NDA 1 2025

LIVE

MATHS

INTEGRATION - 1

MCQS

NAVJYOTI SIR

Crack
EXAMS

19 Feb 2025 Live Classes Schedule

9:00AM	19 FEBRUARY 2025 DAILY DEFENCE UPDATES	DIVYANSHU SIR
10:00AM	19 FEBRUARY 2025 DAILY CURRENT AFFAIRS	RUBY MA'AM

SSB INTERVIEW LIVE CLASSES

9:30AM	MOCK PERSONAL INTERVIEWS	ANURADHA MA'AM

AFCAT 1 2025 LIVE CLASSES

12:00PM	AFCAT 1 2025 MAHA MARATHON - PART 1	

NDA 1 2025 LIVE CLASSES

10:00AM	MATHS - INTEGRATION - CLASS 1	NAVJYOTI SIR
11:30AM	GK - CLIMATOLOGY - CLASS 2	RUBY MA'AM
1:00PM	BIOLOGY - CLASS 8	SHIVANGI MA'AM
4:30PM	ENGLISH - ORDERING OF WORDS - CLASS 2	ANURADHA MA'AM

CDS 1 2025 LIVE CLASSES

11:30AM	GK - CLIMATOLOGY - CLASS 2	RUBY MA'AM
1:00PM	BIOLOGY - CLASS 8	SHIVANGI MA'AM
4:30PM	ENGLISH - ORDERING OF WORDS - CLASS 2	ANURADHA MA'AM
5:30PM	MATHS - STATISTICS	NAVJYOTI SIR

SSBCrack EXAMS

What is $\int_{-1}^{1}(3\sin x-\sin 3x)\cos^2 x\,dx$ equal to ?

PYQ – 2024 - I

(a) $-\frac{1}{4}$

(b) 0

(c) $\frac{1}{2}$

(d) $\frac{1}{4}$

$$\int_{-a}^{a} f(x)\,dx = \begin{cases} 0, & \text{if } f(x) \text{ is odd.} \\ 2a, & \text{if } f(x) \text{ is even.} \end{cases}$$

$$f(x) = (3\sin x - \sin 3x)\cos^2 x$$

$$f(-x) = (3\sin(-x) - \sin 3(-x))(\cos(-x))^2$$

$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos x$$

$$= (-3\sin x + \sin 3x)(\cos^2 x)$$

$$= -(3\sin x - \sin 3x)\cos^2 x = -f(x)$$

$$f(-x) = -f(x)$$

$$\int_{-a}^{a} (3\sin x - \sin 3x)\cos^2 x \, dx = 0$$

PYQ – 2024 - I

What is $\int_{-1}^{1}(3\sin x-\sin 3x)\cos^2 x\,dx$ equal to ?

(a) $-\frac{1}{4}$

(b) 0

(c) $\frac{1}{2}$

(d) $\frac{1}{4}$

ANS : (b)

PYQ – 2024 - I

Let $p = \int_a^b f(x)dx$ and $q = \int_a^b |f(x)|dx$.

If $f(x) = e^{-x}$, then which one of the following is correct ?

(a) $p = 2q$

(b) $p = -q$

(c) $4p = q$

(d) $p = q$

$$f(x) = e^{-x} > 0$$

$$|f(x)| = \begin{cases} e^{-x}, & x > 0 \\ e^{-x}, & x < 0 \end{cases}$$

$$p = \int_a^b f(x)dx \qquad q = \int_a^b |f(x)dx = \int_a^b f(x)dx$$

$$\underline{p = q}$$

PYQ – 2024 - I

Let $p = \int_a^b f(x)dx$ and $q = \int_a^b |f(x)|dx$.

If $f(x) = e^{-x}$, then which one of the following is correct ?

(a) $p = 2q$

(b) $p = -q$

(c) $4p = q$

(d) $p = q$

ANS : (d)

Q) What is $\int \tan^{-1}(\sec x + \tan x)\,dx$ equal to?

(a) $\frac{\pi x}{4} + \frac{x^2}{4} + c$

(b) $\frac{\pi x}{2} + \frac{x^2}{4} + c$

(c) $\frac{\pi x}{4} + \frac{\pi x^2}{4} + c$

(d) $\frac{\pi x}{4} - \frac{x^2}{4} + c$

$$\sec x + \tan x = \frac{1 + \sin x}{\cos x} = \frac{\sin^2 x/2 + \cos^2 x/2 + 2\sin x/2 \cos x/2}{\cos^2 x/2 - \sin^2 x/2}$$

$$= \frac{(\sin x/2 + \cos x/2)^2}{(\sin x/2 + \cos x/2)(\cos x/2 - \sin x/2)} = \frac{\cos x/2 + \sin x/2}{\cos x/2 - \sin x/2}$$

$$\frac{\cos x/2 + \sin x/2}{\cos x/2 - \sin x/2}$$

$$= \frac{1 + \tan x/2}{1 - \tan x/2} = \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$$

$$\frac{1 + \tan A}{1 - \tan A} = \tan\left(\frac{\pi}{4} + A\right)$$

$$\frac{1 - \tan A}{1 + \tan A} = \tan\left(\frac{\pi}{4} - A\right)$$

$$\int\left(\frac{\pi}{4} + \frac{x}{2}\right)dx = \frac{\pi}{4}x + \frac{x^2}{4} + C$$

Q) What is $\int \tan^{-1}(\sec x + \tan x)\,dx$ equal to?

(a) $\frac{\pi x}{4} + \frac{x^2}{4} + c$

(b) $\frac{\pi x}{2} + \frac{x^2}{4} + c$

(c) $\frac{\pi x}{4} + \frac{\pi x^2}{4} + c$

(d) $\frac{\pi x}{4} - \frac{x^2}{4} + c$

Ans: (a)

Q) Let $f(x)$ be an indefinite integral of $\sin^2 x$. Consider the following statements :

Statement 1 : The function $f(x)$ satisfies $f(x + \pi) = f(x)$ for all real x.

Statement 2 : $\sin^2(x+\pi) = \sin^2 x$ for all real x.

Which one of the following is correct in respect of the above statements?

(a) Both the statements are true and Statement 2 is the correct explanation of Statement 1

(b) Both the statements are true but Statement 2 is not the correct explanation of Statement 1

(c) Statement 1 is true but Statement 2 is false

(d) Statement 1 is false but Statement 2 is true

(1)

$$f(x) = \int \sin^2 x \, dx$$

$$= \frac{1}{2}\int (1 - \cos 2x)\, dx$$

$$= \frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right] + c$$

$$= \frac{x}{2} - \frac{1}{4}\sin 2x + c$$

$$f(x) = \frac{x}{2} - \frac{1}{4}\sin 2x + C$$

$$f(x+\pi) = \frac{\pi+x}{2} - \frac{1}{4}\sin 2(\pi+x) + C$$

$$= \frac{x}{2} - \frac{1}{4}\sin 2x + \underbrace{\left(\frac{\pi}{2} + C\right)}$$ also a constant

$$= f(x)$$

(II) $\sin^2(x+\pi) = [\sin(x+\pi)]^2 = [-\sin x]^2 = (\sin x)^2 = \underline{\sin^2 x}$

Q) Let $f(x)$ be an indefinite integral of $\sin^2 x$. Consider the following statements :

Statement 1 : The function $f(x)$ satisfies $f(x + \pi) = f(x)$ for all real x.

Statement 2 : $\sin^2(x + \pi) = \sin^2 x$ for all real x.

Which one of the following is correct in respect of the above statements?

(a) Both the statements are true and Statement 2 is the correct explanation of Statement 1

(b) Both the statements are true but Statement 2 is not the correct explanation of Statement 1

(c) Statement 1 is true but Statement 2 is false

(d) Statement 1 is false but Statement 2 is true

Ans: (b)

Q) What is $\int \frac{xe^x dx}{(x+1)^2}$ equal to?

(a) $(x+1)^2 e^x + c$ (b) $(x+1)e^x + c$

(c) $\frac{e^x}{x+1} + c$ (d) $\frac{e^x}{(x+1)^2} + c$

where c is the constant integration.

$$\int e^x (f(x) + f'(x))\, dx = e^x f(x) + c$$

$$\frac{x}{(x+1)^2} = \frac{1}{x+1} - \frac{1}{(x+1)^2}$$

$$\int e^x \left(\frac{1}{x+1} + \left(\frac{-1}{(x+1)^2} \right) \right) dx = e^x \left(\frac{1}{x+1} \right) + c = \frac{e^x}{x+1} + c$$

Q) What is $\int \frac{xe^x dx}{(x+1)^2}$ equal to?

(a) $(x+1)^2 e^x + c$

(b) $(x+1)e^x + c$

(c) $\frac{e^x}{x+1} + c$

(d) $\frac{e^x}{(x+1)^2} + c$

where c is the constant integration.

Ans: (c)

Q) What is $\int e^{e^x} e^x dx$ equal to ?

(a) $e^{e^x} + c$

(b) $2e^{e^x} + c$

(c) $e^{e^x} e^x + c$

(d) $2e^{e^x} e^x + c$

$t = e^x$

$dt = e^x dx$

$$\int e^t \cdot dt = e^t + c = e^{e^x} + c$$

Q) What is $\int e^{e^x} e^x dx$ equal to ?

(a) $e^{e^x} + c$

(b) $2e^{e^x} + c$

(c) $e^{e^x} e^x + c$

(d) $2e^{e^x} e^x + c$

Ans: (a)

Q) What is $\int_0^{\frac{\pi}{2}} e^{\ln(\cos x)}\, dx$ equal to?

(a) -1

(b) 0

(c) 1

(d) 2

$$I = \int_0^{\pi/2} e^{\ln(\cos x)}\, dx$$

$e^{\ln(\cos x)} = \cos x$

$$I = \int_0^{\pi/2} \cos x\, dx$$

$$= \left[\sin x\right]_0^{\pi/2} = 1$$

Q) What is $\int_0^{\frac{\pi}{2}} e^{\ln(\cos x)}\, dx$ equal to?

(a) -1

(b) 0

(c) 1

(d) 2

Ans: (c)

Q) What is $\int_0^{\pi} \ln\left(\tan\frac{x}{2}\right) dx$ equal to?

(a) 0

(b) $\frac{1}{2}$

(c) 1

(d) 2

$$I = \int_0^{\pi} \ln\left(\tan\frac{x}{2}\right) dx \quad \text{——} \quad (1)$$

$$I = \int_0^{\pi} \ln \tan\left(\frac{\pi - x}{2}\right) dx = \int_0^{\pi} \ln \tan\left(\frac{\pi}{2} - \frac{x}{2}\right) dx$$

$$I = \int_0^{\pi} \ln \cot\frac{x}{2}\, dx \quad \text{——} \quad (2)$$

$$(1) + (2),$$

$$2I = \int_0^{\pi} \left[\ln\left(\tan\frac{x}{2}\right) + \ln\left(\cot\frac{x}{2}\right) \right] dx$$

$$2I = \int_0^{\pi} \ln(1)\, dx$$

$$\log A + \log B = \log(A \cdot B)$$

$$I = \frac{1}{2}(0) = \underline{0}$$

Q) What is $\int_0^{\pi} \ln\left(\tan\frac{x}{2}\right) dx$ equal to?

(a) 0

(b) $\frac{1}{2}$

(c) 1

(d) 2

Ans: (a)

What is $\int_0^{\frac{\pi}{2}} \frac{a+\sin x}{2a+\sin x+\cos x}dx$ equal to ?

PYQ – 2024 - I

(a) $\frac{\pi}{4}$

(b) $\frac{\pi}{2}$

(c) 1

(d) 0

$$I = \int_0^{\pi/2} \frac{a+\sin x}{2a+\sin x+\cos x}dx \quad \text{---} \quad (1)$$

$$I = \int_0^{\pi/2} \frac{a+\sin\left(\frac{\pi}{2}-x\right)}{2a+\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)} = \int_0^{\pi/2} \frac{a+\cos x}{2a+\cos x+\sin x} \quad -(2)$$

$$(1)+(2), \quad 2I = \int_0^{\pi/2} \frac{2a+\sin x+\cos x}{2a+\sin x+\cos x}dx \Rightarrow I = \frac{1}{2}\left(\frac{\pi}{2}\right) = \frac{\pi}{4}$$

PYQ – 2024 - I

What is $\int_0^{\frac{\pi}{2}} \frac{a+\sin x}{2a+\sin x+\cos x}dx$ equal to ?

(a) $\frac{\pi}{4}$

(b) $\frac{\pi}{2}$

(c) 1

(d) 0

ANS : (a)

Q) What is $\int_a^b [x]\,dx + \int_a^b [-x]\,dx$ equal to, where [,] is the greatest integer function?

(a) $b-a$ (b) $a-b$ (c) 0 (d) $2(b-a)$

$$\int_a^b \left([x] + [-x]\right) dx = \int_a^b (-1)\,dx$$

$$= [-x]_a^b = a - b$$

$$\frac{[2.3] + [-2.3]}{2} + (-3) = -1$$

Q) What is $\int_a^b [x]\,dx + \int_a^b [-x]\,dx$ equal to, where [,] is the greatest integer function?

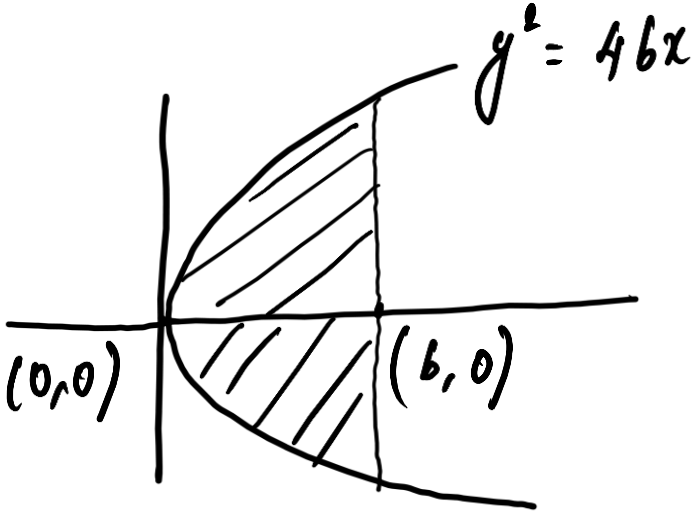
(a) $b-a$ (b) $a-b$ (c) 0 (d) $2(b-a)$

Ans: (b)

Q) What is the area of the parabola $y^2 = 4bx$ bounded by its latus rectum ?

(a) $2b^2/3$ square unit (b) $4b^2/3$ square unit

(c) b^2 square unit (d) $8b^2/3$ square unit



$$\text{Area} = 2\int_0^b \sqrt{4bx}\, dx$$

$$= 2\, 2\sqrt{b}\int_0^b \sqrt{x}\, dx = 4\sqrt{b}\left(\frac{2}{3}\right)\left[x^{3/2}\right]_0^b$$

$$= \frac{8}{3}\sqrt{b}\,(b)^{3/2} = \frac{8}{3}b^2$$

Q) What is the area of the parabola $y^2 = 4bx$ bounded by its latus rectum ?

(a) $2b^2/3$ square unit

(b) $4b^2/3$ square unit

(c) b^2 square unit

(d) $8b^2/3$ square unit

Ans: (d)

Q) The value of $\int_0^{\frac{\pi}{4}} \sqrt{\tan x}\, dx + \int_0^{\frac{\pi}{4}} \sqrt{\cot x}\, dx$ is equal to

(a) $\frac{\pi}{4}$

(b) $\frac{\pi}{2}$

(c) $\frac{\pi}{2\sqrt{2}}$

(d) $\frac{\pi}{\sqrt{2}}$

Q) The value of $\int_0^{\frac{\pi}{4}} \sqrt{\tan x}\, dx + \int_0^{\frac{\pi}{4}} \sqrt{\cot x}\, dx$ is equal to

(a) $\frac{\pi}{4}$

(b) $\frac{\pi}{2}$

(c) $\frac{\pi}{2\sqrt{2}}$

(d) $\frac{\pi}{\sqrt{2}}$

Ans: (d)

Q) What is $\int \frac{dx}{2x^2 - 2x + 1}$ equal to?

(a) $\frac{\tan^{-1}(2x-1)}{2} + c$

(b) $2\tan^{-1}(2x-1) + c$

(c) $\frac{\tan^{-1}(2x+1)}{2} + c$

(d) $\tan^{-1}(2x-1) + c$

$$\frac{1}{2}\int \frac{dx}{x^2 - x + \frac{1}{2}} = \frac{1}{2}\int \frac{dx}{\left(x-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$

$x^2 - x + \frac{1}{2}$

$$\left(x-\frac{1}{2}\right)^2 - \frac{1}{4} + \frac{1}{2} = \left(x-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$= \frac{1}{2} \times \frac{1}{\left(\frac{1}{2}\right)} \tan^{-1}\left(\frac{x-\frac{1}{2}}{\frac{1}{2}}\right) + C$$

$$= \tan^{-1}(2x-1) + C$$

Q) What is $\int \frac{dx}{2x^2 - 2x + 1}$ equal to?

(a) $\frac{\tan^{-1}(2x - 1)}{2} + c$

(b) $2 \tan^{-1}(2x - 1) + c$

(c) $\frac{\tan^{-1}(2x + 1)}{2} + c$

(d) $\tan^{-1}(2x - 1) + c$

Ans: (d)

Q) $\int_0^{\frac{\pi}{2}} |\sin x - \cos x| dx$ is equal to

(a) 0
(b) $2(\sqrt{2}-1)$
(c) $2\sqrt{2}$
(d) $2(\sqrt{2}+1)$

$$|\sin x - \cos x| = \begin{cases} +(\sin x - \cos x) & 0 \le x \le \pi/4 \\ -(\sin x - \cos x) & \frac{\pi}{4} \le x \le \frac{\pi}{2} \end{cases}$$

$$\int_0^{\pi/2} |\sin x - \cos x| dx = \int_0^{\pi/4} (\sin x - \cos x) dx + \int_{\pi/4}^{\pi/2} -(\sin x - \cos x) dx$$

Q) $\int_0^{\frac{\pi}{2}} |\sin x - \cos x| dx$ is equal to

(a) 0
(b) $2(\sqrt{2} - 1)$
(c) $2\sqrt{2}$
(d) $2(\sqrt{2} + 1)$

Ans: (b)

Q) What is $\int \ln(x^2)dx$ equal to?

(a) $2x\ln(x) - 2x + C$

(b) $\frac{2}{x} + C$

(c) $2x\ln(x) + C$

(d) $\frac{2\ln(x)}{x} - 2x + C$

Q) What is $\int \ln(x^2)dx$ equal to?

(a) $2x\ln(x) - 2x + C$

(b) $\frac{2}{x} + C$

(c) $2x\ln(x) + C$

(d) $\frac{2\ln(x)}{x} - 2x + C$

Ans: (a)

Q) If $I_1 = \int_e^{e^2} \frac{dx}{\log x}$ and $I_2 = \int_1^2 \frac{e^x}{x}\, dx$, then

(a) $I_1 = I_2$

(b) $2I_1 = I_2$

(c) $I_2 + I_1 = 0$

(d) $I_1 = 2I_2$

Q) If $I_1 = \int_e^{e^2} \frac{dx}{\log x}$ and $I_2 = \int_1^2 \frac{e^x}{x}\, dx$, then

(a) $I_1 = I_2$
(b) $2I_1 = I_2$
(c) $I_2 + I_1 = 0$
(d) $I_1 = 2I_2$

Ans: (a)

Q) What is $\int_0^1 x(1-x)^n\,dx$ equal to?

(a) $\dfrac{1}{n(n+1)}$

(b) $\dfrac{1}{(n+1)(n+2)}$

(c) 1

(d) 0

Q) What is $\int_0^1 x(1-x)^n\,dx$ equal to?

(a) $\dfrac{1}{n(n+1)}$

(b) $\dfrac{1}{(n+1)(n+2)}$

(c) 1

(d) 0

Ans: (b)

Q) If $\int_0^{\pi/2} \frac{\cot x}{\cot x + \operatorname{cosec} x} dx = m(\pi + n)$, then $m \cdot n$ is equal to

(a) $-\frac{1}{2}$ (b) 1 (c) $\frac{1}{2}$ (d) -1

Q) If $\int_0^{\pi/2} \frac{\cot x}{\cot x + \operatorname{cosec} x} dx = m(\pi + n)$, then $m \cdot n$ is equal to

(a) $-\frac{1}{2}$ (b) 1 (c) $\frac{1}{2}$ (d) -1

Ans: (d)

Q) What is $\int \frac{\left(x^{e-1}+e^{x-1}\right)dx}{x^{e}+e^{x}}$ equal to?

(a) $\frac{x^2}{2}+c$

(b) $\ln(x+e)+c$

(c) $\ln(x^e+e^x)+c$

(d) $\frac{1}{e}\ln\left(x^e+e^x\right)+c$

Q) What is $\int \frac{\left(x^{e-1}+e^{x-1}\right)dx}{x^{e}+e^{x}}$ equal to?

(a) $\frac{x^2}{2}+c$

(b) $\ln(x+e)+c$

(c) $\ln(x^e+e^x)+c$

(d) $\frac{1}{e}\ln\left(x^e+e^x\right)+c$

Ans: (d)

Q) $\int (\ln x)^{-1}\,dx - \int (\ln x)^{-2}\,dx$ is equal to

(a) $x(\ln x)^{-1} + c$

(b) $x(\ln x)^{-2} + c$

(c) $x(\ln x) + c$

(d) $x(\ln x)^{2} + c$

Q) $\int (\ln x)^{-1}\,dx - \int (\ln x)^{-2}\,dx$ is equal to

(a) $x(\ln x)^{-1} + c$

(b) $x(\ln x)^{-2} + c$

(c) $x(\ln x) + c$

(d) $x(\ln x)^{2} + c$

Ans: (a)

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