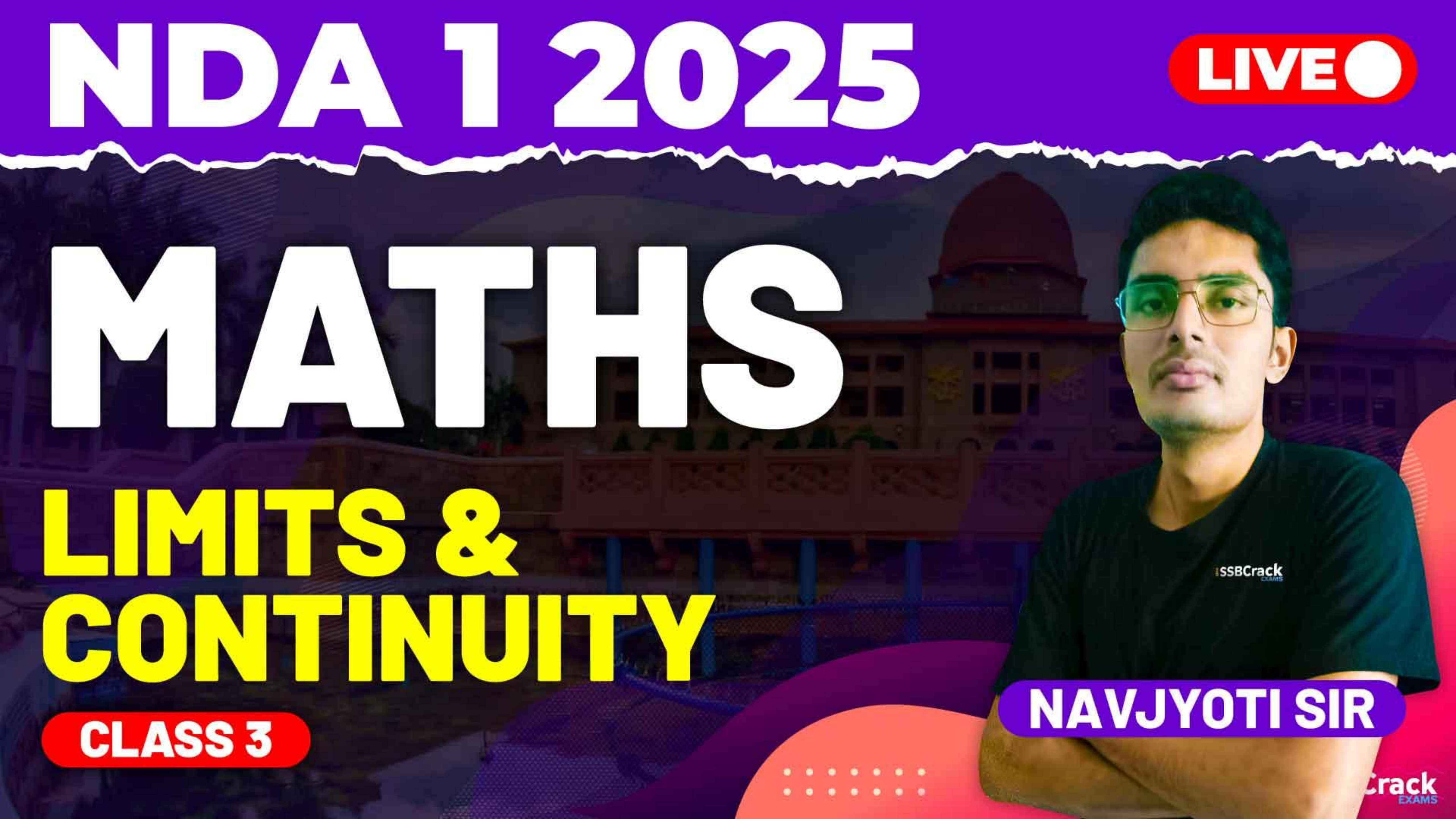
NDA 1 2025
LIVE
MATHS
LIMITS &
CONTINUITY
CLASS 3
SSBCrack EXAMS
NAVJYOTI SIR
Crack EXAMS

SSBCrack EXAMS

02 Dec 2024 Live Classes Schedule

8:00AM	02 DEC 2024 DAILY CURRENT AFFAIRS	RUBY MA'AM
9:00AM	02 DEC 2024 DAILY DEFENCE UPDATES	DIVYANSHU SIR

NDA 1 2025 LIVE CLASSES

1:00PM	PHYSICS - REFRACTION OF LIGHT - CLASS 1	NAVJYOTI SIR
6:30PM	MATHS - LIMITS & CONTINUITY - CLASS 3	NAVJYOTI SIR

CDS 1 2025 LIVE CLASSES

1:00PM	PHYSICS - REFRACTION OF LIGHT - CLASS 1	NAVJYOTI SIR
7:00PM	MATHS - TRIGONOMETRY - CLASS 4	NAVJYOTI SIR

Q) $\lim\limits_{x \to 0} \dfrac{x \cot(4x)}{\sin^2 x \cot^2(2x)}$ is equal to

(a) 0 (b) 1 ✓ (c) 4 (d) 2

$$\frac{x}{\tan 4x} \times \frac{1}{\sin^2 x} \times \tan^2 2x$$

$$\lim_{x \to 0} \frac{\tan x}{x} = \frac{x}{\tan x} = 1$$

$$\frac{1}{4}\left(\frac{4x}{\tan 4x}\right) \times \left(\frac{x^2}{\sin^2 x}\right) \times \frac{1}{\cancel{x^2}} \times \left(\frac{\tan 2x}{2x}\right)^2 \times 4\cancel{x^2}$$

$x \to 0$

$4x \to 0$

$$\frac{1}{4} \times 1 \times 1^2 \times 1^2 \times 4 = 1$$

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Q) $\lim\limits_{x \to 0} \dfrac{x\cot(4x)}{\sin^2 x\cot^2(2x)}$ is equal to

(a) 0 (b) 1 (c) 4 (d) 2

Ans: (b)

Q) The value of $\lim_{x \to 0} \dfrac{\sqrt{\frac{1}{2}(1-\cos^2 x)}}{x}$ is

(a) 1 (b) –1

(c) 0 (d) None of these ✓

$\dfrac{\sqrt{\frac{1}{2}(\sin^2 x)}}{x}$

$\dfrac{1}{\sqrt{2}}\left(\dfrac{\pm \sin x}{x}\right)$

$x < 0 \longrightarrow -\sin x$

$x > 0 \longrightarrow +\sin x$

LHL at $x = 0$

$\lim_{x \to 0^-} \dfrac{1}{\sqrt{2}}\left(\dfrac{-\sin x}{x}\right) = -\dfrac{1}{\sqrt{2}}$

RHL at $x = 0$

$\lim_{x \to 0^+} \dfrac{1}{\sqrt{2}}\left(\dfrac{+\sin x}{x}\right) = +\dfrac{1}{\sqrt{2}}$

As LHL $\neq$ RHL,

limit does not exists.

Q) The value of $\lim_{x \to 0} \dfrac{\sqrt{\frac{1}{2}(1-\cos^2 x)}}{x}$ is

(a) 1

(b) -1

(c) 0

(d) None of these

Ans: (d)

Q) $\lim_{n\to\infty}\left(\frac{1}{1-n^2}+\frac{2}{1-n^2}+\ldots+\frac{n}{1-n^2}\right)$ is equal to

(a) 0

(b) $-\frac{1}{2}$

(c) $\frac{1}{2}$

(d) None of these

$$\frac{1+2+3+4\ldots n}{1-n^2} = \frac{n(n+1)}{2(1+n)(1-n)} = \frac{n}{2(1-n)}$$

$$\lim_{n\to\infty}\frac{\cancel{n}(1)}{2\cancel{n}\left(\frac{1}{n}-1\right)} = \frac{1}{2\left(\frac{1}{n}-1\right)} = \frac{1}{2(0-1)} = -\frac{1}{2}$$

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Q) $\lim_{n \to \infty} \left(\frac{1}{1-n^2} + \frac{2}{1-n^2} + \ldots + \frac{n}{1-n^2} \right)$ is equal to

(a) 0

(b) $-\frac{1}{2}$

(c) $\frac{1}{2}$

(d) None of these

Ans: (b)

Q) If $\lim_{x \to 0} \dfrac{\log(3+x) - \log(3-x)}{x} = k$, the value of k is

(a) $-\dfrac{2}{3}$ (b) 0 (c) $-\dfrac{1}{3}$ (d) $\dfrac{2}{3}$

$$\lim_{x \to 0} \frac{1}{3+x} \cdot (1) - \frac{1}{3-x}(-1)$$

$$\frac{1}{3+x} + \frac{1}{3-x}$$

, putting $x=0$, makes it $\frac{0}{0}$ form,

L-Hopital rule,

$$= \lim_{x \to 0} \frac{6}{9-x^2} = \frac{6}{9} = \frac{2}{3}$$

$\log x = \frac{1}{x}$

$x^n = \underbrace{nx^{n-1}}$

Q) If $\lim\limits_{x \to 0} \dfrac{\log(3+x) - \log(3-x)}{x} = k$, the value of k is

(a) $-\dfrac{2}{3}$ (b) 0 (c) $-\dfrac{1}{3}$ (d) $\dfrac{2}{3}$

Ans: (d)

Q) If $\lim_{x \to a}\left[\frac{f(x)}{g(x)}\right]$ exists, then which one of the following correct ?

(a) Both $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ must exist ✓

(b) $\lim_{x \to a} f(x)$ need not exist but $\lim_{x \to a} g(x)$ must exist

(c) Both $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ need not exist

(d) $\lim_{x \to a} f(x)$ must exist but $\lim_{x \to a} g(x)$ need not exist

If $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exists, then

$\lim_{x \to a} f(x) \begin{matrix} \pm \\ \times \\ \div \end{matrix} \lim_{x \to a} g(x)$ exists and vice - versa,

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Q) If $\lim_{x \to a} \left[\frac{f(x)}{g(x)} \right]$ exists, then which one of the following correct ?

(a) Both $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ must exist

(b) $\lim_{x \to a} f(x)$ need not exist but $\lim_{x \to a} g(x)$ must exist

(c) Both $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ need not exist

(d) $\lim_{x \to a} f(x)$ must exist but $\lim_{x \to a} g(x)$ need not exist

Ans: (a)

Q) What is $\lim_{x \to 0} \dfrac{a^x - b^x}{x}$?

(a) $\log\left(\dfrac{a}{b}\right)$

(b) $\log\left(\dfrac{b}{a}\right)$

(c) ab

(d) $\log(ab)$

$\frac{0}{0}$ form, (L-Hopital rule)

$(a^x)' = a^x \log a$

$(b^x)' = b^x \log b$

$$\lim_{x \to 0} a^x \log a - b^x \log b$$

$$= \log a - \log b = \underbrace{\log\left(\frac{a}{b}\right)}$$

Q) What is $\lim_{x \to 0} \dfrac{a^x - b^x}{x}$?

(a) $\log\left(\dfrac{a}{b}\right)$

(b) $\log\left(\dfrac{b}{a}\right)$

(c) ab

(d) $\log(ab)$

Ans: (a)

Q) What is $\lim_{x \to 0} e^{-\frac{1}{x^2}}$ equal to?

(a) 0
(b) 1
(c) –1
(d) Limit does not exist

$$\lim_{x \to 0} e^{-\frac{1}{0^2}} = e^{-\infty} = 0$$

$$= \underbrace{\left(\frac{1}{e^{\infty}}\right)}$$

$$e^{-\infty} = \frac{1}{e^{\infty}} = 0$$

Q) What is $\lim_{x \to 0} e^{-\frac{1}{x^2}}$ equal to?

(a) 0
(b) 1
(c) –1
(d) Limit does not exist

Ans: (a)

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Q) The function $f(x)$ is given by

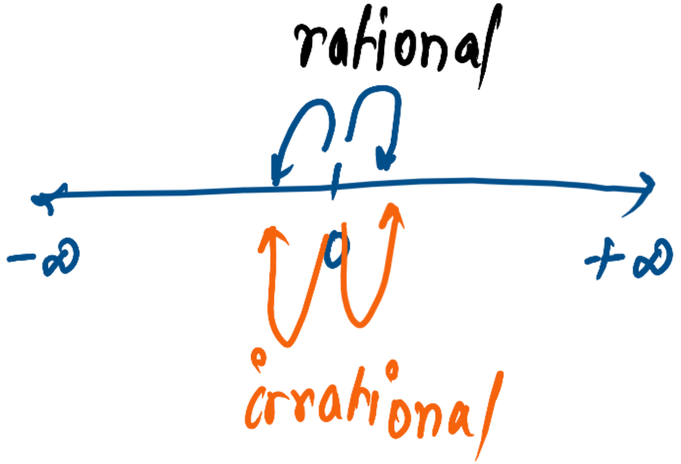
$$f(x)=\begin{cases} x^2, \text{ if } x \text{ is rational} \\ -x^2, \text{ if } x \text{ is irrational}\end{cases}$$ then, it is

(a) continuous at $x=0$

(b) continuous at $x=\frac{1}{2}$

(c) discontinuous at $x=0$

(d) None of the above



Q) The function $f(x)$ is given by $f(x) = \begin{cases} x^2, & \text{if } x \text{ is rational} \\ -x^2, & \text{if } x \text{ is irrational} \end{cases}$ then, it is

(a) continuous at $x = 0$

(b) continuous at $x = \frac{1}{2}$

(c) discontinuous at $x = 0$

(d) None of the above

Ans: (a)

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Q) If the function $f(x) = \dfrac{2x - \sin^{-1} x}{2x + \tan^{-1} x}, (x \neq 0)$ is continuous at each point of its domain, then the value of $f(0)$ is

(a) 2 (b) 1/3 (c) 2/3 (d) −1/3

$\frac{0}{0}$ form,

$$(\sin^{-1}x)' = \frac{1}{\sqrt{1-x^2}}$$

$$(\tan^{-1}x)' = \frac{1}{1+x^2}$$

$$f(0) = \lim_{x\to 0} \frac{2 - \frac{1}{\sqrt{1-x^2}}}{2 + \frac{1}{1+x^2}} = \frac{2 - \frac{1}{1}}{2 + \frac{1}{1+0}} = \frac{1}{3}$$

Q) If the function $f(x) = \dfrac{2x - \sin^{-1} x}{2x + \tan^{-1} x}, (x \neq 0)$ is continuous at each point of its domain, then the value of $f(0)$ is

(a) 2 (b) 1/3 (c) 2/3 (d) –1/3

Ans: (b)

Q) What is $\lim_{\theta\to 0} \frac{\sqrt{1-\cos\theta}}{\theta}$ equal to?

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $-\frac{1}{2\sqrt{2}}$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$\frac{\sqrt{2\sin^2\frac{\theta}{2}}}{\theta} = \frac{\sqrt{2}\sin\frac{\theta}{2}}{\frac{\theta}{2}\times 2} = \frac{\sqrt{2}}{2}\left(\frac{\sin\frac{\theta}{2}}{\frac{\theta}{2}}\right) = \frac{1}{\sqrt{2}}(1)$$

$$= \frac{1}{\sqrt{2}}$$

$$\frac{\theta}{2} \to 0$$

Q) What is $\lim_{\theta \to 0} \frac{\sqrt{1 - \cos\theta}}{\theta}$ equal to?

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $-\frac{1}{2\sqrt{2}}$

Ans: (c)

Q) $f(x) = \cos(|x|)$ is a continuous function because

(a) composition of continuous functions is a continuous function

(b) product of continuous functions is a continuous function

(c) cosine is an even function

(d) sum of continuous functions is continuous

$f(x) = \cos|x|$

continous

continous

$t(x) = \cos x$ $\quad g(x) = |x|$

$\boxed{t \circ g(x) = \cos|x|}$ ← so composition will also be continous,

Q) $f(x) = \cos(|x|)$ is a continuous function because

(a) composition of continuous functions is a continuous function

(b) product of continuous functions is a continuous function

(c) cosine is an even function

(d) sum of continuous functions is continuous

Ans: (a)

Q) What is $\lim_{x \to -1} \dfrac{x^3 + x^2}{x^2 + 3x + 2}$ equal to?

(a) 0 (b) 1 (c) 2 (d) 3

$\frac{0}{0}$ form,

$x^n \longrightarrow nx^{n-1}$

$x^4 \longrightarrow 4x^3$

$$\lim_{x \to -1} \frac{3x^2 + 2x^1}{2x + 3 + 0} = \frac{3-2}{-2+3} = 1$$

Q) What is $\lim_{x \to -1} \dfrac{x^3 + x^2}{x^2 + 3x + 2}$ equal to?

(a) 0 (b) 1 (c) 2 (d) 3

Ans: (b)

Q) Let $f(x)$ be defined as follows

$$f(x) = \begin{cases} 2x+1, & -3 < x < -2 \\ x-1, & -2 \leq x < 0 \\ x+2, & 0 \leq x < 1 \end{cases}$$

Which one of the following statements is correct in respect of the above function?

(a) It is discontinuous at $x = -2$ but continuous at every other point. ✗

(b) It is continuous only in the interval $(-3, -2)$. ✗

(c) It is discontinuous at $x = 0$ but continuous at every other point. ✓

(d) It is discontinuous at every point. ✗

$x = -2$

$f(-2) = -2 - 1 = -3$ ✓

LHL $\longrightarrow$ $2x+1 = 2(-2)+1 = -3$ ✓

RHL $\longrightarrow$ $-2-1 = -3$ ✓

continous at $x = -2$,

Q) Let $f(x)$ be defined as follows

$$f(x)=\begin{cases} 2x+1, & -3<x<-2 \\ x-1, & -2\le x<0 \\ x+2, & 0\le x<1 \end{cases}$$

Which one of the following statements is correct in respect of the above function?

(a) It is discontinuous at $x = -2$ but continuous at every other point.

(b) It is continuous only in the interval $(-3, -2)$.

(c) It is discontinuous at $x = 0$ but continuous at every other point.

(d) It is discontinuous at every point.

Ans: (c)

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Q) If $f(x) = \begin{cases} \dfrac{3x + 4\tan x}{x}; & x \neq 0 \\ k; & x = 0 \end{cases}$ is continuous at $x = 0$,

then the value of k is

(a) 7 (b) 6

(c) – 5 (d) – 1

$$k = \lim_{x \to 0} \frac{3x + 4\tan x}{x}$$

$$\boxed{k = 7}$$

$$= 3 + 4\left(\frac{\tan x}{x}\right)$$

$$= 3 + 4 = 7$$

Q) If $f(x) = \begin{cases} \dfrac{3x + 4\tan x}{x}; x \neq 0 \\ k; \quad x = 0 \end{cases}$ is continuous at $x = 0$,

then the value of k is

(a) 7
(b) 6
(c) – 5
(d) – 1

Ans: (a)

PYQ – 2024 - I

Let $f(x) = |x| + 1$ and $g(x) = [x] - 1$, where [.] is the greatest integer function.

Let $h(x) = \dfrac{f(x)}{g(x)}$

What is $\lim_{x \to 0-} h(x) + \lim_{x \to 0+} h(x)$ equal to ?

(a) $-\frac{3}{2}$

(b) $-\frac{1}{2}$

(c) $\frac{1}{2}$

(d) $\frac{3}{2}$

$$\lim_{x \to 0^-} h(x) = \frac{|x|+1}{[x]-1}$$

$\left(x \to 0^- ; \quad x = 0 - h ; \quad x = -h \right)$

$\lim_{x \to 0^-} \Rightarrow \lim_{h \to 0}$

$$= \lim_{h \to 0} \frac{|0-h|+1}{[0-h]-1} = \frac{1}{-1-1} = \underline{\underline{-\frac{1}{2}}}$$

$[5.7] = 5$

5, 6 (between integers)

$$\lim_{x \to 0^+} h(x) \qquad x = 0 + h$$

$$\lim_{h \to 0} \frac{|0+h| + 1}{[0+h] - 1} = \frac{1}{0-1} = -1$$

$$\lim_{x \to 0^-} h(x) + \lim_{x \to 0^+} h(x) = \frac{-1}{2} - 1 = -\frac{3}{2}$$

Let $f(x) = |x| + 1$ and $g(x) = [x] - 1$, where [.] is the greatest integer function.

PYQ – 2024 - I

Let $h(x) = \dfrac{f(x)}{g(x)}$

What is $\lim_{x \to 0-} h(x) + \lim_{x \to 0+} h(x)$ equal to ?

(a) $-\dfrac{3}{2}$

(b) $-\dfrac{1}{2}$

(c) $\dfrac{1}{2}$

(d) $\dfrac{3}{2}$

Ans: (a)

What is $\lim_{x \to \frac{\pi}{2}} (\sec\theta - \tan\theta)$ equal to?

PYQ – 2024 - II

(a) -1

(b) 0

(c) $1/2$

(d) 1

$$\lim_{x \to \frac{\pi}{2}} \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$

$$(\sin\theta)' = \cos\theta$$

$$(\cos\theta)' = -\sin\theta$$

$\frac{0}{0}$ form,

$$\frac{-\cos\theta}{-\sin\theta} = \frac{0}{1} = 0$$

What is $\lim_{x \to \frac{\pi}{2}} (\sec\theta - \tan\theta)$ equal to?

PYQ – 2024 - II

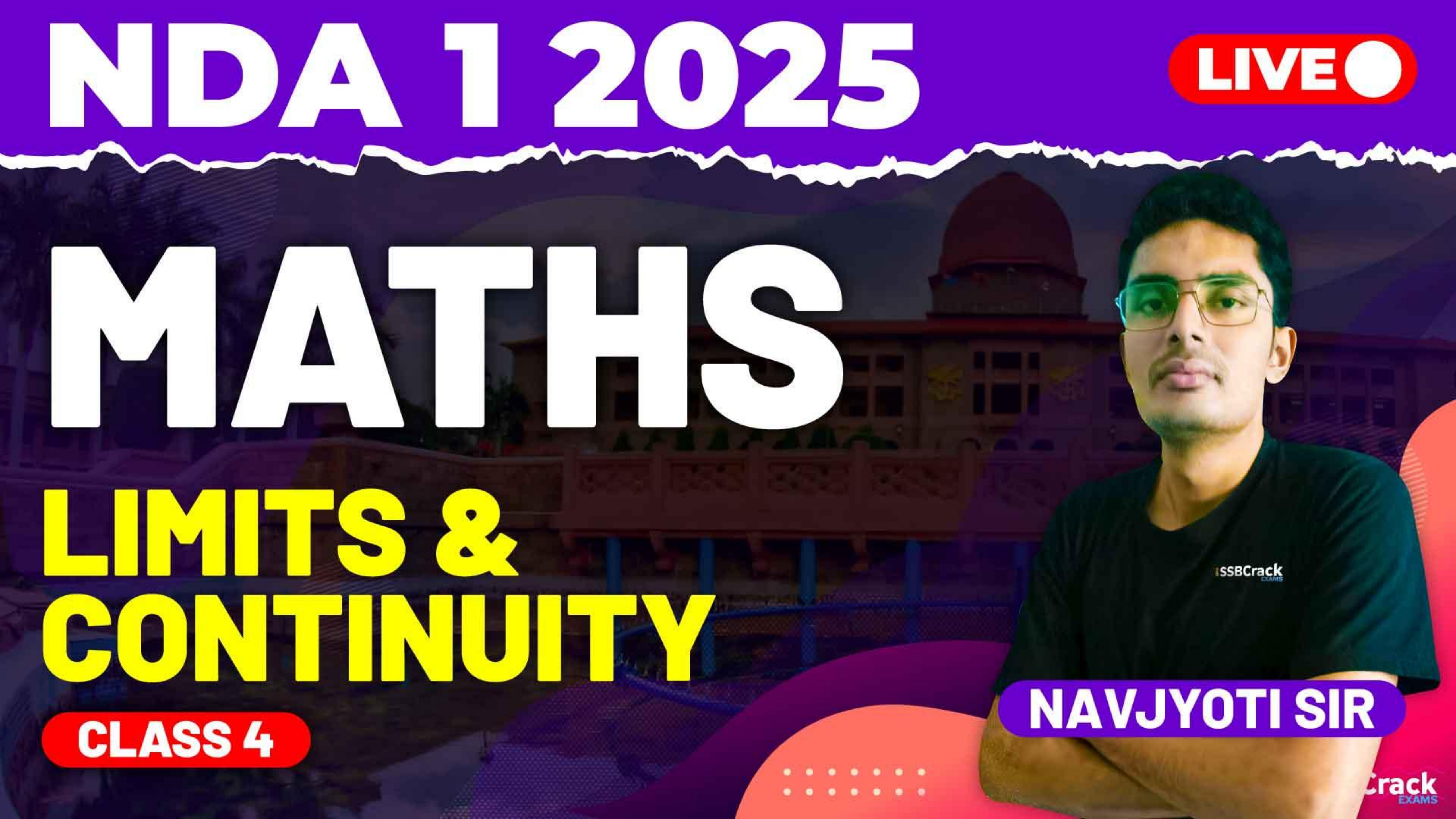
(a) -1

(b) 0

(c) $1/2$

(d) 1

Ans: (b)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