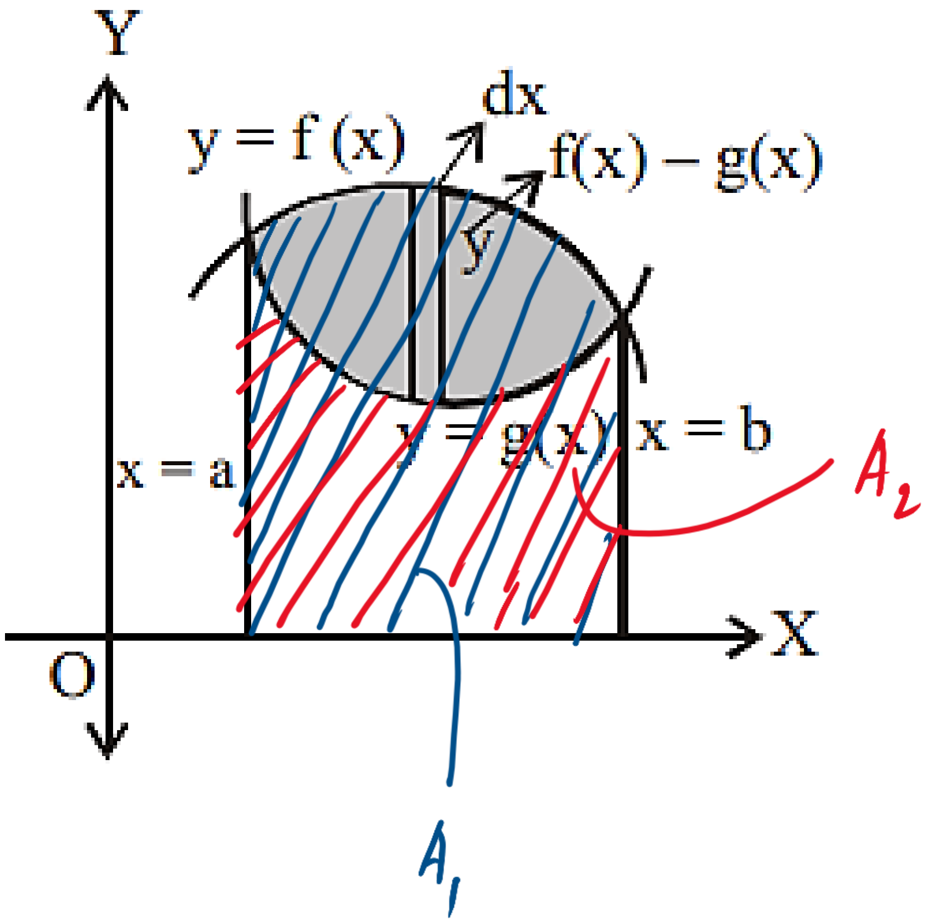
$$= \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

AREA BOUNDED BETWEEN TWO CURVES



$$\text{Shaded area} = \underbrace{\int_a^c f(x)\,dx}_{(A_1)} + \underbrace{\int_c^b g(x)\,dx}_{(A_2)}$$

AREA BOUNDED BETWEEN TWO CURVES



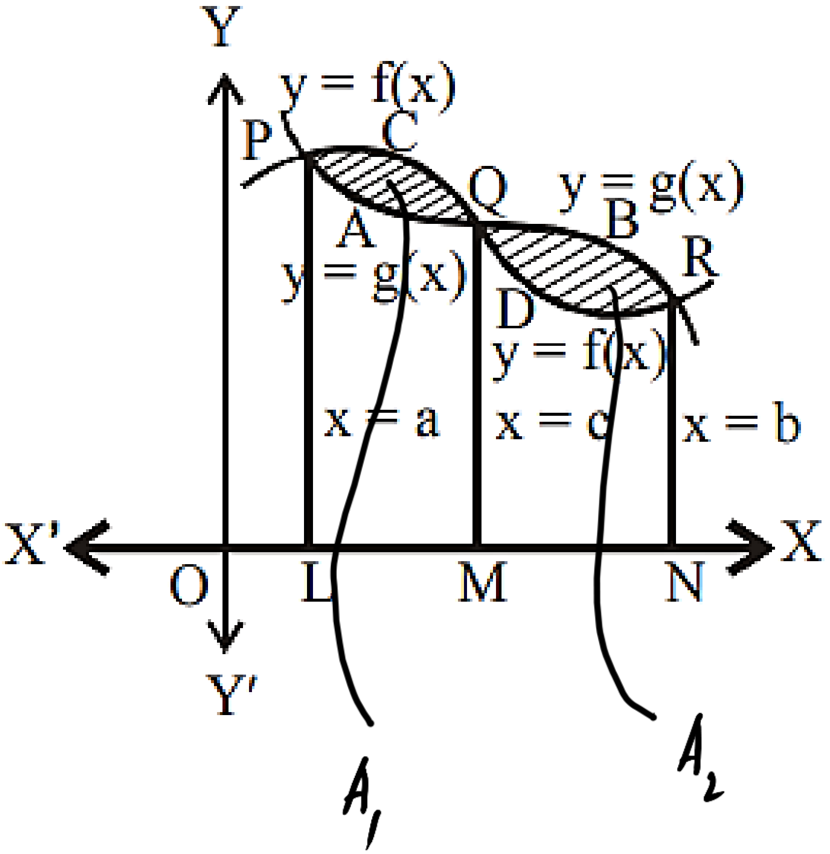
Shaded region $= A_1 - A_2$

$$= \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

$$= \int_a^b \left(f(x) - g(x)\right) dx$$

SSBCrack EXAMS

AREA BOUNDED BETWEEN TWO CURVES



$$A_1 = \int_a^c \left(f(x) - g(x)\right) dx$$

$$A_2 = \int_c^b \left(g(x) - f(x)\right) dx$$

$A_1 + A_2 =$ shaded regim

QUESTION

Find the area of the curve $y = \sin x$ between 0 and π.

$$\int_0^{\pi} y\,dx = \int_0^{\pi} \sin x\,dx$$

$$= \left[-\cos x\right]_0^{\pi} = \left[\cos x\right]_{\pi}^{0} = \cos 0 - \cos \pi$$

$$= 1 - (-1)$$

$$= 2$$

QUESTION

Find the area enclosed by the curve $x = 3\cos t$, $y = 2\sin t$.

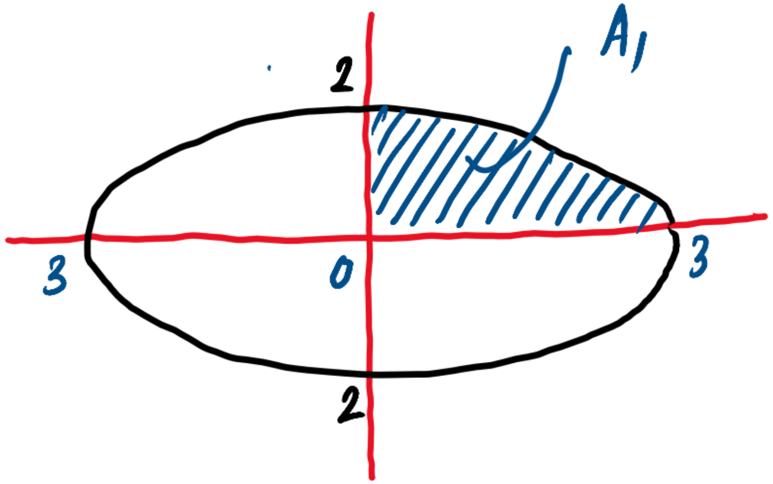
$x^2 = 9\cos^2 t \Rightarrow \cos^2 t = \frac{x^2}{9}$

$y^2 = 4\sin^2 t \Rightarrow \frac{y^2}{4} = \sin^2 t$

$\cos^2 t + \sin^2 t = 1$

$\frac{x^2}{9} + \frac{y^2}{4} = 1$ (eqn. of ellipse)

$\frac{x^2}{3^2} + \frac{y^2}{2^2} = 1$



Area enclosed by curve $= 4A_1$

$$\frac{x^2}{9} + \frac{y^2}{4} = 1 \Rightarrow y^2 = 4\left(1 - \frac{x^2}{9}\right) \Rightarrow y = \frac{2}{3}\sqrt{9 - x^2}$$

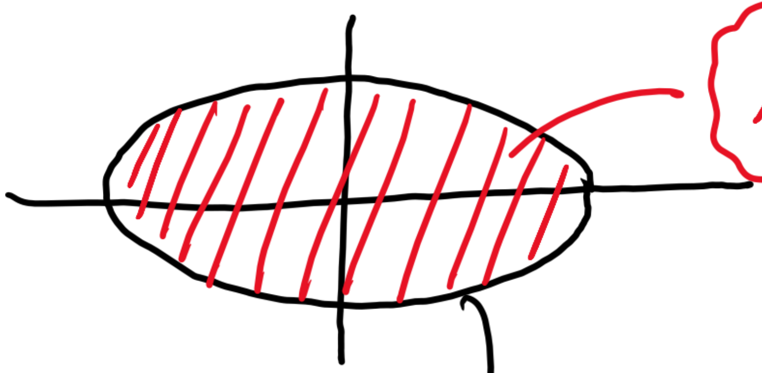
$$A_1 = \int_0^3 \frac{2}{3}\sqrt{9 - x^2}\, dx$$

$$= \frac{2}{3}\int_0^3 \sqrt{(3)^2 - (x)^2}\, dx$$

$$\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$$

Full area

$$\hookrightarrow 4 \times \frac{2}{3}\left[\frac{x}{2}\sqrt{9 - x^2} + \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right)\right]_0^3 = \frac{8}{3}\left(\frac{9}{2} \times \frac{\pi}{2}\right) = \frac{8}{3} \times \frac{9}{4}\pi = 6\pi$$



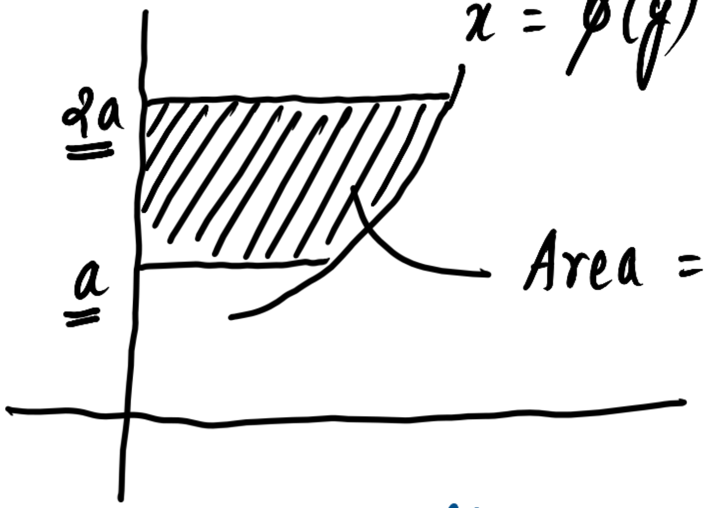
$A = \pi ab$ sq. units

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

QUESTION

Find the area of the region bounded by the curve $ay^2 = x^3$, the y-axis and the lines $y = a$ and $y = 2a$.

$$x = (ay^2)^{\frac{1}{3}} = a^{\frac{1}{3}} y^{2/3}$$



$x = \phi(y)$

$$\text{Area} = \int_a^{2a} x\,dy = \int_a^{2a} \phi(y)\,dy$$

$$= \int_a^{2a} a^{\frac{1}{3}} y^{2/3}\,dy = a^{\frac{1}{3}} \int_a^{2a} y^{2/3}\,dy = a^{\frac{1}{3}} \left[\frac{y^{2/3+1}}{2/3+1} \right]_a^{2a}$$

$$a^{\frac{1}{3}}\left[\frac{y^{2/3+1}}{2/3+1}\right]_a^{2a}$$

$$= \frac{3}{5}a^{\frac{1}{3}}\left[y^{5/3}\right]_a^{2a}$$

$$= \frac{3}{5}a^{\frac{1}{3}}\left[(2a)^{5/3}-(a)^{5/3}\right]$$

$$= \frac{3}{5}a^{\frac{1}{3}}\cdot a^{5/3}\left(2^{5/3}-1\right)$$

$$\boxed{= \frac{3}{5}a^{2}\left(2^{5/3}-1\right)}$$

QUESTION

Find the area of the region included between the parabola $y = \frac{3x^2}{4}$ and the line $3x - 2y + 12 = 0$.

$y = \frac{3x^2}{4}$

$3x - 2y + 12 = 0$

$3x - 2\left(\frac{3x^2}{4}\right) + 12 = 0$

$3x - \frac{3x^2}{2} + 12 = 0$

$6x - 3x^2 + 24 = 0$

$3x^2 - 6x - 24 = 0$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

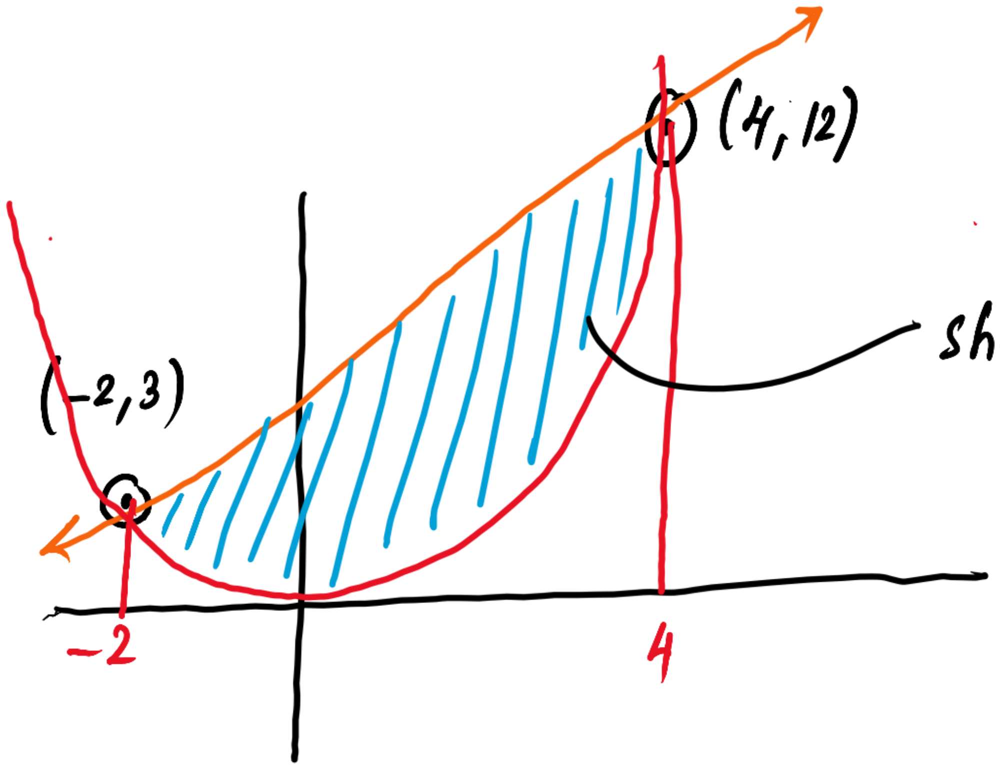
$x = 4, -2$

$y = \frac{3x^2}{4}$

$= \frac{3(4)^2}{4} = 12$

$y = \frac{3(-2)^2}{4} = 3$

Two points of intersection $(4,12), (-2,3)$



$$y = \frac{3x^2}{4}$$

shaded region $= \displaystyle\int_{-2}^{4} \left(\frac{3x+12}{2} - \frac{3x^2}{4} \right) dx$

$$\left[\frac{3}{2}\frac{x^2}{2}\right]_{-2}^{4} + 6\left[x\right]_{-2}^{4} - \frac{3}{4}\left[\frac{x^3}{3}\right]_{-2}^{4}$$

$$3x - 2y + 12 = 0 \Rightarrow y = \frac{3x+12}{2}$$

$$= \frac{3}{4}\left(4^2 - (-2)^2\right) + 6\left(4 - (-2)\right) - \frac{1}{4}\left(4^3 - (-2)^3\right)$$

$$= \frac{3}{4}\left(4^2 - (-2)^2\right) + 6\left(4 - (-2)\right) - \frac{1}{4}\left(4^3 - (-2)^3\right)$$

$$= \frac{3}{4}\left(16 + 4\right) + 6\left(6\right) - \frac{1}{4}\left(64 + 8\right)$$

$$= \frac{3}{4}\left(20\right) + 36 - \frac{1}{4}\left(72\right)$$

$$= 15 + 36 - 18$$

$$= 15 + 18 = \boxed{33 \text{ sq. units}}$$

QUESTION

Find the area of the region bounded by the parabola $y^2 = 2x$ and the straight line $x - y = 4$.

$y^2 = 2x$

$x = \frac{y^2}{2}$

$\frac{y^2}{2} - y - 4 = 0$

$y^2 - 2y - 8 = 0$

$(y-4)(y+2) = 0$

$y = 4, -2$

$y = 4 \Rightarrow x = \frac{4^2}{2} = 8$

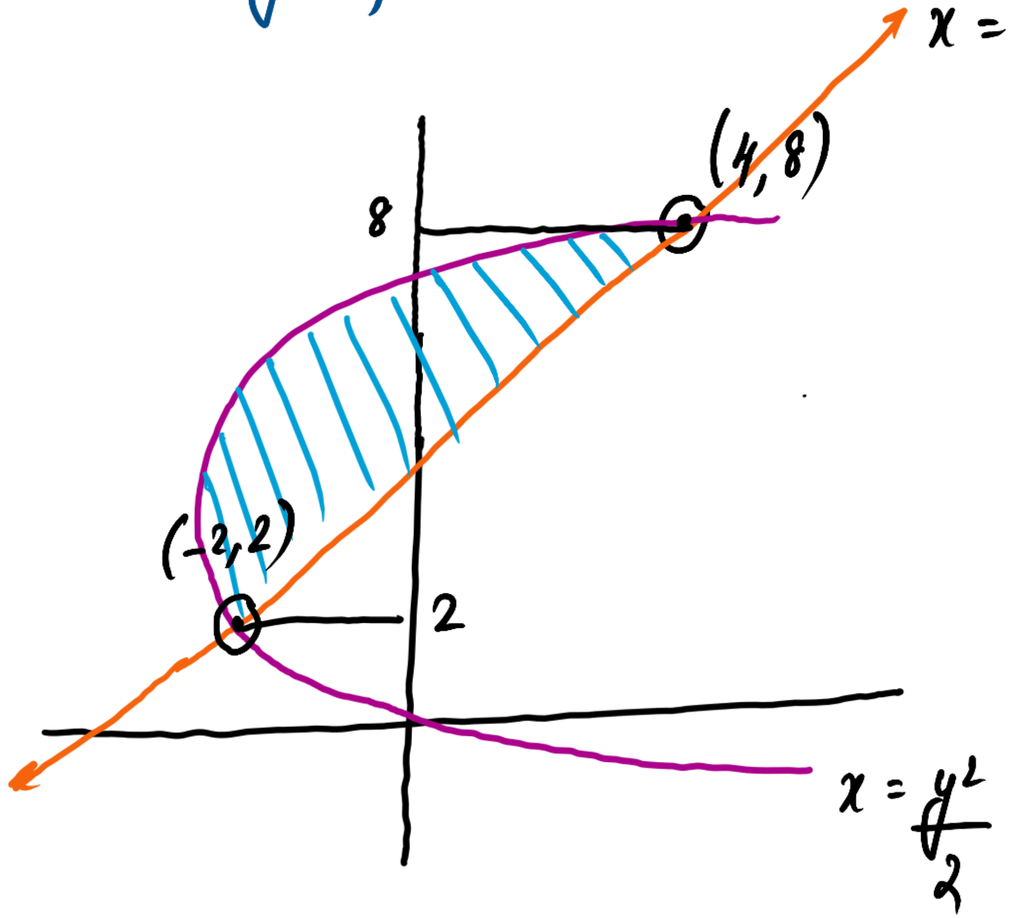
$y = -2 \Rightarrow x = \frac{(-2)^2}{2} = 2$

Intersecting points $= (4, 8)$ and $(-2, 2)$.

Intersecting points $= (4,8)$ and $(-2,2)$.

$y^2 = 2x$

$x - y = 4$



shaded region $= \int \left(\text{curve on right} - \text{curve on left} \right)$

$$= \int_2^8 \left(4 + y - \frac{y^2}{2}\right) dy$$

$$= \int_2^8 \left(4 + y - \frac{y^2}{2}\right) dy$$

$$= \left[4y\right]_2^8 + \frac{1}{2}\left[y^2\right]_2^8 - \frac{1}{2}\left[\frac{y^3}{3}\right]_2^8$$

$$= 4(8-2) + \frac{1}{2}(8^2 - 2^2) - \frac{1}{6}(8^3 - 2^3)$$

$$= 4 \times 6 + \frac{1}{2}(64-4) - \frac{1}{6}(512-8)$$

$$= 24 + 30 - \frac{1}{6}(504)$$

$$= 54 - 84$$

$$= -30$$

(Area can't be -ve)

Answer $\Rightarrow$ 30 sq. units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