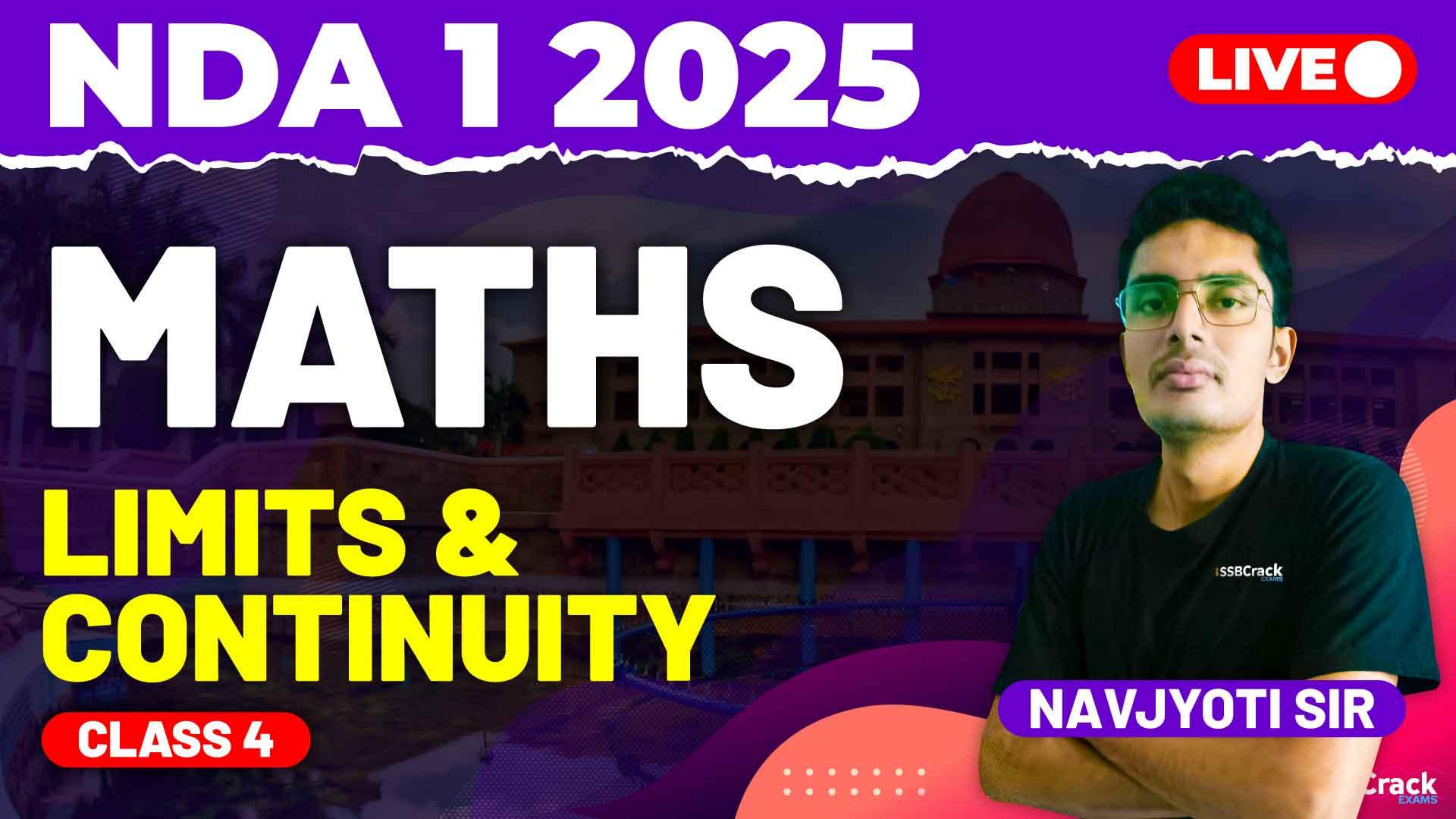
NDA 1 2025
LIVE
MATHS
LIMITS &
CONTINUITY
CLASS 4
NAVJYOTI SIR
SSBCrack EXAMS
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SSBCrack EXAMS

# 03 Dec 2024 Live Classes Schedule

| | | |
|---|---|---|
| 9:00AM | 03 DEC 2024 DAILY DEFENCE UPDATES | DIVYANSHU SIR |

## SSB INTERVIEW LIVE CLASSES

| | | |
|---|---|---|
| 9:30AM | ONLINE COURSE INTRO | ANURADHA MA'AM |

## NDA 1 2025 LIVE CLASSES

| | | |
|---|---|---|
| 1:00PM | PHYSICS - REFRACTION OF LIGHT - CLASS 2 | NAVJYOTI SIR |
| 4:30PM | ENGLISH - ADAPTATION OF BORROWED WORDS - CLASS 1 | ANURADHA MA'AM |
| 5:30PM | MATHS - LIMITS & CONTINUITY - CLASS 4 | NAVJYOTI SIR |

## CDS 1 2025 LIVE CLASSES

| | | |
|---|---|---|
| 1:00PM | PHYSICS - REFRACTION OF LIGHT - CLASS 2 | NAVJYOTI SIR |
| 4:30PM | ENGLISH - ADAPTATION OF BORROWED WORDS - CLASS 1 | ANURADHA MA'AM |
| 7:00PM | MATHS - TRIGONOMETRY - CLASS 5 | NAVJYOTI SIR |

**Q)** If $f(x) = \dfrac{[x]}{|x|}, x \neq 0,$

where [ ] denotes the greatest integer function, then what is the right-hand limit of $f(x)$ at $x = 1$?

(a) – 1

(b) 0

(c) 1 ✓

(d) Right-hand limit of $f(x)$ at $x = 1$ does not exist

$$\lim_{x \to 1^+} f(x)$$

$$= \frac{[x]}{|x|}$$

$$= \lim_{h \to 0} \frac{[1+h]}{|1+h|} = \frac{1}{1} = 1$$

**Q)** If $f(x) = \frac{[x]}{|x|}, x \neq 0,$

where [ ] denotes the greatest integer function, then what is the right-hand limit of $f(x)$ at $x = 1$?

(a) $-1$

(b) 0

(c) 1

(d) Right-hand limit of $f(x)$ at $x = 1$ does not exist

**Ans: (c)**

**Q)** Consider the following function $f: R \to R$ such that $f(x) = x$ if $x \geq 0$ and $f(x) = -x^2$ if $x < 0$. Then, which one of the following is correct?

(a) $f(x)$ is continuous at every $x \in R$ ✓

(b) $f(x)$ is continuous at $x = 0$ only

(c) $f(x)$ is discontinous at $x = 0$ only

(d) $f(x)$ is discontinuous at every $x \in R$

LHL = RHL = $f(0) = 0$ $\Rightarrow$ $f(x)$ is continous at $x = 0$.

**Q)** Consider the following function $f : R \to R$ such that $f(x) = x$ if $x \geq 0$ and $f(x) = -x^2$ if $x < 0$. Then, which one of the following is correct?

(a) $f(x)$ is continuous at every $x \in R$

(b) $f(x)$ is continuous at $x = 0$ only

(c) $f(x)$ is discontinous at $x = 0$ only

(d) $f(x)$ is discontinuous at every $x \in R$

**Ans: (a)**

SSBCrack EXAMS

**Q)** A function $f$ is defined as follows

$$f(x) = x^p \cos\left(\frac{1}{x}\right), x \neq 0, f(0) = 0.$$

What conditions should be imposed on $p$, so that $f$ may be continuous at $x = 0$?

(a) $p = 0$ ✗ (b) $p > 0$

(c) $p < 0$ (d) No value of $p$

$$0 = \lim_{x \to 0} x^p \underbrace{\cos\left(\frac{1}{x}\right)}_{-1 \text{ to } 1}$$

$x^p = 0$ $(x \sim 0)$

$(0.001)^3$

$\left(\frac{1}{(0.001)^3}\right.$ nowhere near to zero$)$

SSBCrack EXAMS

**Q)** A function $f$ is defined as follows

$$f(x) = x^p \cos\left(\frac{1}{x}\right), x \neq 0, f(0) = 0.$$

What conditions should be imposed on $p$, so that $f$ may be continuous at $x = 0$?

(a) $p = 0$ (b) $p > 0$

(c) $p < 0$ (d) No value of $p$

**Ans: (b)**

**Q)** Let [.] denotes the greatest integer function and $f(x) = [\tan^2 x]$, then

(a) $\lim_{x \to 0} f(x)$ does not exist ✗

✓ (b) $f(x)$ is continuous at $x = 0$

(c) $f(x)$ is not differentiable at $x = 0$

(d) $f'(0) = 1$

$$\lim_{x \to 0} [\tan^2 x] = 0 = f(0)$$

**Q)** Let [.] denotes the greatest integer function and $f(x) = [\tan^2 x]$, then

(a) $\lim_{x \to 0} f(x)$ does not exist

(b) $f(x)$ is continuous at $x = 0$

(c) $f(x)$ is not differentiable at $x = 0$

(d) $f'(0) = 1$

**Ans: (b)**

**Q)** The function $f(x) = \dfrac{\log(1 + ax) - \log(1 - bx)}{x}$ is not defined at $x = 0$. The value which should be assigned to $f$ at $x = 0$, so that it is continuous at $x = 0$, is

(a) $a - b$
(b) $a + b$
(c) $\log a + \log b$
(d) None of these

$$f(0) = \lim_{x \to 0} \frac{\log(1+ax) - \log(1-bx)}{x}$$

$\left(\frac{0}{0} \text{ form}\right)$

L - Hopital rule,

$$\lim_{x \to 0} \frac{\frac{1}{1+ax} \cdot a - \frac{1}{1-bx}(-b)}{1}$$

$$= \frac{a}{1} + \frac{b}{1}$$

$$= a + b$$

SSBCrack EXAMS

Q) The function $f(x) = \dfrac{\log(1 + ax) - \log(1 - bx)}{x}$

is not defined at $x = 0$. The value which should be assigned to $f$ at $x = 0$, so that it is continuous at $x = 0$, is

(a) $a - b$
(b) $a + b$
(c) $\log a + \log b$
(d) None of these

Ans: (b)

**Q)** $\lim_{x\to 0} \dfrac{\sin\left(\pi \cos^2 x\right)}{x^2}$ is equal to:

(a) $-\pi$ (b) $\pi$ (c) $\dfrac{\pi}{2}$ (d) 1

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$\sin(\pi - \theta) = \underline{\sin\theta}$$

$$\lim_{x\to 0} \frac{\sin\left(\pi\left(1 - \sin^2 x\right)\right)}{x^2} \qquad \frac{\sin\left(\pi - \pi\sin^2 x\right)}{x^2} = \frac{\sin(\pi\sin^2 x)}{x^2}$$

$$\left[\lim_{\pi\sin^2 x\to 0} \frac{\sin(\pi\sin^2 x)}{\pi\sin^2 x}\right] \times \frac{\pi\sin^2 x}{x^2} = 1 \times \lim_{x\to 0} \pi\left(\frac{\sin x}{x}\right)^2 = \pi(1)^2$$

$$= \pi$$

**Q)** $\lim_{x \to 0} \frac{\sin\left(\pi \cos^2 x\right)}{x^2}$ is equal to:

(a) $-\pi$ (b) $\pi$ (c) $\frac{\pi}{2}$ (d) $1$

**Ans: (b)**

**Q)** What is $\lim_{x \to \frac{\pi}{6}} \frac{2\sin^2 x + \sin x - 1}{2\sin^2 x - 3\sin x + 1}$ equal to?

(a) $-\frac{1}{2}$ (b) $-\frac{1}{3}$ (c) $-2$ (d) $-3$

$\frac{0}{0}$ form,

$$\frac{2\sin^2 x + 2\sin x - \sin x - 1}{2\sin^2 x - 2\sin x - \sin x + 1}$$

$$= \frac{(2\sin x - 1)(\sin x + 1)}{(2\sin x - 1)(\sin x - 1)} = \frac{\sin x + 1}{\sin x - 1}$$

$$\lim_{x \to \frac{\pi}{6}} \frac{\sin x + 1}{\sin x - 1}$$

$$= \frac{\frac{1}{2} + 1}{\frac{1}{2} - 1} = \frac{\frac{3}{2}}{-\frac{1}{2}}$$

$$= -3$$

**Q)** What is $\lim_{x \to \frac{\pi}{6}} \dfrac{2\sin^2 x + \sin x - 1}{2\sin^2 x - 3\sin x + 1}$ equal to?

(a) $-\frac{1}{2}$ (b) $-\frac{1}{3}$ (c) $-2$ (d) $-3$

Using L-Hopital rule,

$$\lim_{x \to \frac{\pi}{6}} \frac{4\sin x\cos x + \cos x}{4\sin x\cos x - 3\cos x}$$

$$\frac{4 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}}{\sqrt{3} - \frac{3\sqrt{3}}{2}} = \frac{\sqrt{3} + \frac{\sqrt{3}}{2}}{\sqrt{3} - \frac{3\sqrt{3}}{2}}$$

$$\frac{\sqrt{3}\left(1 + \frac{1}{2}\right)}{\sqrt{3}\left(1 - \frac{3}{2}\right)}$$

$$\frac{\frac{3}{2}}{-\frac{1}{2}} = -3$$

**Ans: (d)**

Q) $\lim\limits_{x \to 0} \dfrac{\sin^{-1} x - \tan^{-1} x}{x^3}$ is equal to

(a) 0 (b) 1 (c) $-1$ (d) $1/2$

$x^{-1} = (-1)x^{-1-1}$

$= -x^{-2} = -1/x^2$

$\frac{0}{0}$ form, L-Hopital rule,

$$\lim_{x\to 0} \frac{\frac{1}{\sqrt{1-x^2}} - \frac{1}{1+x^2}}{3x^2} = \lim_{x\to 0} \frac{-\frac{1}{2}(1-x^2)^{-\frac{3}{2}} \cdot (-2\cancel{x}) - \frac{-1}{(1+x^2)^2}(2\cancel{x})}{6\cancel{x}}$$

$$= \lim_{x\to 0} \frac{\frac{1}{(1-x^2)^{3/2}} + \frac{2}{(1+x^2)^2}}{6} = \frac{1+2}{6} = \frac{1}{2}$$

SSBCrack EXAMS

**Q)** $\lim\limits_{x \to 0} \dfrac{\sin^{-1} x - \tan^{-1} x}{x^3}$ is equal to

(a) 0 (b) 1 (c) –1 (d) 1/2

**Ans: (d)**

**Q)** Consider the function

$$f(x)=\begin{cases} ax-2 & \text{for } -2<x<-1 \\ -1 & \text{for } -1\le x\le 1 \\ a+2(x-1)^2 & \text{for } 1<x<2 \end{cases}$$

What is the value of a for which f(x) is continuous at $x=-1$ and $x=1$?

(a) $-1$ ✓ (b) $1$

(c) $0$ (d) $2$

At $x=-1$

$f(-1) = \text{RHL at } x=-1 = -1$

$\text{LHL} = -1$

$\lim_{x \to -1^-} f(x) = -1$

$\lim_{x \to -1} ax-2 = -1$

$a(-1)-2 = -1$

$-a-2 = -1$

$a = -1$

Q) Consider the function

$$f(x)=\begin{cases} ax-2 & \text{for} \quad -2<x<-1 \\ -1 & \text{for} \quad -1\le x\le 1 \\ a+2(x-1)^2 & \text{for} \quad 1<x<2 \end{cases}$$

What is the value of a for which f(x) is continuous at $x=-1$ and $x=1$?

(a) $-1$

(b) $1$

(c) $0$

(d) $2$

Ans: (a)

**Q)** If $\lim_{x \to \infty} \left[ \dfrac{x^3+1}{x^2+1} - (ax+b) \right] = 2$, then

(a) $a = 1$ and $b = 1$

(b) $a = 1$ and $b = -1$

(c) $a = 1$ and $b = -2$

(d) $a = 1$ and $b = 2$

$$\frac{x^3+1}{x^2+1} - (ax+b)$$

$$\frac{x^3+1-ax^3-bx^2-ax-b}{x^2+1} = \frac{x^3(1-a)-bx^2-ax-b+1}{x^2+1}$$

$$\frac{x^3(1-a) - bx^2 - ax - b + 1}{x^2+1}$$

$$\lim_{x \to \infty} \frac{x^2\left(x(1-a) - b - \frac{a}{x} - \frac{b}{x^2} + \frac{1}{x^2}\right)}{x^2\left(1 + \frac{1}{x^2}\right)} = 2$$

$$x(1-a) - b = 2$$

$$(1-a) = 0 \Rightarrow \boxed{a = 1}$$

$$0 - b = 2 \Rightarrow \boxed{b = -2}$$

**Q)** If $\lim_{x \to \infty} \left[ \frac{x^3 + 1}{x^2 + 1} - (ax + b) \right] = 2$, then

(a) $a = 1$ and $b = 1$

(b) $a = 1$ and $b = -1$

(c) $a = 1$ and $b = -2$

(d) $a = 1$ and $b = 2$

**Ans: (c)**

**Q)** What is $\lim\limits_{n \to \infty} \dfrac{a^n + b^n}{a^n - b^n}$

where $a > b > 1$, equal to?

(a) – 1 (b) 0

(c) 1

(d) Limit does not exist

$\frac{b}{a} < 1$

$\left(\frac{b}{a}\right)^n \longrightarrow 0$

$$\lim_{n \to \infty} \frac{a^n\left(1 + \left(\frac{b}{a}\right)^n\right)}{a^n\left(1 - \left(\frac{b}{a}\right)^n\right)} = \frac{1+0}{1-0} = 1$$

**Q)** What is $\lim_{n \to \infty} \dfrac{a^n + b^n}{a^n - b^n}$

where $a > b > 1$, equal to?

(a) – 1 (b) 0

(c) 1

(d) Limit does not exist

**Ans: (c)**

**Q)** Let $f(x) = \begin{cases} 1 + \dfrac{x}{2k}, & 0 < x < 2 \\ kx, & 2 \le x < 4 \end{cases}$

If $\lim_{x \to 2} f(x)$ exists, then what is the value of $k$?

(a) – 2 (b) – 1

(c) 0 (d) 1 ✓

$$(\text{LHL at } x = 2) = (\text{RHL at } x = 2)$$

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$$

$$\lim_{x \to 2} 1 + \frac{x}{2k} = \lim_{x \to 2} kx$$

$$1 + \frac{1}{k} = 2k$$

$$2k^2 - k - 1 = 0$$

(a) $k = -2$ ✗

(b) $k = -1$ ✗

(c) $k = 0$ ✗

(d) $k = 1$ ✓

put options and check

**Q)** Let $f(x) = \begin{cases} 1 + \dfrac{x}{2k}, & 0 < x < 2 \\ kx, & 2 \le x < 4 \end{cases}$

If $\lim\limits_{x \to 2} f(x)$ exists, then what is the value of $k$?

(a) – 2 (b) – 1

(c) 0 (d) 1

**Ans: (d)**

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**Q)** Consider the following statements in respect of the function.

$f(x) = \sin\left(\frac{1}{x^2}\right), x \neq 0.$

1. It is continuous at $x = 0$, if $f(0) = 0$. ✗
2. It is continuous at $x = \frac{2}{\sqrt{\pi}}$. ✓

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

$$\lim_{x \to 0} \sin\left(\frac{1}{x^2}\right) = \sin(\infty)$$

$-1$ —— $1$ (not necessarily 0)

$$f\left(\frac{2}{\sqrt{\pi}}\right) = \sin\left(\frac{1}{\frac{4}{\pi}}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\lim_{x \to \frac{2}{\sqrt{\pi}}} f(x) = \frac{1}{\sqrt{2}}$$

SSBCrack EXAMS

**Q)** Consider the following statements in respect of the function.

$$f(x) = \sin\left(\frac{1}{x^2}\right),\ x \neq 0.$$

1. It is continuous at $x = 0$, if $f(0) = 0$.
2. It is continuous at $x = \dfrac{2}{\sqrt{\pi}}$.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

**Ans: (b)**

**Q)** If $f(x)=\sqrt{25-x^2}$, then what is $\lim_{x\to 1}\frac{f(x)-f(1)}{x-1}$ equal to?

(a) $-\frac{1}{\sqrt{24}}$ (b) $\frac{1}{\sqrt{24}}$ (c) $-\frac{1}{4\sqrt{3}}$ (d) $\frac{1}{\sqrt{4\sqrt{3}}}$

$$\lim_{x\to 1}\frac{\sqrt{25-x^2}-\sqrt{24}}{x-1}$$

$\frac{0}{0}$ form, L-Hopital rule,

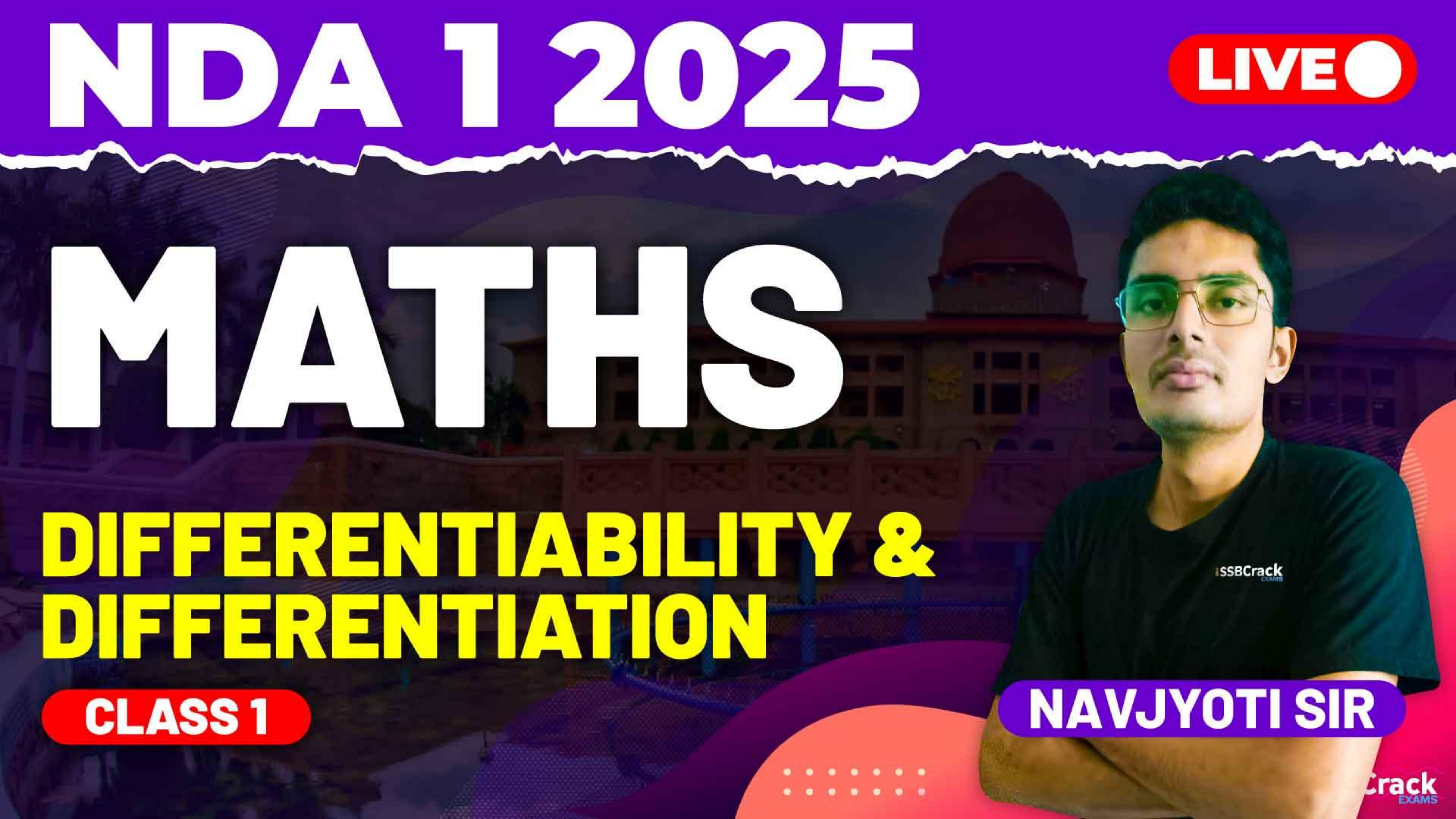
$$\lim_{x\to 1}\frac{\frac{1}{2\sqrt{25-x^2}}(-2x)-0}{1}$$

$$\lim_{x\to 1}-\frac{x}{\sqrt{25-x^2}}=\frac{-1}{\sqrt{24}}$$

**Q)** If $f(x)=\sqrt{25-x^2}$, then what is $\lim_{x\to 1}\frac{f(x)-f(1)}{x-1}$ equal to?

(a) $-\frac{1}{\sqrt{24}}$ (b) $\frac{1}{\sqrt{24}}$ (c) $-\frac{1}{4\sqrt{3}}$ (d) $\frac{1}{\sqrt{4\sqrt{3}}}$

**Ans: (a)**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