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LIVE

MATHS

PROBABILITY

CLASS 2

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Crack EXAMS

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CONDITIONAL PROBABILITY

If E and F are two events associated with the same sample space of a random experiment, then the conditional probability of the event E under the condition that the event F has occurred, written as P (E | F), is given by

$$P(E \mid F) = \frac{P(E \cap F)}{P(F)}, \quad P(F) \neq 0$$

E such that F has occured,

$$\frac{n(E \cap F)}{n(F)}$$

$$= \frac{\dfrac{n(E \cap F)}{n(S)}}{\dfrac{n(F)}{n(S)}} = \frac{P(E \cap F)}{P(F)}$$

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CONDITIONAL PROBABILITY

$$P(E \mid F) = \frac{P(E \cap F)}{P(F)}, \quad P(F) \neq 0$$

Example - You have a fair six-sided die. You want to determine the probability of rolling an even number, given that the number rolled is greater than four.

E : rolling an even number

F : number rolled is greater than 4.

$\frac{1}{2}$

$S = \{1, 2, 3, 4, 5, 6\}$

reduction in

$F = \{5, 6\}$ sample space,

$E \cap F$ = rolling an even number greater than 4.

$E \cap F = \{6\} \longrightarrow$

$$E \cap F = \{6\}$$

$$P(E \cap F) = \frac{n(E \cap F)}{n(S)} = \frac{1}{6}$$

$$F = \{5, 6\}$$

$$P(F) = \frac{2}{6} = \frac{1}{3}$$

$$P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{1/6}{1/3} = \frac{3}{6} = \frac{1}{2}$$

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CONDITIONAL PROBABILITY

Example - Event A: Drawing a four Event B: Drawing a red card

A/B : getting a four from red cards

B/A : getting a red card out of (cards of four)

$P(A) = \frac{4}{52} = \frac{1}{13}$

$P(B) = \frac{26}{52} = \frac{1}{2}$

$P(A \cap B) = \frac{n(A \cap B)}{52} = \frac{2}{52} = \frac{1}{26}$

$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1/26}{1/2} = \frac{1}{13}$

$P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{P(A \cap B)}{P(A)} = \frac{1/26}{1/13} = 1/2$

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MULTIPLICATION THEOREM

Let E and F be two events associated with a sample space of an experiment. Then

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \qquad P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A \cap B) = P(B) \cdot P(A/B) = P(A) \cdot P(B/A)$$

MULTIPLICATION THEOREM

If E, F and G are three events associated with a sample space, then

$$\boxed{P(E \cap F \cap G) = P(E)\,P(F \mid E)\,P(G \mid E \cap F)}$$

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INDEPENDENT EVENTS

Let E and F be two events associated with a sample space S. If the probability of occurrence of one of them is not affected by the occurrence of the other, then we say that the two events are independent.

Thus, two events E and F will be independent, if

(a) $P(F \mid E) = P(F)$, provided $P(E) \neq 0$

(b) $P(E \mid F) = P(E)$, provided $P(F) \neq 0$

$$P(E \cap F) = P(E)\,P(F/E) = P(E) \cdot P(F)$$

INDEPENDENT EVENTS

$$\boxed{P(A \cap B) = P(A)\,P(B)}$$

$$\boxed{P(A \cap B \cap C) = P(A)\,P(B)\,P(C)}$$

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THEOREM OF TOTAL PROBABILITY

$$P(A) = \sum_{j=1}^{n} P(E_j)P(A|E_j)$$

$$P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + \ldots\ldots P(E_n)P(A/E_n)$$

$E_1, E_2 \ldots\ldots E_n$

$$\left\{ \begin{array}{l} E_i \cap E_j = \phi \quad (i \neq j) \\ E_1 \cup E_2 \cup E_3 \cup \ldots E_n = S \end{array} \right.$$

mutually exclusive and exhaustive,

BAYES' THEOREM

If $E_1, E_2, \ldots, E_n$ are mutually exclusive and exhaustive events associated with a sample space, and A is any event of non zero probability, then

$$\boxed{P(E_i \mid A) = \frac{P(E_i)P(A \mid E_i)}{\sum_{i=1}^{n} P(E_i)P(A \mid E_i)}}$$

reverse conditional probability,

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EXAMPLE

The chances of defective screws in three boxes A, B and C are $\frac{1}{5}$, $\frac{1}{6}$ and $\frac{1}{7}$, respectively. A box is selected at random and a screw drawn from it at random is found to be defective. Find the probability that it came from box A is

(a) $\frac{42}{107}$ (b) $\frac{41}{141}$ (c) $\frac{42}{243}$ (d) None of these

D : getting a defective screw

A : Selecting box A

B : " " B

C : " " C

$P(A) = \frac{1}{3}$; $P(B) = \frac{1}{3}$; $P(C) = \frac{1}{3}$

$P(D/A) = \frac{1}{5}$; $P(D/B) = \frac{1}{6}$; $P(D/C) = \frac{1}{7}$

$P(A/D) = ?$

$$P(A) = \frac{1}{3} \; ; \; P(B) = \frac{1}{3} \; ; \; P(C) = \frac{1}{3}$$

$$P(D/A) = \frac{1}{5} \; ; \; P(D/B) = \frac{1}{6} \; ; \; P(D/C) = \frac{1}{7}$$

$$P(A/D) = ?$$

Using Bayes' Theorem,

$$P(A/D) = \frac{P(A)\,P(D/A)}{P(A)\,P(D/A) + P(B)\,P(D/B) + P(C)\,P(D/C)} = \frac{\frac{1}{3} \times \frac{1}{5}}{\frac{1}{3} \times \frac{1}{5} + \frac{1}{3} \times \frac{1}{6} + \frac{1}{3} \times \frac{1}{7}}$$

$$\frac{\frac{1}{3}\times\frac{1}{5}}{\frac{1}{3}\times\frac{1}{5}+\frac{1}{3}\times\frac{1}{6}+\frac{1}{3}\times\frac{1}{7}}$$

$$=\frac{\frac{1}{15}}{\frac{1}{3}\left(\frac{1}{5}+\frac{1}{6}+\frac{1}{7}\right)} = \frac{\frac{1}{5}}{\frac{42+35+30}{5\times6\times7}} = \quad 42 \times \frac{1}{107} = \frac{42}{107}$$

EXAMPLE

The chances of defective screws in three boxes A, B and C are $\frac{1}{5}$, $\frac{1}{6}$ and $\frac{1}{7}$, respectively. A box is selected at random and a screw drawn from it at random is found to be defective. Find the probability that it came from box A is

(a) $\frac{42}{107}$ (b) $\frac{41}{141}$ (c) $\frac{42}{243}$ (d) None of these

Ans: (a)

RANDOM VARIABLE AND PROBABILITY DISTRIBUTION

A random variable is a real valued function whose domain is the sample space of a random experiment.

The probability distribution of a random variable X is the system of numbers

X :	x_1		x_2	...	x_n
P(X) :	p_1		p_2	...	p_n

where $p_i > 0,\ i = 1, 2, \ldots, n,\ \sum_{i=1}^{n} p_i = 1$.

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x : number of heads when 2 coins are tossed

x	0	1	2
$P(x)$ probability	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

HH

HT

TH

TT

Sum of $p(x)$: $\frac{1}{4} + \frac{1}{2} + \frac{1}{4} = \frac{1+2+1}{4} = 1$

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MEAN AND VARIANCE OF RANDOM VARIABLE

Let X be a random variable assuming values $x_1, x_2, \ldots, x_n$ with probabilities $p_1, p_2, \ldots, p_n$, respectively such that $p_i \geq 0$, $\sum_{i=1}^{n} p_i = 1$. Mean of X, denoted by μ [or expected value of X denoted by E (X)] is defined as

$$\mu = E(X) = \sum_{i=1}^{n} x_i p_i$$

and variance, denoted by σ^2, is defined as

$$\sigma^2 = \sum_{i=1}^{n} (x_i - \mu)^2 p_i = \sum_{i=1}^{n} x_i^2 p_i - \mu^2$$

or equivalently

$$\sigma^2 = E(X - \mu)^2$$

STANDARD DEVIATION OF RANDOM VARIABLE

Standard deviation of the random variable X is defined as

$$\sigma = \sqrt{\text{variance (X)}} = \sqrt{\sum_{i=1}^{n} (x_i - \mu)^2 \ p_{\text{i}}}$$

BINOMIAL DISTRIBUTION

A random variable X taking values 0, 1, 2, ..., n is said to have a binomial distribution with parameters n and p, if its probability distibution is given by

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$$\boxed{P(X=r) = {}^{n}c_{r}\, p^{r}\, q^{n-r}}$$

$p \longrightarrow$ probability of success

$q \longrightarrow$ probability of failure

$q = 1-p$

There should be a finite number of trials

The trials should be independent

Each trial has exactly two outcomes: success or failure

The probability of success (or failure) remains the same in each trial.

Bernoulli's trials

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QUESTION

A committee of 4 students is selected at random from a group consisting 8 boys and 4 girls. Given that there is at least one girl on the committee, calculate the probability that there are exactly 2 girls on the committee.

A : atleast one girl in committee

$$P(\text{no girls in committee}) = \frac{{}^{8}C_4}{{}^{12}C_4} = \frac{\cancel{8}^{\cancel{4}\,2} \times 7 \times \cancel{6} \times \cancel{5}}{\cancel{12} \times 11 \times \cancel{10}_{2} \times 9} = \frac{2 \times 7}{11 \times 9} = \frac{14}{99}$$

A′

$$P(A) = 1 - \frac{14}{99} = \frac{85}{99}$$

B : exactly two girls in committee,

$$P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{\left\{\dfrac{{}^4C_2 \times {}^8C_2}{{}^{12}C_4}\right.}{\left(\dfrac{85}{99}\right)} = \frac{\dfrac{4\times3}{2} \times \dfrac{8\times7}{2}}{\dfrac{12\times11\times10\times9}{4\times3\times2}} \times \frac{99}{85}$$

$P(B \cap A)$

exactly two girls

atleast one girl

$P(B)$

$$= \frac{6\times28}{\cancel{55}\times\cancel{9}} \times \frac{\cancel{99}}{85}$$

5, 11

$$= \frac{6\times28}{5\times85} = \frac{168}{425}$$ //

QUESTION

Three machines E1 , E2 , E3 in a certain factory produce 50%, 25% and 25%, respectively, of the total daily output of electric tubes. It is known that 4% of the tubes produced one each of machines E1 and E2 are defective, and that 5% of those produced on E3 are defective. If one tube is picked up at random from a day's production, calculate the probability that it is defective.

Total probability Theorem,

D : electric tube is defective ; E_1, E_2, E_3 : Events that electric tube is produced from E_1, E_2 and E_3.

$P(D) = ?$

$P(E_1) = 50\% = \frac{50}{100}$

$P(E_2) = \frac{25}{100}$ $\quad P(E_3) = \frac{25}{100}$

$P(D/E_1) = 4/100$ $\quad P(D/E_2) = 4/100$ $\quad P(D/E_3) = 5/100$

$$P(E_1) = 50\% = \frac{50}{100}$$

$$P(E_2) = \frac{25}{100} \qquad P(E_3) = \frac{25}{100}$$

$$P(D/E_1) = 4/100 \qquad P(D/E_2) = 4/100 \qquad P(D/E_3) = 5/100$$

$$P(D) = P(E_1)P(D/E_1) + P(E_2)P(D/E_2) + P(E_3)P(D/E_3)$$

$$= \frac{50}{100} \times \frac{4}{100} + \frac{25}{100} \times \frac{4}{100} + \frac{25}{100} \times \frac{5}{100}$$

$$= \frac{200 + 100 + 125}{10000} = \frac{425}{10000} = \frac{85}{2000} = \frac{17}{400}$$

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QUESTION

Find the probability that in 10 throws of a fair die a score which is a multiple of 3 will be obtained in at least 8 of the throws.

$P(\text{getting multiple of } 3) = \frac{2}{6} = \frac{1}{3}$

${}^{n}C_{r}\, p^{r} q^{n-r}$

$p = \frac{1}{3}$

$q = \frac{2}{3}$

$P(X \geq 8) = P(X=8) + P(X=9) + P(X=10)$

$= {}^{10}C_{8}\left(\frac{1}{3}\right)^{8}\left(\frac{2}{3}\right)^{2} + {}^{10}C_{9}\left(\frac{1}{3}\right)^{9}\left(\frac{2}{3}\right)^{1} + {}^{10}C_{10}\left(\frac{1}{3}\right)^{10}\left(\frac{2}{3}\right)^{0}$

$= {}^{10}C_{2}\,\frac{4}{3^{10}} + 10\left(\frac{2}{3^{10}}\right) + \frac{1}{3^{10}} = \frac{180+20+1}{3^{10}}$

$= \frac{201}{3^{10}}$

QUESTION

A discrete random variable X has the following probability distribution:

X	1	2	3	4	5	6	7
P (X)	C	2C	2C	3C	C^2	$2C^2$	$7C^2 + C$

Find the value of C. Also find the mean of the distribution.

$c + 2c + 2c + 3c + c^2 + 2c^2 + 7c^2 + c = 1$ (sum of probabilities $= 1$)

$9c + 10c^2 - 1 = 0$

$10c^2 + 9c - 1 = 0$

$10c^2 + 10c - c - 1 = 0$

$10c(c+1) - 1(c+1) = 0$

$(10c-1)(c+1) = 0$

$c = \frac{1}{10}$; $c = -1$

rejected, probabilities -1, -2 cannot be possible

$$c = \frac{1}{10}$$

x	1	2	3	4	5	6	7
$p(x)$:	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{1}{100}$	$\frac{2}{100}$	$\frac{7}{100} + \frac{1}{10}$

$$\text{Mean, } \mu \text{ or } E(x) = \sum x\,p(x) = 1 \times \frac{1}{10} + 2 \times \frac{2}{10} + 3 \times \frac{2}{10} + 4 \times \frac{3}{10} + 5 \times \frac{1}{100} +$$

$$6 \times \frac{2}{100} + 7\left(\frac{7}{100} + \frac{1}{10}\right)$$

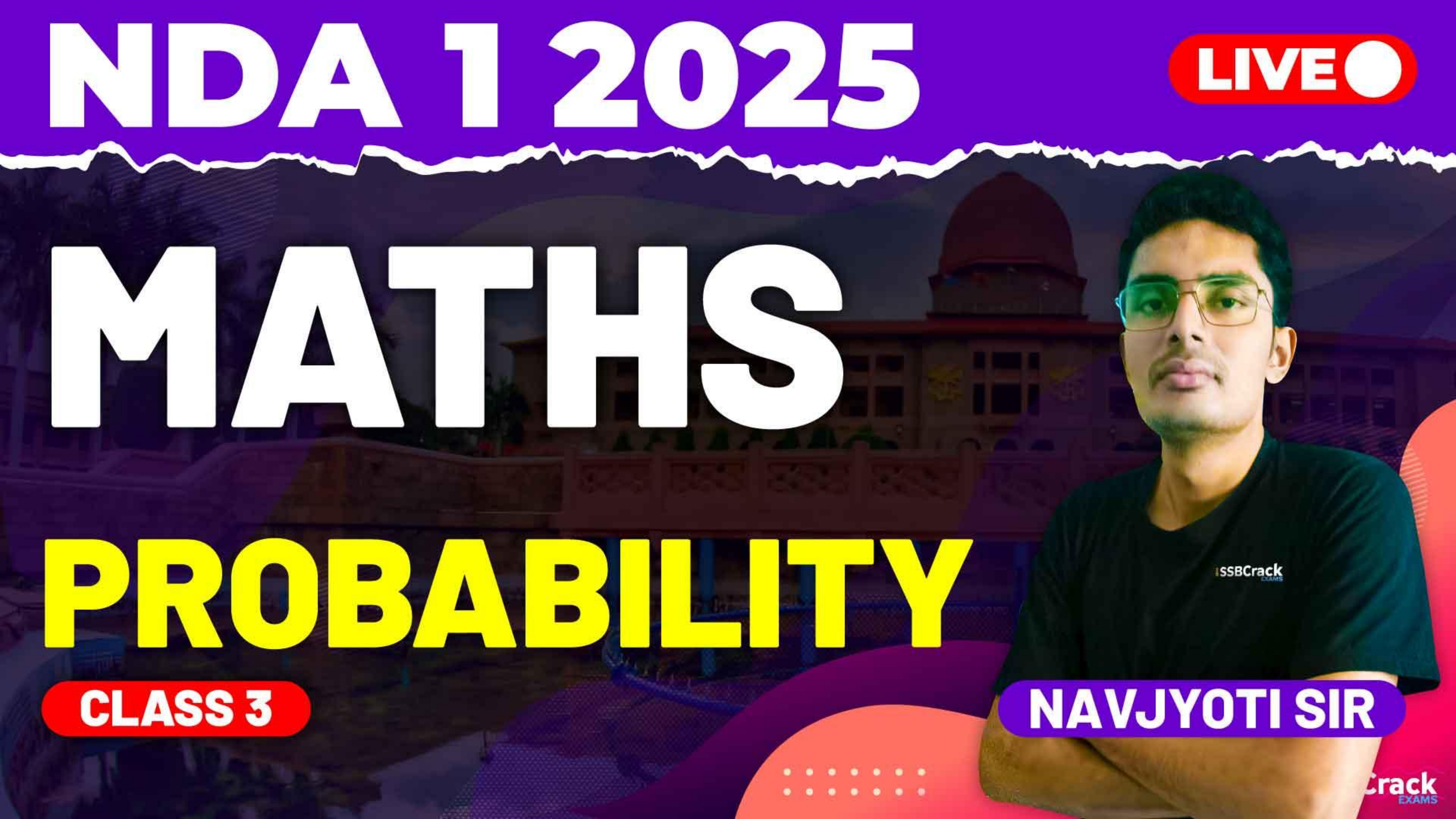
$$\sum x\,p(x) = 1\times\frac{1}{10} + 2\times\frac{2}{10} + 3\times\frac{2}{10} + 4\times\frac{3}{10} + 5\times\frac{1}{100},$$

$$6\times\frac{2}{100} + 7\left(\frac{7}{100}+\frac{1}{10}\right)$$

$$= \frac{1}{10} + \frac{4}{10} + \frac{6}{10} + \frac{12}{10} + \frac{5}{100} + \frac{12}{100} + \frac{49}{100} + \frac{7}{10}$$

$$= \frac{10+40+60+120+5+12+49+70}{100}$$

$$= \frac{366}{100} = 3.66$$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