NDA 1 2025

LIVE

MATHS

INTEGRATION

CLASS 5

NAVJYOTI SIR

What is $\int_{-1}^{1}(3\sin x - \sin 3x)\cos^2 x\,dx$ equal to ?

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(a) $-\frac{1}{4}$

(b) 0

(c) $\frac{1}{2}$

(d) $\frac{1}{4}$

$\int_{-a}^{a} f(x)\,dx$ } To check whether $f(x)$ is odd or even.

$f(x) = (3\sin x - \sin 3x)\cos^2 x$

$f(-x) = (3\sin(-x) - \sin 3(-x))(\cos(-x))^2$

$= (-3\sin x + \sin 3x)(\cos x)^2$

$= -[(3\sin x - \sin 3x)\cos^2 x]$

$= -f(x) \Rightarrow f(x)$ is odd.

$\sin(-x) = -\sin x$

$\cos(-x) = \cos x$

$$\int_{-a}^{a} f(x)\,dx = 0 \quad \text{if } f(x) \text{ is odd.}$$

What is $\int_{-1}^{1}(3\sin x - \sin 3x)\cos^2 x\,dx$ equal to ?

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(a) $-\frac{1}{4}$

(b) 0

(c) $\frac{1}{2}$

(d) $\frac{1}{4}$

ANS : (b)

Let $p = \int_a^b f(x)dx$ and $q = \int_a^b |f(x)|dx$.

If $f(x) = e^{-x}$, then which one of the following is correct ?

(a) $p = 2q$

(b) $p = -q$

(c) $4p = q$

(d) $p = q$

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$f(x) = e^{-x} > 0$ (for any x, $e^{-x} > 0$)

$|f(x)| = |e^{-x}| = e^{-x}$

$p = q$

Let $p = \int_a^b f(x)dx$ and $q = \int_a^b |f(x)|dx$.

If $f(x) = e^{-x}$, then which one of the following is correct ?

(a) $p = 2q$

(b) $p = -q$

(c) $4p = q$

(d) $p = q$

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ANS : (d)

Q) What is $\int \tan^{-1}(\sec x + \tan x)\,dx$ equal to?

(a) $\frac{\pi x}{4} + \frac{x^2}{4} + c$

(b) $\frac{\pi x}{2} + \frac{x^2}{4} + c$

(c) $\frac{\pi x}{4} + \frac{\pi x^2}{4} + c$

(d) $\frac{\pi x}{4} - \frac{x^2}{4} + c$

$$\sec x + \tan x = \frac{1}{\cos x} + \frac{\sin x}{\cos x} = \frac{1+\sin x}{\cos x} = \frac{\sin^2 x/2 + \cos^2 x/2 + 2\sin x/2 \cos x/2}{\cos^2 x/2 - \sin^2 x/2}$$

$$= \frac{(\sin x/2 + \cos x/2)^2}{(\cos x/2 + \sin x/2)(\cos \frac{x}{2} - \sin \frac{x}{2})} = \frac{\cos x/2 + \sin x/2}{\cos x/2 - \sin x/2}$$

$$\frac{\cos x/2 + \sin x/2}{\cos x/2 - \sin x/2}$$

Dividing by $\cos x/2$

$$\frac{1 + \tan x/2}{1 - \tan x/2} = \frac{\tan \pi/4 + \tan x/2}{1 - \tan\frac{\pi}{4} \cdot \tan\frac{x}{2}} = \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$$

↑

$\sec x + \tan x$

$$\frac{1 + \tan A}{1 - \tan A} = \tan\left(\frac{\pi}{4} + A\right)$$

$$I = \int \tan^{-1}(\sec x + \tan x)\,dx = \int \tan^{-1}\left(\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right)dx$$

$$I = \int \left(\frac{\pi}{4} + \frac{x}{2}\right)dx$$

$$= \frac{\pi}{4}\int dx + \frac{1}{2}\int x\,dx$$

$$= \frac{\pi}{4}x + \frac{1}{2}\left(\frac{x^2}{2}\right) + c = \frac{\pi}{4}x + \frac{x^2}{4} + c$$

Q) What is $\int \tan^{-1}(\sec x + \tan x)dx$ equal to?

(a) $\frac{\pi x}{4} + \frac{x^2}{4} + c$

(b) $\frac{\pi x}{2} + \frac{x^2}{4} + c$

(c) $\frac{\pi x}{4} + \frac{\pi x^2}{4} + c$

(d) $\frac{\pi x}{4} - \frac{x^2}{4} + c$

Ans: (a)

Q) Let $f(x)$ be an indefinite integral of $\sin^2 x$. Consider the following statements :

Statement 1 : The function $f(x)$ satisfies $f(x + \pi) = f(x)$ for all real x.

Statement 2 : $\sin^2(x+\pi) = \sin^2 x$ for all real x.

Which one of the following is correct in respect of the above statements?

(a) Both the statements are true and Statement 2 is the correct explanation of Statement 1

(b) Both the statements are true but Statement 2 is not the correct explanation of Statement 1

(c) Statement 1 is true but Statement 2 is false

(d) Statement 1 is false but Statement 2 is true

$$f(x) = \int \sin^2 x \, dx$$

$$= \frac{1}{2}\int 2\sin^2 x \, dx$$

$$= \frac{1}{2}\int (1 - \cos 2x) \, dx$$

$$= \frac{x}{2} - \frac{1}{2}\left(\frac{\sin 2x}{2}\right) + C$$

$$f(x) = \frac{x}{2} - \frac{1}{4}\sin 2x + C$$

$$f(x) = \frac{x}{2} - \frac{\sin 2x}{4} + c$$

$$f(x+\pi) = \frac{x+\pi}{2} - \frac{1}{4}\left(\sin(2(x+\pi))\right) + c$$

$$= \frac{x}{2} + \frac{\pi}{2} - \frac{1}{4}\left(\sin(2\pi + 2x)\right) + c$$

$$= \frac{x}{2} - \frac{1}{4}\sin 2x + \frac{\pi}{2} + c$$

$\frac{\pi}{2} + c$ can be taken as another constant,

$$= \frac{x}{2} - \frac{1}{4}\sin 2x + C = \underline{f(x)}$$

II) $\sin^2(x+\pi) = \sin^2 x$

$LHS = (\sin(x+\pi))^2$

$= (-\sin x)^2 = -\sin x \times -\sin x$

$= \sin^2 x = \underline{RHS}$

Q) Let $f(x)$ be an indefinite integral of $\sin^2 x$. Consider the following statements :

Statement 1 : The function $f(x)$ satisfies $f(x + \pi) = f(x)$ for all real x.

Statement 2 : $\sin^2(x + \pi) = \sin^2 x$ for all real x.

Which one of the following is correct in respect of the above statements?

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(b) Both the statements are true but Statement 2 is not the correct explanation of Statement 1

(c) Statement 1 is true but Statement 2 is false

(d) Statement 1 is false but Statement 2 is true

Ans: (b)

Q) What is $\int \frac{xe^x dx}{(x+1)^2}$ equal to?

(a) $(x+1)^2 e^x + c$

(b) $(x+1)e^x + c$

(c) $\frac{e^x}{x+1} + c$

(d) $\frac{e^x}{(x+1)^2} + c$

where c is the constant integration.

$$\int e^x \left(f(x) + f'(x) \right) dx = e^x f(x) + c$$

$$\int e^x \left(\frac{1}{x+1} - \frac{1}{(x+1)^2} \right) dx$$

$$\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$$

$$= \int e^x \left(f(x) + f'(x) \right) dx = e^x f(x) + c = e^x \left(\frac{1}{x+1} \right) + c = \frac{e^x}{x+1} + c$$

Q) What is $\int \frac{xe^x dx}{(x+1)^2}$ equal to?

(a) $(x+1)^2 e^x + c$

(b) $(x+1)e^x + c$

(c) $\frac{e^x}{x+1} + c$

(d) $\frac{e^x}{(x+1)^2} + c$

where c is the constant integration.

Ans: (c)

Q) What is $\int e^{e^x} e^x dx$ equal to ?

(a) $e^{e^x} + c$

(b) $2e^{e^x} + c$

(c) $e^{e^x} e^x + c$

(d) $2e^{e^x} e^x + c$

$t = e^x$

$dt = e^x dx$

$I = \int e^{e^x} e^x dx$

$= \int e^t dt = e^t + c$

$= e^{e^x} + c$

Q) What is $\int e^{e^x} e^x dx$ equal to ?

(a) $e^{e^x} + c$

(b) $2e^{e^x} + c$

(c) $e^{e^x} e^x + c$

(d) $2e^{e^x} e^x + c$

Ans: (a)

Q) What is $\int_0^{\frac{\pi}{2}} e^{\ln(\cos x)}\, dx$ equal to?

(a) -1

(b) 0

(c) 1

(d) 2

$$I = \int_0^{\pi/2} \cos x \, dx$$

$$e^{\ln x} = x$$

$$a^{\log_a y} = y$$

$$= [\sin x]_0^{\pi/2} = \sin\frac{\pi}{2} - \sin 0 = 1 - 0 = 1$$

Q) What is $\int_0^{\frac{\pi}{2}} e^{\ln(\cos x)}\, dx$ equal to?

(a) -1

(b) 0

(c) 1

(d) 2

Ans: (c)

Q) What is $\int_0^{\pi} \ln\left(\tan\frac{x}{2}\right) dx$ equal to?

(a) 0

(b) $\frac{1}{2}$

(c) 1

(d) 2

$$I = \int_0^{\pi} \ln\left(\tan\frac{x}{2}\right) dx \quad \text{—— (1)}$$

$$\int_0^{a} f(x)\,dx = \int_0^{a} f(a-x)\,dx$$

$$I = \int_0^{\pi} \ln\left(\tan\frac{(\pi - x)}{2}\right) dx$$

$$= \int_0^{\pi} \ln\left(\tan\left(\frac{\pi}{2} - \frac{x}{2}\right)\right) dx = \int_0^{\pi} \ln\left(\cot\frac{x}{2}\right) dx \quad \text{—— (2)}$$

(1) + (2),

$$2I = \int_0^{\pi} \ln\left(\tan\frac{x}{2}\right)dx + \int_0^{\pi} \ln\left(\cot\frac{x}{2}\right)dx$$

$$= \int_0^{\pi} \left[\ln\left(\tan\frac{x}{2}\right) + \ln\left(\cot\frac{x}{2}\right)\right]dx$$

$$= \int_0^{\pi} \ln\left(\tan\frac{x}{2}\cot\frac{x}{2}\right)dx = \int_0^{\pi} \ln(1)\,dx = \int_0^{\pi} 0\,dx = 0$$

21 = 0

1 = 0

Q) What is $\int_0^{\pi} \ln\left(\tan\frac{x}{2}\right) dx$ equal to?

(a) 0

(b) $\frac{1}{2}$

(c) 1

(d) 2

Ans: (a)

What is $\int_0^{\frac{\pi}{2}} \frac{a+\sin x}{2a+\sin x+\cos x}dx$ equal to ?

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(a) $\frac{\pi}{4}$

(b) $\frac{\pi}{2}$

(c) 1

(d) 0

$$I = \int_0^{\pi/2} \frac{a + \sin x}{2a + \sin x + \cos x} dx \quad \text{--- (1)}$$

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

$$I = \int_0^{\pi/2} \frac{a + \sin\left(\frac{\pi}{2} - x\right)}{2a + \sin\left(\frac{\pi}{2}-x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx$$

$$I = \int_0^{\pi/2} \frac{a + \cos x}{2a + \cos x + \sin x} dx \quad \text{--- (2)}$$

(1) + (2),

$$2I = \int_0^{\pi/2} \frac{2a + \sin x + \cos x}{2a + \sin x + \cos x} \, dx$$

$$2I = \int_0^{\pi/2} 1 \, dx$$

$$2I = \left[x \right]_0^{\pi/2} \Rightarrow 2I = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$$

What is $\int_0^{\frac{\pi}{2}} \frac{a+\sin x}{2a+\sin x+\cos x}dx$ equal to ?

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(a) $\frac{\pi}{4}$

(b) $\frac{\pi}{2}$

(c) 1

(d) 0

ANS : (a)

Q) What is $\int_a^b [x]\,dx + \int_a^b [-x]\,dx$ equal to, where [,] is the greatest integer function?

(a) $b-a$ (b) $a-b$ (c) 0 (d) $2(b-a)$

$$\int_a^b [x]\,dx + \int_a^b [-x]\,dx = \int_a^b \left([x] + [-x]\right) dx$$

$$= \int_a^b (-1)\,dx = [-x]_a^b = [x]_b^a$$

$$= a - b$$

$$[2.3] + [-2.3]$$

$$= 2 + (-3) = -1$$

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Q) What is $\int_a^b [x]\,dx + \int_a^b [-x]\,dx$ equal to, where [,] is the greatest integer function?

(a) $b - a$ (b) $a - b$ (c) 0 (d) $2(b - a)$

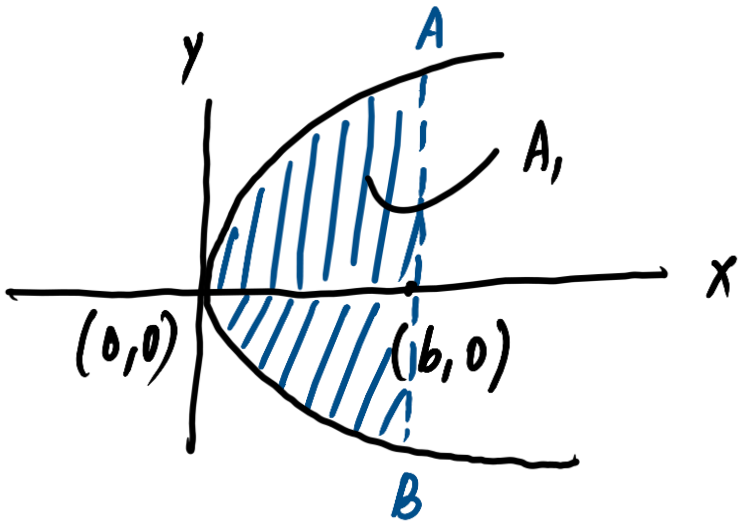
Ans: (b)

Q) What is the area of the parabola $y^2 = 4bx$ bounded by its latus rectum ?

(a) $2b^2/3$ square unit (b) $4b^2/3$ square unit

(c) b^2 square unit (d) $8b^2/3$ square unit

$y^2 = 4bx$



AB – latus rectum ;

$y = 2\sqrt{bx}$

Required area $= 2A_1$

$$= 2\int_{x=0}^{b} 2\sqrt{bx}\,dx$$

$$= 4\sqrt{b}\left[\frac{x^{3/2}}{3/2}\right]_0^b$$

$$4\sqrt{b}\left[\frac{x^{3/2}}{3/2}\right]_0^b$$

$$= 4\sqrt{b} \times \frac{2}{3}\left[x^{3/2}\right]_0^b$$

$$= \frac{8}{3} b^{1/2}\left(b^{3/2} - 0^{3/2}\right)$$

$$= \frac{8}{3} b^{1/2} \cdot b^{3/2}$$

$$= \frac{8}{3} b^2 \Rightarrow \boxed{\frac{8}{3} b^2 \text{ sq. units}}$$

→ Area bounded by $y^2 = 4ax$ with latus rectum $= \frac{8a^2}{3}$

→ Area bounded by $y^2 = 4ax$ with latus rectum and x-axis, $= \frac{4a^2}{3}$

Q) What is the area of the parabola $y^2 = 4bx$ bounded by its latus rectum ?

(a) $2b^2/3$ square unit

(b) $4b^2/3$ square unit

(c) b^2 square unit

(d) $8b^2/3$ square unit

Ans: (d)

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Q) The value of $\int_0^{\frac{\pi}{4}} \sqrt{\tan x}\, dx + \int_0^{\frac{\pi}{4}} \sqrt{\cot x}\, dx$ is equal to

(a) $\frac{\pi}{4}$

(b) $\frac{\pi}{2}$

(c) $\frac{\pi}{2\sqrt{2}}$

(d) $\frac{\pi}{\sqrt{2}}$

$$I = \int_0^{\pi/4} \sqrt{\tan x} + \sqrt{\cot x}\, dx$$

$$= \int_0^{\pi/4} \frac{\sqrt{\sin x}}{\sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x}}\, dx$$

$$\frac{\sin x + \cos x}{\sqrt{\sin x \cos x}}$$

$$\frac{\sin x + \cos x}{\sqrt{\sin x \cos x}} = \frac{\sqrt{2}(\sin x + \cos x)}{\sqrt{2\sin x \cos x}}$$

$$\sqrt{1 - (\sin^2 x + \cos^2 x - 2\sin x \cos x)}$$

$$\sqrt{1 - (\sin x - \cos x)^2}$$

$$I = \int_0^{\pi/4} \frac{\sqrt{2}(\sin x + \cos x)}{\sqrt{1 - (\sin x - \cos x)^2}}\, dx$$

Let $t = \sin x - \cos x$

$dt = (\cos x + \sin x)\, dx$

$x = 0 \Rightarrow t = -1$

$x = \frac{\pi}{4} \Rightarrow t = 0$

$$= \sqrt{2} \int_{-1}^{0} \frac{dt}{\sqrt{1-t^2}} = \sqrt{2}\left[\sin^{-1}(t)\right]_{-1}^{0} = \sqrt{2}\left(\sin^{-1}(0) - \sin^{-1}(-1)\right)$$

$\left(\sin^{-1}(0) + \sin^{-1}\left(\frac{\pi}{2}\right)\right)$

$$= \sqrt{2}\left(0 + \frac{\pi}{2}\right) = \frac{\pi}{\sqrt{2}} \text{ sq. units}$$

Q) The value of $\int_0^{\frac{\pi}{4}} \sqrt{\tan x}\, dx + \int_0^{\frac{\pi}{4}} \sqrt{\cot x}\, dx$ is equal to

(a) $\frac{\pi}{4}$

(b) $\frac{\pi}{2}$

(c) $\frac{\pi}{2\sqrt{2}}$

(d) $\frac{\pi}{\sqrt{2}}$

Ans: (d)

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Q) What is $\int \frac{dx}{2x^2 - 2x + 1}$ equal to?

(a) $\frac{\tan^{-1}(2x-1)}{2} + c$

(b) $2\tan^{-1}(2x-1) + c$

(c) $\frac{\tan^{-1}(2x+1)}{2} + c$

(d) $\tan^{-1}(2x-1) + c$

Q) What is $\int \frac{dx}{2x^2 - 2x + 1}$ equal to?

(a) $\frac{\tan^{-1}(2x-1)}{2} + c$

(b) $2 \tan^{-1}(2x-1) + c$

(c) $\frac{\tan^{-1}(2x+1)}{2} + c$

(d) $\tan^{-1}(2x-1) + c$

Ans: (d)

Q) $\int_0^{\frac{\pi}{2}} |\sin x - \cos x| dx$ is equal to

(a) 0
(b) $2(\sqrt{2} - 1)$
(c) $2\sqrt{2}$
(d) $2(\sqrt{2} + 1)$

$$\int_0^{\pi/4} (\cos x - \sin x)\, dx + \int_{\pi/4}^{\pi/2} (\sin x - \cos x)\, dx$$

Q) $\int_0^{\frac{\pi}{2}} |\sin x - \cos x| dx$ is equal to

(a) 0
(b) $2(\sqrt{2} - 1)$
(c) $2\sqrt{2}$
(d) $2(\sqrt{2} + 1)$

Ans: (b)

Q) What is $\int \ln(x^2)dx$ equal to?

(a) $2x\ln(x) - 2x + C$

(b) $\frac{2}{x} + C$

(c) $2x\ln(x) + C$

(d) $\frac{2\ln(x)}{x} - 2x + C$

$I = \int 2 \ln x \, dx$

$= 2\int \ln x \, dx$

Q) What is $\int \ln(x^2)dx$ equal to?

(a) $2x\ln(x) - 2x + C$

(b) $\frac{2}{x} + C$

(c) $2x\ln(x) + C$

(d) $\frac{2\ln(x)}{x} - 2x + C$

Ans: (a)

Q) If $I_1 = \int_e^{e^2} \frac{dx}{\log x}$ and $I_2 = \int_1^2 \frac{e^x}{x} dx$, then

(a) $I_1 = I_2$ (b) $2I_1 = I_2$

(c) $I_2 + I_1 = 0$ (d) $I_1 = 2I_2$

$$I_1 = \int_e^{e^2} \frac{dx}{\log x}$$

Let $\log x = t \Rightarrow x = e^t$

$dx = e^t\, dt$

$x = e \Rightarrow t = 1$ & $x = e^2 \Rightarrow t = 2$

$$\int_1^2 \frac{e^t dt}{t} \qquad I_2 = \int_1^2 \frac{e^x dx}{x}$$

just different variable

$(I_1 = I_2)$

Q) If $I_1 = \int_e^{e^2} \frac{dx}{\log x}$ and $I_2 = \int_1^2 \frac{e^x}{x} dx$, then

(a) $I_1 = I_2$

(b) $2I_1 = I_2$

(c) $I_2 + I_1 = 0$

(d) $I_1 = 2I_2$

Ans: (a)

Q) What is $\int_0^1 x(1-x)^n\,dx$ equal to?

(a) $\dfrac{1}{n(n+1)}$

(b) $\dfrac{1}{(n+1)(n+2)}$

(c) 1

(d) 0

Q) What is $\int_0^1 x(1-x)^n \, dx$ equal to?

(a) $\frac{1}{n(n+1)}$

(b) $\frac{1}{(n+1)(n+2)}$

(c) 1

(d) 0

Ans: (b)

Q) If $\int_0^{\pi/2} \frac{\cot x}{\cot x + \operatorname{cosec} x} dx = m(\pi + n)$, then $m \cdot n$ is equal to

(a) $-\frac{1}{2}$ (b) 1 (c) $\frac{1}{2}$ (d) -1

Q)If $\int_0^{\pi/2} \frac{\cot x}{\cot x + \operatorname{cosec} x} dx = m(\pi + n)$, then $m \cdot n$ is equal to

(a) $-\frac{1}{2}$ (b) 1 (c) $\frac{1}{2}$ (d) -1

Ans: (d)

Q) What is $\int \frac{\left(x^{e-1} + e^{x-1}\right)dx}{x^e + e^x}$ equal to?

(a) $\frac{x^2}{2} + c$

(b) $\text{In}(x + e) + c$

(c) $\text{In}(x^e + e^x) + c$

(d) $\frac{1}{e}\text{In}\left(x^e + e^x\right) + c$

Q) What is $\int \frac{\left(x^{e-1}+e^{x-1}\right)dx}{x^{e}+e^{x}}$ equal to?

(a) $\frac{x^2}{2}+c$

(b) $\ln(x+e)+c$

(c) $\ln(x^{e}+e^{x})+c$

(d) $\frac{1}{e}\ln\left(x^{e}+e^{x}\right)+c$

Ans: (d)

Q) $\int (\text{In } x)^{-1} dx - \int (\text{In } x)^{-2} dx$ is equal to

(a) $x(\text{In } x)^{-1} + c$

(b) $x(\text{In } x)^{-2} + c$

(c) $x(\text{In } x) + c$

(d) $x(\text{In } x)^{2} + c$

Q) $\int (\text{In } x)^{-1} dx - \int (\text{In } x)^{-2} dx$ is equal to

(a) $x (\text{In } x)^{-1} + c$

(b) $x (\text{In } x)^{-2} + c$

(c) $x (\text{In } x) + c$

(d) $x (\text{In } x)^{2} + c$

Ans: (a)


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